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Climate Action Game Experiment v1.00 Code Design and Parameters

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Climate Action Game Experiment v1.00 Code Design and Parameters

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Abstract: This is a Climate Action Game Experiment modular coding design and programmer's guide. A coding module flow pattern, input and output files, and module contents are described. That is preceded by guidelines for changing the programming. Also included are varied parameter inputs lists for Figures 1–6 and Tables 1–6 of Regional Welfare Impacts from Options for Limiting Global Average Temperature. A summary of the derivation and calibration of the model is also included. The present document aims at allowing a reader to reproduce any of the results in the manuscript independently, and/or to produce new versions of the model in their own preferred coding platform, either in cooperation with or independently from the authors of the version described here. The coding itself is not open source material but may be provided in the whole or in parts to interested parties pursuant to requests sent to the author.

1. Varied Parameters

For readers of Regional Welfare Impacts from Options for Limiting Global Average Temperature ("the manuscript") who may want to know exactly what was calculated for each figure and table, Table S0 contains values or ranges or varied parameter values, particularly when they are different from the sample values listed below in Table S1. The values of the Green Deal fraction parameters g_{1r} used in the manuscript are the same for all sixteen of the model's geographic regions for each case calculated, except that the Green Deal fraction for the JPK region is 0 for case in Table 3 of the manuscript with not fund transfers to that region. Values of parameters that are the same for all of the results in the manuscript are listed in the Master Input File Contents section of the present document.

Table S0. Parameter Values by Table and Figure

Table	1	2 (no SRM)	2 (SRM)	3	4	5	6
Figure		1 (no SRM)	1 (SRM)	4	5	6	
g_{1r}	1	1		0–0.8	0.22–0.23		1.0456
g_2	36	36–48		36	36		66
g_4	10	10		10	10		10
t_2	2025–2043	2031		2031	2031	2031	2031
t_s	2031	2025–2043		2031	2031	2031	2031
g_{s1}	0	0	-1			-1 – -3.4947	-1
g_{s2}			2052			2037	2037
g_{s4}			2084			2067	2067
g_{s6}			2025			2031	2031
F_{type}			6.2693, 15.545			6.2693, 15.545	6.2693
$f_{\text{temp,ref}}$	0	0	0	0, 0.05	0.05	0, 0.05	0, 0.05

Results in Tables 3 and 4 of the manuscript were obtained using cubic interpolations of results from ranges of parameters listed in Table S1 for g_{1r} and g_{s1}. For Table 3, the parameter range used for that purpose for g_{1r} was 0–0.5 in increments of 0.05. All cubic interpolations used the Hermite polynomials $\{1, 2x, -2+4x^2, -12x+8x^3\}$. Interpolation using up through third powers of x should give very similar results.

2. Guidelines for Programming

This document aims at precisely describing the design of a modular approach to computing results for the Climate Action Gaming Experiment (CAGE). The primary purpose of this document is to assist programmers who are interested in re-writing or updating the coding in the version described here. That version was written with Mathematica and is designated CAGE 1.0.0. Such work should start with updating the design document, making it clear how that update differs from the present document. Readers interested in what the model does but not how it is programmed may want to skip to the Section 4 on Modules below.

A re-write that incudes minor stylistic changes or corrections should be given version numbers starting with 1.0.1. A version that changes the overall coding approach (e.g. by writing for Python) but aims at the same results should be given a different version number, starting with 1.1.0. Versions that introduce substantial new capabilities (e.g. limits on emissions of minor greenhouse gases) should be given version numbers that start with 2 (e.g. 2.0.0 if in Mathematica or 2.1.0 if in Python.)

Different versions should use the same or very similar constant and variable names, particularly for global constants for symbols in the coding that are in the Master Input file described below. Particular attention should be paid to coding style for operations that "thread" through a list or list of lists (e.g. multiplying each element of a list by a constant). To assist in identifying which lists can be threaded by some operation upon them, a readily recognizable last letter in the name of a list (and the last two letters in the name of a list of lists, etc.) should be included. Care should be taken to verify that each operation on a list has threaded properly. When not, then each element of the list must be operated on separately and the resulting list then assembled. If it helps with clarity, that may be done even when the operation can in fact be more compactly threaded.

A systematic approach should be used for naming all modules and data files. On the date of each creation or modification of each file, two digits in the name thereof should indicate the year, three letters the month, and one or two letters the day of month. (To way distinguish modules or data files they create between Mathematica and Python coding, use lower case month abbreviations for Mathematica and at least partly upper case for Python.)

Each sheet of each data file contains two header rows and one footer row. The first header row contains a description of the contents of columns below it. The second header row usually contains the units of those columns, with Julian meaning Julian year. Numbers of Julian years are usually expected to be integers, and are sometimes rounded to the nearest integer to ensure that. The footer row contains the name of the module that produced it, preceded by as much information on the input files used by that module as fits in the same number of columns as in the header row.

3. Overview

The part of radiative forcing from other than $<N_2O>$, $<CH_4>$, $<CO_2>$, and solar radiation management (SRM) is prescribed and not meant to be changed by users without recoding the "Prescribed" module. (Here, angular brackets indicate atmospheric concentration in parts by volume per billion for $<N_2O>$ and $<CH_4>$ and per million for $<CO_2>$. τ is the change in global average temperature compared to that in equilibrium with zero radiative forcing as calculated using the formulas described herein.) The atmospheric concentrations $<CH_4>$ and $<N_2O>$ are also prescribed, but the concentrations of those gases are computed in a separate module in case future work modifies that module to explore implementation of policies for limiting their emissions. Population referred to here for each of sixteen geographic regions is total population less the population in 1820. Per capita gross domestic product in the background economy ("Background") for each region as used here is the difference between per historical per capita GDP and a constant value from fits to historical data, in the approximation of neglecting historical effects of climate change.

Each of the modules inputs a file with a name of the form new23r10masterTypeMonthDay.xlsx. Here, new is a placeholder for the model name. Type is a short descriptor for a set of examples (e.g. NoDeal), Month is a three-letter abbreviation, and Day is a number from 1 through 31. The notebook file names are of the form cage23r10moduleTypeDayMonthDay.nb where Day is a two digit number from 01 through 31. The different Month and Day order distinguishes between xlsx and nb files that would otherwise have the same name except for that suffix. (The ...prescribed...nb, ...n2och4,...nb and ...background ...nb modules produce xlsx files suitable for all example calculations, so the Type part of the related filenames is omitted.) With similar file names, with prescribed replaced by other module names and xlsx replaced by nb for Mathematica notebooks (or another descriptor for a difference coding platform), flow of information through the modules is as follows:

...prescribed...nb → ...prescribed...xlsx
...prescribed...xlsx → ...n2och4...nb → ...n2och4...xlsx
...prescribed...xlsx → ...co2...nb → ...co2...xlsx
{...prescribed...xlsx, ...n2och4...xlsx, ...co2...xlsx}→ ...forcing...nb → ...forcing..xlsx
...forcing..xlsx → ...seatau...nb → ...seatau...xlsx
...background.nb → ...background...xlsx
{...background...xlsx, ...co2...xlsx, ...seatau...xlsx} → ...impacts...nb → ...impacts...xlsx
{ ...background...xlsx, ...impacts...xlsx} → ...welfare...nb → ...welfare...xlsx

Climate change is operationally defined as changes in τ and $<CO_2>$. Impacts for each region are percentage changes in economic productivity due to climate change, less their economic productivity impacts in 1990. Changes to historical economic productivity are approximated as proportional to percentage changes in historical GDP. Computed welfare for each region ("Welfare") is population times discounted utility of per capita consumption, integrated from specified start to end years. A graphics post-processor module, not described in the present document, will be needed to output a set of graphics, which users may want to choose from or modify to meet their own needs. The eight modules described here are thus named as the upper case parts of: (1) Prescribed radiative forcing, (2) N2OCH4, (3) CO2, (4) total radiative Forcing, (5) Seatau for sea level and τ, (6) Background, (7) Impacts, and (8) Welfare.

Any information passed from a module to one or more succeeding modules is via xlsx files with lists of numbers for specified sets of years. Sheet 2 of the ...master...xlsx file contains parameters used in the Prescribed module for specifying those years to be chosen for computing the numbers in output files. That output of the Prescribed module has to be imported directly or indirectly into all of the other modules so those sets of years can be extracted.

For some application examples, execution of one of more of the earlier modules in the above list can be replaced by reading an input file from a previous run. In particular, examples with solar radiation management (SRM) compute the radiative shielding needed to meet a target rate of change of τ. That radiative shielding is then used in the Impacts module. Input files for previous results for total radiative forcing can then be used rather than recomputing total radiative forcing that the SRM modifies. Also, the Background module is designed to be executed only once for most sets of examples.

The eight modules are designed to allow for execution of a wide variety of examples without any recoding of those modules except for changing the names of changed input files. There is a single master input parameters file for all eight of these modules. The first sheet in the master input file contains all of the parameters needed to execute many interesting examples without changing the parameters in the other sheets in the master input file.

For simplicity of coding, those eight modules are all designed to execute only one example at a time. Some useful information can be obtained by inspection of the module output files. However, to produce analysis of results from multiple examples, a user may need to construct a driver for executing multiple examples and/or an analysis tool for combining and analyzing the results.

All CAGE module and data file names start with cage2n (where e.g. n=3 for year 2023) and end with the month and day of last modification, with a three-letter abbreviation for the month. Importing a list from full columns from module output files requires dropping the two header rows and one footer row. The present document contains a list of the contents of the Master Input file by sheet number.

Although not needed for reproducing the model, the last section of the present document describes the derivation and calibration of the model. That may be helpful for readers who want more insight into its connection with relevant literature, and also to anyone who wants to use all or parts the model in the likely event that they would want to modify the equations and/or the values of parameters that were not varied for the results in the manuscript. More detail is included in a research report (Singer, 2024) and eight other research reports referenced in that report. Anyone interested in revising or updating those calibrations would be advised to both download those reports and possibly also contact the Corresponding Author of the manuscript for suggestions on how to proceed.

4. Modules

This section contains program design information for each of the eight modules. The numbers in parentheses in the lists of quantities imported from the master input file are (sheet, row, column) numbers. The sheet number is also the number of the table at the end of this document that lists all of the parameter values in each sheet of the master input file. The column number is omitted for sheets S3 and S4, because only the second column in each of those sheets contains any parameter values. Though it is suggested that imports be collected near the beginning of each coded module, some are listed below with their values near equations for which they are used in order to make it clearer what the equations are meant to do.

The parameter values used for calculations in the manuscript in many cases have more significant figures (typically up to six digits) than listed in tables here. That is to avoid clutter here that would make it more difficult to remember the approximate values of the parameters. An Excel file with the exact values used to produce the results in the manuscript will be available from the Corresponding Author of that manuscript upon request.

4.1. **Prescribed.** Import from ...master...xlsx by (sheet, row, column) numbers:
t_2 (S1,3,14), t_1=Julian year 2019 (S3,3), $t_{\text{long}} = 300$ yr (S3,4), $\hat{c}_a = 0.3709$ (S3,5)
b_{0n} (S2,4–8,2), b_{1n} (S2,4–8,3), b_{2n} (S2,4–8,4), and b_{3n} (S2,4–8,5).

Set $t_{\text{annual},i} = t_1 + i, i = 0 \ldots t_2 - t_1 + t_{\text{long}} + 1$. Define the function

$$u(b_2, b_3) = 1/(1 + e^{-(t-b_2)/b_3}) \tag{4.1}$$

Radiative forcing including effects (other than on CO_2 emissions) of changes in land use F_4, contrails and cirrus clouds F_5, halogens F_6, ozone plus black carbon on snow F_7, and tropospheric aerosols F_8, are

$$F_4 = b_{40} + b_{41}u(b_{42}, b_{43}) \tag{4.2}$$
$$F_5 = \text{Max}[0, b_{50} + b_{51}u(b_{52}, b_{53})] \tag{4.3}$$
$$F_6 = \text{Max}[0, b_{60} + b_{61}u(b_{62}, b_{63})] \tag{4.4}$$
$$F_7 = b_{70} + b_{71}u(b_{72}, b_{73})(1 - u(b_{72}, b_{73})) \tag{4.5}$$
$$F_8 = (b_{80} + b_{81}u(b_{82}, b_{83})(1 - u(b_{82}, b_{83})))\hat{c}_a \tag{4.6}$$

Import from ...master...xlsx: b_{smn} (S2,1–3,9–11) for m=1–3, n=1–3

Contributions to solar radiative forcing are

$$F_{s1} = b_{s11}\cos(2\pi(t-b_{s12})/b_{s13}) \tag{4.7}$$
$$F_{s2} = b_{s21}\cos(2\pi(t-b_{s22})/b_{s23}) \tag{4.8}$$
$$F_{s3} = b_{s31}\cos(2\pi(t-b_{s32})/b_{s33}) \tag{4.9}$$

Total solar forcing is

$$F_{10} = F_{s1} + F_{s2} + F_{s3} - (F_{s1} + F_{s2} + F_{s3})|_{t=t_{\rm pre}} \tag{4.10}$$

Total prescribed radiative forcing is

$$F_{\rm prescribed} = F_{10} + \sum_{n=4}^{8} F_n \tag{4.11}$$

Output columns with $t_{\rm annual}$ and $F_{\rm prescribed}$.

4.2. **N2OCH4.** Import $t_{\rm annual}$ from ...prescribed...xlsx.
From ...master...xlsx, import $t_{\rm pre}$=Julian year 1750 (S3,6); $\{t_N = 116$ yr, $t_M = 9.1$ yr, t_H=2 yr$\}$ (S3,7–9); $b_{\rm Npre}$; $b_{\rm Mpre}$; b_{Nn} (2,13,1–3); and b_{Mn} (2,14,1–3).

Atmospheric concentrations G =<N_2O> and G =<CH_4> evolve according to the following equations:

$$\delta_G = b_{G3}/t_G \tag{4.12}$$
$$x_G = (t - b_{G2})/b_{G3} \tag{4.13}$$
$$x_{G\rm pre} = (t_{\rm pre} - b_{G2})/b_{G3} \tag{4.14}$$
$$I_G = {}_2F_1[1, 1+\delta_G, 2+\delta, -e^{x_G}]e^{(1+\delta_G)x_G}/(1+\delta_G)] \tag{4.15}$$
$$I_{G\rm pre} = {}_2F_1[1, 1+\delta_G, 2+\delta, -e^{x_{G\rm pre}}]e^{(1+\delta_G)x_{G\rm pre}}/(1+\delta_G)] \tag{4.16}$$
$$u_{G\rm pre} = 1/(1+e^{x_{G\rm pre}}) \tag{4.17}$$
$$H_G = e^{-\delta_G x_G}(I_G - I_{G\rm pre}) - (u_{G\rm pre}/\delta_G)(1 - e^{-(x_G - x_{G\rm pre})\delta_G}) \tag{4.18}$$
$$G = b_{G\rm pre} + b_{G1}b_{G3}H_G \tag{4.19}$$

where ${}_2F_1$ is a hypergeometric function. Let $M_{\rm lagged}(t)$ be the value G for <CH_4> with the argument t replaced a time t_H years earlier:

$$M_{\rm lagged} = <{\rm CH}_4>|_{t-t_H} \tag{4.20}$$

Output columns with $t_{\rm annual}$, and G =<N_2O>, G =<CH_4>, and $M_{\rm lagged}$ evaluated for the times in the list $t_{\rm annual}$.

4.3. **CO2.** Import $t_{\rm annual}$ from ...prescribed...xlsx. Set $g_{e1} = 0$.
From ...master...xlsx, import g_{en} (S1,3,2–5) for n=2–5; t_s (S1,3,13); g_{1r} (S1,4,2–17); and t_1 (S3,3).
As in ...prescribed...nb, define the function $u(b_2, b_3) = 1/(1+e^{-(t-b_2)/b_3})$.
Extrapolation of historical global anthropogenic carbon emissions in the form of CO_2 gives $e_{\rm old}$:

$$e_{\rm early} = b_{10} + b_{11}u(b_{12}, b_{13})(1 - u(b_{12}, b_{13})) \tag{4.21}$$
$$e_{\rm late} = b_{21}u(b_{22}, b_{23})(1 - u(b_{22}, b_{23})) \tag{4.22}$$
$$e_{\rm land} = e_{\rm early} + e_{\rm late} \tag{4.23}$$
$$e_{\rm ind} = b_{30} + b_{31}u(b_{32}, b_{33}) \tag{4.24}$$
$$e_{\rm old} = e_{\rm ind} + e_{\rm land} \tag{4.25}$$

From ...master...xlsx, import: $\{b_d = 0.6781$ TtonneC^{-1}, $\beta_f = -0.35$, $U_1 = 0.4386$ TtonneC, $f_c = 0.41\}$ (S3,12–15).

Prepare a correction factor, f_d for post-2019 depletion of global fluid fossil fuel resources, and multiply it by region-dependent Partial Green Deal factors turned on with a smoothed step function with inflection time t_s and smoothing width b_{s3}. For each region, multiply by a constant fraction of global carbon emissions with constant (and thus also long-term limit) input fractions of global emissions f_r for all but the CAN, JPK, ANZ, and USA regions. For those regions, compute time-dependent fractions. Calling all of the resulting fractions F_r, add up the global emissions to find e_c, including a correction included using the factor f_d to account for depletion of global fluid fossil fuel resources.

$$U = b_{31}(b_{33}\ln[e^{b_{32}/b_{33}} + e^{t/b_{33}}] - b_{32}) \tag{4.26}$$

$$f_d = 1 - u(b_{s2}, b_{s3}) + u(b_{s2}, b_{s3})\left(\frac{1 + b_d U}{1 + b_d U_1}\right)^{\beta_f} \tag{4.27}$$

For carbon emission limitations policies, compute

$$e_{23} = e^{g_2}/e^{g_3} \tag{4.28}$$
$$f_{23} = 1/e_{23} \tag{4.29}$$
$$e_{45} = e^{g_4}/e^{g_5} \tag{4.30}$$
$$f_{45} = 1/e_{45} \tag{4.31}$$
$$e_{y3} = e^{(t-t_1)/g_3} \tag{4.32}$$
$$e_{y5} = e^{(t-t_1)/g_5} \tag{4.33}$$
$$f_{p1} = (1 + f_{23})g_3\ln[1 + e_{23}] - (1 + f_{45})g_5\ln[1 + e_{45}] \tag{4.34}$$
$$f_p = (f_{45} - f_{23})(t - t_1) + (1 + f_{23})g_3\ln[e_{y3} + e_{23}] - (1 + f_{45})g_5\ln[e_{y5} + e_{45}] \tag{4.35}$$
$$f_{gr} = 1 - u_{23}(t_s, b_{s3}) + u_{23}(t_s, b_{s3})(1 - g_{1r} + g_{1r}f_p/f_{p1}) \tag{4.36}$$

Set $F_{er} = f_{er}$ for all regions r except USA, CAN, JPK, and ANZ. Then let

$$f_{\text{eD}} = f_{\text{eUSA}} + f_{\text{eCAN}} + f_{\text{eJPK}} + f_{\text{eANZ}} \tag{4.37}$$
$$F_{\text{eCAN}} = f_{eD}(b_{C0} + b_{C1}u(b_{C2}, b_{C3})) \tag{4.38}$$
$$F_{\text{eJPK}} = f_{eD}(b_{J0} + b_{J1}u(b_{J2}, b_{J3})) \tag{4.39}$$
$$F_{\text{eANZ}} = f_{eD}(b_{A0} + b_{A1}u(b_{A2}, b_{A3})) \tag{4.40}$$
$$F_{\text{eUSA}} = f_{eD} - F_{\text{eCAN}} - F_{\text{eJPK}} - F_{\text{eANZ}} \tag{4.41}$$

With emissions limitations, regional and global carbon emissions are then respectively

$$e_{cr} = F_{er}f_{gr}(f_c e_{\text{ind}} + (1 - f_c)f_d e_{\text{ind}} + e_{\text{land}}) \tag{4.42}$$
$$e_c = \sum_r e_{cr} \tag{4.43}$$

The sum is over all $n_r = 16$ regions.

From ...master...xlsx, import: $\{f_m = 0.5813$, $a_{\text{pre}} = 0.5920$ TtonneC, $a_3 = 0.5244$ TtonneC, ν_c=0.1285 yr^{-1}, $r_{sa} = 1.5331$ yr^{-1}, $a_{c2019} = 0.8709$ TtonneC, $s_{c2019} = 1.0759$ TtonneC, $c_1 = 0.02124$ TtonneC/ppm$\}$ (S3,16–23).

To compute $< CO_2 >$, set

$$f_e = 1 + (f_m - 1)e^{-(a_c - a_{\text{pre}})/a_3} \tag{4.44}$$

and, with $'=d/dt$,

$$a_c' = f_e e_c - s_c' \tag{4.45}$$

$$s_c' = \nu_c(r_{\mathrm{sa}} a_c - s_c) \tag{4.46}$$

starting at input values a_{c2019} and s_{c2019} at time $t_1 = 2019$ and integrating up to the last time in the list t_{annual}, and set

$$< \mathrm{CO}_2 >= a_c/c_1 \tag{4.47}$$

Output columns with t_{annual}, $< \mathrm{CO}_2 >$, and e_c.

4.4. **Forcing.** Import the lists t_{annual} and $F_{\mathrm{presribed}}$ from ...prescribed...xlsx. Import from ...n2och4...xlsx and rename as G_N the list $<\mathrm{N_2O}>$, as G_M the list $<\mathrm{CH_4}>$, and as M_{lagged} the list $<\mathrm{CH_4}>_{\mathrm{lagged}}$. Import from ...co2...xlsx the list $< \mathrm{CO}_2 >$, and rename as G_C.
Import from ...master...xlsx: $a_H = 0.000048$ (W/m^2)/ppb (S3,24), $\{C_0 = 277.15$ ppm, $N_0 = 273.87$ ppb, $M_0 = 731.41$ ppb$\}$ (S3,25–28); $\{F_{1,\mathrm{pre}} = 0.029,\ F_{2,\mathrm{pre}} = -0.013,\ F_{3,\mathrm{pre}} = 0.008\}$ W/m^2 (S3,28–30); $M_{\mathrm{measured,pre}} = 742.60$ ppb (S3,31); $ABCD$ (S2,26–28,2–5). Denoting $A_n = ABCD_{1n}, B_n = ABCD_{2n}, C_n = ABCD_{3n}, D_n = ABCD_{4n}$ for n=1–3, set

$$F_1 = (D_1 + A_1(G_C - C_0)^2 + B_1(G_C - C_0) + C_1\sqrt{G_N})\ln(G_C/C_0) - F_{1,\mathrm{pre}} \tag{4.48}$$

$$F_2 = (D_2 + A_2\sqrt{G_C} + B_2\sqrt{G_N} + C_2\sqrt{G_M})(\sqrt{G_N} - \sqrt{N_0}) - F_{2,\mathrm{pre}} \tag{4.49}$$

$$F_3 = (D_3 + A_3\sqrt{G_M} + B_3\sqrt{G_N})(\sqrt{G_M} - \sqrt{M_0}) - F_{3,\mathrm{pre}} \tag{4.50}$$

$$F_9 = a_H(M_{\mathrm{lagged}} - M_{\mathrm{measured,pre}}) \tag{4.51}$$

$$F_\Sigma = F_{\mathrm{prescribed}} + F_1 + F_2 + F_3 + F_9 \tag{4.52}$$

Output columns with t_{annual} and F_Σ, for consistency with the way radiative forcing was computed when calibrating parameters used in the global heat balance equation.

4.5. τ **and Sea Level.** Import the list t_{annual} from ...prescribed...xlsx.
Import g_s (S1,3,6–12); F_{type} (S1,3,15); $S_{\max}$ (S1,3,16);
$\{c_{th} = 28.49$ (W/m^2)/°C, $\lambda = 0.5175$ °C/(W/m^2), $\tau_1 = 1.3087$ °C, S_{ref}=26.695, $F_w = 0.0135$ MtonneS/yr(S3,32–36$\}$; and t_1 (S3, 3).
Define the function $u(b_2, b_3) = 1/(1 + e^{-(t-b_2)/b_3})$ as in ...prescribed...nb.
Let F_S be cubic interpolation of the input values of F_Σ. Integrating from τ_1 at time t_1 to the last element of the list t_{annual}, solve

$$\tau'_{\mathrm{noSRM}} = (F_S - \tau_{\mathrm{noSRM}}/\lambda)/c_{th} \tag{4.53}$$

Then, if $g_{s1} \neq 0$, find the radiative shielding ΔF needed to limit τ by computing

$$g_s = 1 - u(g_{s6}, g_{s7}) + u(g_{s6}, g_{s7})(1 + g_{s1}u(g_{s2}, g_{s3}) - (1 + g_{s1})u(g_{s4}, g_{s5})) \tag{4.54}$$

$$\tau'_{SRM} = g_s\tau'_{\mathrm{noSRM}} \tag{4.55}$$

$$\tau_{\mathrm{SRM}} = \int_{t_1}^{t} \tau'_{SRM} dt \tag{4.56}$$

$$\Delta F_{\mathrm{unlimited}} = F_S - (c_{th}\tau'_{\mathrm{SRM}} + \tau_{\mathrm{SRM}}/\lambda) \tag{4.57}$$

$$\Delta F_{\max} = F_{\mathrm{type}}(1 - e^{-S_{\max}/S_{\mathrm{ref}}}) \tag{4.58}$$

$$\Delta F = \Delta F_{\mathrm{unlimited}} - (\Delta F_{\mathrm{unlimited}} - \Delta F_{\max})/(1 + e^{-(\Delta F_{\mathrm{unlimited}} - \Delta F_{\max})/F_w}) \tag{4.59}$$

$$S_{\mathrm{SRM}} = -S_{\mathrm{ref}}\ln[1 - \Delta F/F_{\mathrm{type}}] \tag{4.60}$$

Then, starting again from τ_1 at time t_1, solve

$$\tau' = (F_S - \Delta F - \tau/\lambda)/c_{th} \tag{4.61}$$

From ...master...xlxs, import: $\{a_S = 0.003266$ (m/yr)/°C, $\tau_S = 0.1626$°C, $S_1 = 0.08015$°C, and $H_0 = 0.26$ m$\}$ (S3,36–39).

For sea level change S from 1990, and H from 1750, solve

$$S' = a_S(\tau - \tau_S) \tag{4.62}$$

starting from S_1 at time t_1 and set

$$H = H_0 + S \tag{4.63}$$

Output columns with t_{annual}, τ, τ_{noSRM}, τ', τ'_{noSRM}, S_{SRM}, ΔF, and H, setting $\Delta F = 0$ if $g_{s1} = 0$.

4.6. **Background.** Import from ...master...xlsx: t_2 (S1,3,14); t_{long} (S3,4); $\{n_{in} = 2$, $\omega = 0.675$, $\bar{t} = 7.76$ yr, $\theta = 1.345$, $\bar{\rho} = 0.023$ yr^{-1}, $t_0 =$ Julian year 1990$\}$ (S4, 3–8); B_{1r} (S5,3–18,3); B_{2r}(S5,3–18,4); B_{3r}(S5,3–18,5); b_{1r} (S5,3–18,7); b_{2r} (S5,3–18,8); and b_{3r} (S5,3–18,9).

Set

$$n_e = [\text{Min}[2, \text{Max}[n_{\text{in}}, \text{Log}_2[3] + 0.001]] \tag{4.64}$$
$$n_{\text{out}} = \text{Floor}[t_{\text{long}}^{1/n_e}] + 1 \tag{4.65}$$
$$t_{\text{out}} = \text{Join}[\text{Table}[\text{Ceiling}[t_2 + k^{n_e} - 1], \{k, 1, n_{\text{out}} - 1\}], \{t_2 + t_{\text{long}}\}] \tag{4.66}$$
$$s_{\text{out}} = (t_{\text{out}} - t_0)/\bar{t} \tag{4.67}$$

where Floor rounds non-integers down and Ceiling rounds up, both producing integers. Define the function $u(b_2, b_3) = 1/(1 + e^{-(t-b_2)/b_3})$ as in ...prescribed...nb. Then set

$$\alpha = 1 - \omega \tag{4.68}$$
$$\delta_r = (\bar{t}/b_{3r})\theta/\omega \tag{4.69}$$
$$\rho = \bar{t}\bar{\rho} \tag{4.70}$$
$$r_{\text{dep}} = 1 - \rho \tag{4.71}$$
$$\mu_r = \bar{t}/B_{3r} \tag{4.72}$$

For use in computing dimensionless welfare integrands, set

$$L_r = 1/(1+e^{-(t_0+\bar{t}s-B_{2r})/B_{3r}}) \tag{4.73}$$
$$M_r = 1-L_r \tag{4.74}$$
$$a_{sr} = 1/(1+e^{-(t_0+\bar{t}s-b_{2r})/b_{3r}}) \tag{4.75}$$
$$z_{sr} = 1-a_{sr} \tag{4.76}$$
$$K_{0r} = (a_{sr}/(1+\delta_r z_{sr}))^{1/\omega} L_r \tag{4.77}$$
$$\dot{K}_{0r} = dK_{0r}/ds \tag{4.78}$$
$$Y_{0r} = a_s K_0^{\alpha} L_r^{\omega} \tag{4.79}$$
$$\dot{Y}_{0r} = dY_{0r}/ds \tag{4.80}$$
$$F_{0r} = Y_{0r}/K_{0r} \tag{4.81}$$
$$\dot{F}_{0r} = dF_{0r}/ds \tag{4.82}$$
$$R_{0r} = (F_{0r}-1+\mu\theta M)/\theta \tag{4.83}$$
$$C_{0r} = Y_{0r}/\alpha + r_{\text{dep}}K_{0r} - \dot{K}_{0r} \tag{4.84}$$
$$c_{kr} = R_{0r}(F_{0r}-r_{\text{dep}}) - \dot{F}_{0r} - (\omega/\theta)F_{0r}C_{0r}/K_{0r} \tag{4.85}$$
$$c_{pr} = \dot{Y}_{0r}/\alpha - R_{0r}Y_{0r}/\alpha - F_{0r}C_{0r}/\theta \tag{4.86}$$
$$c_{Dr} = c_{pr}/c_{kr} \tag{4.87}$$
$$c_{Yr} = (Y_{0r}/\alpha)/c_{kr} \tag{4.88}$$
$$\dot{c}_{Dr} = dc_{Dr}/ds \tag{4.89}$$
$$\dot{c}_{Yr} = dc_{Yr}/ds \tag{4.90}$$

If a symbolic derivative calculator is not available, then finite difference expressions of the form $\dot{Q} = (Q|_{t=t_{\text{annual}}+1} - Q|_{t=t_{\text{annual}}-1})/2$ can be used to compute values of $\dot{c}_{Dr}$ and $\dot{c}_{Dr}$ in order to avoid coding long expressions for those derivatives that are worked out separately.

Output a file of six sheets, with each sheet having n_r+2 columns and $n_{\text{out}}+3$ rows that include two header rows, one footer row, and n_{out} data rows. The first row of the first column of each sheet contains the name of what the sheet contains: K0, C0, cD, cY, cDdot, cYdot, respectively, followed by "tout" and then the region name abbreviations. The second row of each sheet contains headers "sout," "Julian", and the region name abbreviations. The footer row contains the input file name, "Julian," all but the last of the region name abbreviations, and the module name.

4.7. **Impacts.** Set $g_{e1}=0$. From ...master...xlsx, import g_{en} (S1,1,2–5) for n=2 ... 5; t_s (S1,3,13); t_2 (S1, 3,14); F_{type} (S1,3,15); g_{1r} (S1,4,2–17); b_{s2} (2,20,4); b_{s3} (S2,20,5); t_{long} (S3,4); $f_{\text{pay},r}$ (S1,5,2–17); f_{Tr} (S1,6,2–17); t_1 (S3,3); $S_{\text{ref}} = 23.695$ MtonneS/yr (S3,31); τ_S (S3,34); H_0 (S3,36); t_0=Julian tear 1990 (S4,8); $\{\omega, \bar{t}, \theta\}$ (S4,4–6); and $\{\tau_0$, $< CO_2 >_{OC}$=280 ppm, $\alpha_{OC} = 0.00569$ pHunit/ ppm$^{1/\beta_{OC}}$, $\beta_{OC}=0.67$, $\gamma_{OC}=0.56$ pHunit$^{-1}\}$ (S4,9–13).
B_{0r} (S5,3–18,2) are included only to allow for a modification for output to plot total population. b_{0r} (S5,3–18,6) are additive constants to per capita GDP also included but not used.
Import B_{1r} (S5,3–18,3); B_{2r}(S5,3–18,4); B_{3r}(S5,3–18,5); b_{1r} (S5,3–18,7); b_{2r} (S5,3–18,8); b_{3r} (S5,3–18,9); σ_r (S6,3–18,2); ζ_r (S6,3–18,3). Set $\epsilon_n=0$ and $t_{1/2,n}=0$ for n=1–4.
Import ϵ_n (S7,3–15,2) and $t_{1/2,n}$ (S7,3–15,3) for n=5–17.
Import the climate impacts in percent of GDP in 2019, c_r^{XY}, from all but column 1 and all but the header and footer rows from Sheet 8.
From ...background...xlsx, import $t_{\text{out},j}$ by dropping the header and footer rows from column 2.
Set n_{out} equal to the length of t_{out}. From ...co2...xlsx, import t_{annual} from column 1 and annual $<CO_2>$ from column 2 by dropping header and footer rows. From ...seatau...xlsx, import annual

values of τ, τ', ΔF, and sea level H from columns 4–7.

4.7.1. *Climate Change Impacts.* Let j_{out} be the positions in t_{annual} of the elements of the list t_{out}, and denote by subscripts jk the values of parameters at times t_{jk} one year before, at, and one year after time $t_{\text{out},j}$. Define u as in...prescribed...nb and α, a_r, z_r, δ_r, and L_r as in ...background...nb. Set

$$y_r = b_{1r}(a_r/(1+\delta_r z_r)^{\alpha})^{1/\omega} \tag{4.91}$$

$$P_r = B_{1r} L_r \tag{4.92}$$

Set y_{r1} and P_{r1} equal to the values of y_r and P_r respectively at time t_1.

Set

$$A_{OC,jk} = \alpha_{OC}(<\mathrm{CO}_2>_{jk} - <\mathrm{CO}_2>_{OC})^{\beta_{OC}} \tag{4.93}$$

$$A_{OC0} = \alpha_{OC}(<\mathrm{CO}_2>_{0} - <\mathrm{CO}_2>_{OC})^{\beta_{OC}} \tag{4.94}$$

$$A_{OC1} = \alpha_{OC}(<\mathrm{CO}_2>_{1} - <\mathrm{CO}_2>_{OC})^{\beta_{OC}} \tag{4.95}$$

$$R_{jk} = \gamma_{OC} A_{OC,jk}/(1+\gamma_{OC} A_{OC,jk}) \tag{4.96}$$

$$R_0^{OC} = \gamma_{OC} A_{OC0}/(1+\gamma_{OC} A_{OC0}) \tag{4.97}$$

$$R_1 = \gamma_{OC} A_{OC1}/(1+\gamma_{OC} A_{OC1}) \tag{4.98}$$

and, for each region r,

$$\Sigma_{1r} = (1+\sigma_r)(H_1/1m)^{\sigma_r}(\tau_1 - \tau_S) \tag{4.99}$$

$$\Sigma_{r} = (1+\sigma_r)(H/1m)^{\sigma_r}(\tau - \tau_S) \tag{4.100}$$

$$\Sigma_{0r} = (1+\sigma_r)(H_0/1m)^{\sigma_r}(\tau_0 - \tau_S) \tag{4.101}$$

The $1m$ in the denominators here are a reminder that a dimensionless quantity is raised to the powers σ_r.

Import from ...master...xlsx: $\{\tau_0' = 0.01659°/\text{yr}, <\mathrm{CO}_2>_0 = 353.3\ \text{ppm}\}$ (S4,14–15). Then set

$$T_{jk} = \tau_{jk} - \tau_0 \tag{4.102}$$

For the seventeen impact types n, and sixteen regions r set

$$G_{rjk}^{XY} = T_{jk} \tag{4.103}$$

where superscript examples of XY are impact type identifiers, not exponents. Then, for ten of the impact types, for every r, overwrite G_{rjk}^{XY} with

$$G_{rjk}^{RT} = (\tau_{jk}')^2 - (\tau_0')^2 \tag{4.104}$$

$$G_{rjk}^{QT} = T_{jk}^2 \tag{4.105}$$

$$G_{rjk}^{AC} = \ln(<\mathrm{CO}_2>_{jk} / <\mathrm{CO}_2>_0) \tag{4.106}$$

$$G_{rjk}^{FC} = G_{rjk}^{AC} \tag{4.107}$$

$$G_{rjk}^{WT} = (P_{rjk}/P_{r1})^{\epsilon_{WT}} T_{jk} \tag{4.108}$$

$$G_{rjk}^{HT} = \mathrm{ArcTan}(T_{jk}) \tag{4.109}$$

$$G_{rjk}^{CT} = \tau_{jk}^2 - \tau_0^2 \tag{4.110}$$

$$G_{rjk}^{VC} = <\mathrm{CO}_2>_{jk} - <\mathrm{CO}_2>_0 \tag{4.111}$$

$$G_{rjk}^{OT} = \Sigma_{rjk} - \Sigma_{0r} \tag{4.112}$$

$$G_{rjk}^{OC} = R_{jk} - R_0^{OC} \tag{4.113}$$

For all r and XY, let G_{1r}^{XY} be the expressions for G_{rjk}^{XY} evaluated with quantities with subscripts jk replaced by values of those quantities at time t_1. For XY being RT, LT, QT, and AC, set

$$f_{rjk}^{XY} = c_r^{XY}(y_{rjk}/y_{1r})^{\zeta_r} G_{rjk}^{XY}/G_{1r}^{XY} \tag{4.114}$$

For all other XY, set

$$f_{rjk}^{XY} = c_r^{XY}(y_{rjk}/y_{1r})^{\epsilon^{XY}} 2^{-(t-t_1)/t_{1/2}^{XY}} G_{rjk}^{XY}/G_{1r}^{XY} \tag{4.115}$$

Set

$$D_{Crjk} = \omega \sum_{XY} f_{rjk}^{XY} \tag{4.116}$$

where the sum is over all seventeen of the impact types XY.

4.7.2. *Carbon Emissions Limitations Impacts, Transfers, SRM, and Total Impacts.* Import from ...master...xlsx: $\{\alpha_E = 3.76,\ \beta_E = 1.86,\ \epsilon = 0.01,\ c_{\text{SRM}} = 0.0046$ T\$2019ppp/TtonneS$\}$ (S4,16–19) and f_{Tr} (S1,6,2–17).

Define $e_{23}, f_{23}, e_{45}, f_{45}, e_{y3}, e_{y5}, f_{p1}, f_{gr}$ as in ...co2...nb, and evaluate f_{gr} at time t_{jk} to get $f_{g,rjk}$. Set

$$f_{\text{get},r} = \text{Min}[f_{Tr}, 0] \tag{4.117}$$

$$f_{\text{give},r} = \text{Max}[f_{Tr}, 0] \tag{4.118}$$

The $f_{\text{give},r}$ are meant to add to 1. If not rescale them to add to 1. With $\sum_r$ the sum over all regions, set

$$G_{DP,rjk} = y_{rjk}P_{rjk} \tag{4.119}$$

$$\Sigma_{\text{pay},jk} = \alpha_E \sum_r ((1 - f_{g,rjk})^{\beta_E} f_{\text{get},r} G_{DP,rjk}) \tag{4.120}$$

$$D_{Trjk} = \omega \Sigma_{\text{pay},jk} f_{\text{give},r}/G_{DP,rjk} \tag{4.121}$$

$$D_{Erjk} = -\omega\alpha_E(1 - f_{g,rjk})^{\beta_E}(1 + f_{\text{get},r}) \tag{4.122}$$

$$S_{\text{Sjk}} = -S_{\text{ref}}\text{Ln}[1 - \Delta F_{jk}/F_{\text{type}}] \tag{4.123}$$

$$D_{Srjk} = -\omega(c_{SRM}/\epsilon) f_{\text{pay},r} S_{\text{Sjk}}/G_{DP,rjk} \tag{4.124}$$

$$D_{rjk} = D_{Crjk} + D_{Erjk} + D_{Trjk} + D_{Srjk} \tag{4.125}$$

Extract D_{rj} from the middles of the triplets D_{rjk}, and evaluate the dimensionless derivatives

$$\dot{D}_{rj}(s) = \bar{t}(D_{rj3} - D_{rj1})/2 \tag{4.126}$$

$$\ddot{D}_{rj}(s) = \bar{t}^2(D_{rj3} - 2D_{rj2} + D_{rj1}) \tag{4.127}$$

where an overdot denotes d/ds.

Output a file with five sheets, each with values at dimensionless times ‘ and otherwise formatted as in ...background...xlsx except with ‘Lr”, “ar”, “Dofsr”, “Ddotofsr”, and “Ddotdotofsf” in the top left cell respecitvely in the five sheets.

4.8. **Welfare.** From ...master...xlsx, import t_2 (S1,3,14), $f_{\text{emp,ref}}$ (S1,3,17), t_{long} (S3,4), $\{\omega, \bar{t}, \theta, \bar{\rho}, t_0\}$ (S4,4–8); ϵ (S4,18); $\bar{y}_s = 0.601$ k\$2019ppp/person (S4,20); B_{1r} (S5,3–18,3); B_{2r}(S5,3–18,4); B_{3r}(S5,3–18,5); b_{1r} (S5,3–18,7); b_{2r} (S5,3–18,8); and b_{3r} (S5,3–18,9). From ...background...xslx, import $s_{\text{out},j}$ as in ...impacts...nb but using column 1. Set n_{out} equal to the length of s_{out}. Import region abbreviations as in ...impacts...nb and set n_r equal to the length of that list. From sheets 1–6, drop the header and footer rows and first two columns, and extract respectively $\{K_{0rj}, C_{0rj}, c_{Drj}, c_{Yrj}, \dot{c}_{Drj}, \dot{c}_{Yrj}\}$. From ...impacts...xlsx sheets 1–5 respectively, drop the header and footer rows and first two columns and extract respectively $\{L_{rj}, a_{rj}, D_{rj}, \dot{D}_{rj}, \ddot{D}_{rj}\}$. Define δ_r as in ...background...nb.

Define α, ρ, and r_{dep} as in ...background...nb.

Set

$$s_2 = (t_2 - t_0)/\bar{t} \tag{4.128}$$
$$\bar{K}_r = b_{1r}\bar{t}/\alpha \tag{4.129}$$
$$\beta_r = ((\bar{K}_r/\bar{t})/(B_{1r}\bar{y}_s))^{1-\theta} \tag{4.130}$$
$$c_{Wr} = B_{1r}\bar{t}\beta_r e^{\rho s_2} \tag{4.131}$$

4.8.1. *Without Empathy.* Let s_m be the last element of s_{out}.

$$M_{rj} = 1 - L_{rj} \tag{4.132}$$
$$z_{rj} = 1 - a_{rj} \tag{4.133}$$
$$K_{1rj} = c_{Drj}D_{rj} + c_{Yrj}\dot{D}_{rj} \tag{4.134}$$
$$\dot{K}_{1rj} = \dot{c}_{Drj}D_{rj} + (c_{Drj} + \dot{c}_{Yrj})\dot{D}_{rj} + c_{Yrj}\ddot{D}_{rj} \tag{4.135}$$
$$C_{1rj} = a_{rj}K_{0rj}^{\alpha}L_{rj}^{\omega}(D_{rj}/\alpha + K_{1rj}/K_{0rj}) - r_{\text{dep}}K_{1rj} - \dot{K}_{1rj} \tag{4.136}$$
$$I_{1rj} = e^{-\rho s_{\text{out},j}}L_{rj}^{\theta}C_{0rj}^{-\theta}C_{1rj} \tag{4.137}$$
$$I_{1r} = \text{Interpolation}[s_{\text{out},j}, I_{1rj}] \tag{4.138}$$
$$\Delta\bar{W}_r = 1000\,\epsilon\, c_{Wr}\int_{s_2}^{s_m} I_{1r}\,ds \tag{4.139}$$

where "Interpolation" denotes cubic interpolation of pairs of dimensionless times and integrand values. The factor of 1000 converts from Gperson-yr to Mperson-yr.

4.8.2. *With Empathy.* Set

$$y_{rj} = b_{1r}(a_{rj}/(1+\delta_r z_{rj})^{\alpha})^{1/\omega} \tag{4.140}$$
$$P_{rj} = B_{1r}L_{rj} \tag{4.141}$$

Unless $f_{\text{emp,ref}} = 0$ define an empathy integrand matrix

$$I_{1qrj} = f_{\text{emp,ref}}(P_{rj}/P_{\text{USA},j})(y_{rj}/y_{\text{USA},j})^{\theta}I_{1qj} \tag{4.142}$$
$$I_{1qr} = \text{Interpolation}[s_{\text{out},j}, I_{1qrj}] \tag{4.143}$$
$$E_{qr} = 1000\,\epsilon\, c_{Wr}\int_{s_2}^{s_m} I_{1qr}\,ds \tag{4.144}$$
$$E_r = \Delta\bar{W}_r - E_{rr} + \sum_r E_{qr} \tag{4.145}$$

where the sum in the equation for E_r is over all sixteen regions.

Output a file with nineteen rows with the first row containing descriptors for ΔW_r and ΔE_r and the values thereof for the sixteen regions in rows 3–18. Put the three letter abbreviations for the regions in the first column.

4.8.3. *Background Economy without Empathy.* For viewing within the coding only but not programmed for output, welfare for the background economy is computed as follows

$$I_{\max,rj} = e^{-\rho s_{\text{out},j}}L_{rj}/(\theta - 1) \tag{4.146}$$
$$I_{0rj} = e^{-\rho s_{\text{out},j}}L_{rj}^{\theta}C_{0rj}^{\theta}/(\theta - 1) \tag{4.147}$$
$$\bar{W}_{0r} = 1000\,B_1\,\bar{t}\,e^{\rho s_2}\int_{s_2}^{s_m}(I_{\max,rj} - \beta_r\text{Interpolation}[s_{\text{out},j}, I_{0rj}])\,ds \tag{4.148}$$

5. Master Input File Contents

The following description of the contents of the Master Input file provide an overview of what it contains sheet by sheet. For some of these lists, but not all, a module where they are used is indicated. The subsequent tables contain parameter values and numbers of some of equations where parameters are used.

5.1. **Variable Inputs.** Sheet 1 contains all of the inputs that normally vary between multiple executions of one or more modules for a series of calculations. Row 3 contains global constants, and rows 4–6 contain region-dependent constants. Global constants include $\{g_{e2} \dots g_{e5}, g_{s1} \dots g_{s7}, t_s, t_2\}$ and $\{F_{\text{type}}, S_{\text{max}}, F_{\text{type}}, f_{\text{emp,ref}}\}$. Region-dependent constants include $\{g_{1r}, f_{\text{pay,r}}, f_{Tr}\}$.

5.2. **Logistic, Cosine, and Greenhouse Gas Forcing Formula Constants.** Sheet 2 contains $\{b_{n0}, b_{n1}, b_{n2}, b_{n3}\}$ for n=4–8; $\{b_{sn1}, b_{sn2}, b_{sn3}; b_{G1}, b_{G2}, b_{G3}\}$ with $G = N$ and $G = M$. It also contains $\{b_{cn0}, b_{cn1}, b_{cn2}, b_{cn3}\}$ for carbon emissions; b_{s2} and b_{s3}; $\{b_{Cn}, b_{Jn}, b_{An}\}$ for time-dependent regional fractions of global carbon emissions; and forcing formula parameters $\{A_n, B_n, C_n, D_n\}$. (There are entries for b_{sn0} and b_{s2} in Sheet 2, but these are not needed and not used.)

5.3. **Times and Physical Model Global Constants.** Sheet 3 contains t_1, $\{t_{\text{long}}, \hat{c}_a, t_{\text{pre}}, t_N, t_M\}$; $\{b_{\text{Npre}}, b_{\text{Mpre}}, t_H\}$; and $\{b_d, \beta_f, U_1, f_c, f_m, a_{\text{pre}}, a_3, \nu_c, r_{sa}\}$; $\{t_N, t_M, b_{\text{Npre}}, b_{\text{Mpre}}, t_H\}$;
and $\{a_{\text{c2019}}, s_{\text{c2019}}, c_1\}$ for ...co2...nb. It also contains $\{a_H, n_{in}, C_0, N_0, M_0, F_{1,\text{pre}}, F_{2,\text{pre}}, F_{3,\text{pre}}\}$ and $M_{\text{measured,pre}}$ for ...forcing...nb; $\{c_{th}, \lambda, S_{\text{ref}}, F_w, a_S, \tau_S, S_1, H_0\}$ for seatau...nb;
and H_0 for ...impacts...nb.

5.4. **Economics Constants not Part of Lists.** Sheet 4 contains, with all values in Column 2, $\{\omega, \bar{t}, \theta, \bar{\rho}\}$, for ...background...nb; t_0; $\{\tau_0, CO2OC, \alpha_{OC}, \beta_{OC}, \gamma_{OC}, \tau_0', CO20, \alpha_E, \beta_E, \epsilon, c_{\text{SRM}}\}$ for ...impacts...nb; and $\bar{y}_s$ for ...welfare...nb.

5.5. **Population, Productivity, and per capita GDP.** Sheet 5 contains B_{nr}, b_{nr} with n=0–3.

5.6. **Other Region-dependent Constants.** Sheet 6 contains f_r for ...co2...nb and σ_r and ζ_r for ...impacts...nb.

5.7. **Constants by Non-agricultural Impact Type.** Sheet 7 contains ϵ_{XY}, $t_{1/2,XY}$
for ...impacts...nb.

5.8. **Impacts in 2019.** Sheet 8 contains c_r^{XY} for ...impacts...nb.

Table S1. Sheet 1, Default Values Deal for Variable Parameters

Symbol	Value	Units	Eq. #	Type
g_2	36	yr	4.28	Final emissions multiplier at $t_1 + g_2$ as $g_3 \to 0$
g_3	8	yr	4.28	Smoothing width around time $t_1 + g_2$
g_4	10	yr	4.30	Emissions limitation starts at $t_1 + g_4$ as $g_5 \to 0$
g_5	4	yr	4.30	Smoothing width around time $t_1 + g_4$
g_{s1}	0	1	4.54	-1 for $\tau \to$ constant; $<$ -1 for decrease
g_{s2}	2037	Julian	4.54	SRM start time in $g_{s3} \to 0$ limit
g_{s3}	6	yr	4.54	Smoothing width around time g_{s2}
g_{s4}	2067	Julian	4.54	τ stabilization time in $g_{s5} \to 0$ limit if $g_{s1} = -1$
g_{s5}	6	yr	4.54	Smoothing width around time g_{s4}
g_{s6}	2031	Julian	4.54	No SRM before g_{s6} in $g_{s7} \to 0$ limit
g_{s7}	2	yr	4.54	Smoothing width around time g_{s6}
t_s	2031	1/yr	4.36	No emissions limit before t_s in $b_{s3} \to 0$ limit
t_2	2031	Julian	4.128	Welfare integral lower time limit
$F_{\rm type}$	6.2693	W/m^2	4.123	$F_{\rm type}(1 - e^{S_S/S_{\rm max}})$ W/m^2 with S_S MtonneS/yr
$S_{\rm max}$	100	MtonneS/yr	4.123	$F_{\rm type}(1 - e^{S_S/S_{\rm max}})$ W/m^2 with S_S MtonneS/yr
g_{1r}	1	1	4.36	Full Green Deal emissions limit fractions by region
$f_{\rm pay,r}$	0	1	4.124	Fractions of direct cost SRM for region r
f_{Tr}	0	1	4.117	$f_{Tr} > 0$=pay fraction; < 0, mitigation $\times(1 + f_{Tr})$

Table S2: Sheet 2, Logistic and other Function Parameters

Subscript:	0	1	2	3	Eq.	**Modules**
Units	W/m^2	W/m^2	Julian	Years		**Prescribed**
b_4	0.0021	-0.2128	1916.49	35.96	4.2	Land use (albedo)
b_5	-0.0016	0.2342	2040.36	20.15	4.3	Contrails cirrus
b_6	-0.0019	0.4051	1979.83	7.41	4.4	Halogens
b_7	-0.2057	2.3835	2041.44	42.99	4.5	O_3 +BC on snow
b_8	0.0026	-5.4233	1994.81	32.11	4.6	Tropospheric aerosols
Units	W/m^2	W/m^2	Julian	Years		*Solar cosines + constant*
b_{s1}	0	-0.046095	1650	842	4.7	Grand minimum cycle
b_{s2}	0	0.03227	1772.23	87.53	4.8	Gleissberg cycle
b_{s3}	0.0232	-0.02044	1927.00	269.95	4.9	Amplitude modulation
Units	ppb/yr	ppb/yr	Julian	Years		**N_2O CH_4**
b_N	0	4.95	2059.82	50.76	4.10	N_2O emissions
b_M	0	135.21	1954.50	27.03	4.10	CH_4 emissions
Units	TtonneC/yr	-TtonneC/yr	Julian	Years		*Carbon emissions*
b_1	-0.000075	0.005940	1950.98	46.20	4.21	Land use early
b_2	0	0.002967	2021.63	8.91	4.22	Land use late
b_3	-0.000002	0.015285	2002.57	27.82	4.23	Industrial
Units	1	1	Julian	Years		**CO2**
C	0.03415	0.05461	2031.82	17.72	4.38	CAN carbon emissions
J	0.03864	0.18873	2000.15	28.43	4.39	JPK carbon emissions
A	0.00926	0.03999	1998.42	19.61	4.40	ANZ carbon emissions
b_s	0	1	2020	2	4.36	Smoothed step functions
Units	Various	Various	Various	Various		**Forcing**
1	$-2.4785\times10^{-}7$	0.00075906	-0.0021492	5.2488	4.48	<CO2>
2	-00034197	0.00025455	-0.00024357	0.12173	4.49	<N_2O>
3	-0.000089603	-0.00012642	0	0.045194	4.50	<CH_4>

Table S3. Sheet 3, Model Scalar Parameters

Symbol	Value	Units	Eq. #	Type
t_1	2019	Julian	4.45	Calibration database last year
t_{long}	300	yr	4.66	Welfare integral timespan
$\hat{c}_a$	0.3709	1	4.6	Tropospheric aerosol forcing multiplier
t_{pre}	1750	Juilan	4.10	Preindustrial base year
t_N	116	yr	4.12	Inverse of <N_2O> clearance rate
t_M	9.1	yr	4.12	Inverse of <CH_4> clearance rate
t_H	2	yr	4.20	Delay for stratospheric H_2O vapor forcing
b_d	0.6781	1/TtonneC	4.27	Fossil fuel depletion coefficient
β_f	-0.35	1	4.27	Fluid fossil fuel price elasticity
U_1	0.4386	TtonneC	4.27	Cumulative industrial carbon emitted by 2019
f_c	0.41	1	4.45	Coal fraction of industrial carbon emissions
f_m	0.5813	1	4.44	Maximum carbon sequestration escape fraction
a_{pre}	0.5920	TtonneC	4.44	Atmospheric carbon in 1750
a_3	0.5244	TtonneC	4.44	Carbon sequestration formula parameter
ν_c	0.1285	1/yr	4.46	Atmosphere/ocean transfer rate coefficient
r_{sa}	1.5331	1/yr	4.46	Atmospheric carbon coefficient
a_{c2019}	0.8709	TtonneC	4.45	Atmospheric carbon in 2019
s_{c2019}	1.0759	TtonneC	4.46	Upper ocean exchangeable carbon in 2019
c_1	0.002124	TtonneC/ppm	4.47	Atmospheric carbon to concentration ratio
a_H	0.000048	(W/m^2)/ppb	4.52	Stratospheric H_2O (W/m^2) per ppb <CH_4>
C_0	277.15	ppm	4.48	Radiative forcing formulas parameter
N_0	273.87	ppb	4.49	Radiative forcing formulas parameter
M_0	731.41	ppb	4.50	Radiative forcing formulas parameter
$F_{1,\text{pre}}$	0.029	W/m^2	4.48	Subtract to zero F_1 in 1750
$F_{2,\text{pre}}$	-0.013	W/m^2	4.49	Subtract to zero F_2 in 1750
$F_{3,\text{pre}}$	0.088	W/m^2	4.50	Subtract to zero F_3 in 1750
$M_{\text{measured,pre}}$	742.60	ppb	4.52	Preindustrial <CH_4> for stratospheric H_2O
c_{th}	28.49	$(W/m^2)/°C$	4.53	Thermal inertia coefficient
λ	0.5175	$°C/(W/m^2)$	4.58	Equilibrium climate sensitivity
τ_1	1.3087	°C	4.53	τ in 1991
S_{ref}	26.695	MtonneS/yr	4.58	Sulfur injection rate coefficient
F_w	0.0135	W/m^2	4.59	Width parameter for $S_{\max}$ limit
a_S	0.003266	(m/yr)/°C	4.62	Sea level coefficient
τ_S	0.1626	°C	4.62	τ for sea level equilibrium
S_1	0.08015	m	4.62	Initial condition for S at time t_1
H_0	0.26	m	4.63	Sea level rise from 1750 to 1990

Table S4. Sheet 4, Economic Model Scalar Parameters

Symbol	Value	Units	Eq. #	Type
n_{in}	2	1	4.64	Exponent for list of computation times
ω	0.675	1	4.68	Labor fraction of production
$\bar{t}$	7.76	yr	4.69	Capitalization time
θ	1.345	1	4.83	Utility exponent is $1-\theta$
$\bar{\rho}$	0.023	1/yr	4.70	Social discount rate
t_0	1990	Julian year	4.128	Base year for dimensionless time
CO2OC	280	ppm	4.93	Year 1750 <CO_2> for reef damage
α_{OC}	0.00569	pHunit/ppm$^{1/\beta_{OC}}$	4.93	Upper ocean acidity coefficient
β_{OC}	0.67	1	4.93	Upper ocean acidity exponent
γ_{OC}	0.56	1/pHunit	4.96	Coral reef damage coefficient
τ_0'	0.1659	°C	4.104	$d\tau/dt$ in 1990
CO20	353.3	TtonneC	4.106	<CO_2> in 1990
α_E	3.76	%	4.120	Decarbonization GDP impact coefficient
β_E	1.86	1	4.120	Decarbonization GDP impact exponent
ϵ	0.01	1	4.139	Climate impact expansion parameter
c_{SRM}	0.0046	T\$2019ppp/TtonneS	4.124	Direct cost of SRM per TtonneS
$\bar{y}_s$	0.601	k\$2019ppp/person	4.130	Per capita consumption defining 0 welfare

Table S5. Sheet 5, Population and Per Capita GDP Parameters

Region	$\bar{B}_{r0}$	$\bar{B}_{r1}$	$\bar{B}_{r2}$	$\bar{B}_{r3}$	b_{r0}	b_{r1}	b_{r2}	b_{r3}
USA	0.010	0.448	1980.33	43.47	2.91	139.15	1999.93	43.97
CAN	0.001	0.057	1994.06	40.31	2.55	79.99	1978.89	35.13
WEU	0.135	0.318	1935.90	41.51	4.30	59.74	1972.71	24.75
JPK	0.041	0.133	1948.88	19.28	1.66	44.93	1973.41	15.40
ANZ	0.0004	0.054	2011.22	42.73	7.24	83.22	1992.81	31.28
CEE	0.037	0.086	1913.99	30.69	2.10	59.74	2007.24	40.28
FSU	0.054	0.263	1939.88	35.72	2.38	17.92	1953.36	15.69
MDE	0.026	0.534	2009.02	23.77	1.85	29.57	1975.81	29.99
CAM	0.008	0.224	1993.45	24.41	1.86	20.49	1961.68	30.24
SAM	0.010	0.545	1988.03	27.25	1.34	24.27	1975.47	40.07
SAS	0.225	2.537	2004.11	25.83	1.46	14.26	2014.99	12.06
SEA	0.041	0.946	1995.90	30.28	1.19	64.06	2031.38	25.42
CHI	0.388	1.081	1974.40	16.22	0.61	74.20	2023.41	14.00
NAF	0.011	0.380	2018.84	30.47	1.46	42.62	2036.64	48.12
SAS	0.065	4.643	2054.55	29.14	0.98	1.90	1941.26	30.88
SIS	0.005	0.061	1982.91	30.60	1.41	20.35	1981.77	37.88
Eq. #		4.92	4.73	4.72		4.91	4.75	4.69

Table S6. Sheet 6, σ_r, ζ_r, and f_r

Region	f_r	σ_2	ζ_r
USA	0.0888	-0.417	-0.769
CAN	0.0255	-0.739	-0.669
WEU	0.0738	-0.727	-1.002
JPK	0.0199	-0.588	-0.942
ANZ	0.0654	-0.452	-0.894
CEE	0.0142	-0.807	-1.045
FSU	0.0790	-0.445	-0.573
MDE	0.0732	-0.372	-0.919
CAM	0.0140	-0.322	-0.577
SAM	0.0315	-0.244	-0.218
SAS	0.0821	-0.070	-0.715
SEA	0.0541	-0.188	-0.558
CHI	0.3348	-0.292	-0.530
NAF	0.0174	-0.663	-0.568
SAS	0.0222	-0.201	-0.711
SIS	0.0040	-0.333	-1.446
Eq. #	4.37	4.104	4.114

Table S7. Sheet 7, ϵ_{XY}, and $1/\nu_{XY}$

XY	ϵ_{XY}	$1/t_{1/2}^{XY}$ (yr)
FT	-0.31	0
FC	-0.31	0
WT	-0.15	138.6
HT	-0.20	0
CT	-0.20	0
VC	-1	0
OT	0	0
OC	0	0
MT	-1.58	30
DT	-0.42	30
VT	-2.65	16
ST	-0.514	0
KT	-0.501	0
Eq. #	4.37	4.104

Table S8A. Sheet 8 Part A of % GDP Impacts in 2019

Type	USA	CAN	WEU.	JPK	ANZ	CEE	FSU	MDE
AR	-.0097	-.0028	-.0013	-.0004	-.0011	-.0016	-.0020	-.0010
AL	.0342	.2010	.0214	.0171	.0985	.0839	.1205	.0549
AQ	-.0111	-.3026	-.0090	-.0055	-.0224	-.0216	-.0348	-.0151
AC	.0666	.0405	.0903	.0554	.1809	.0097	.1016	.0724
FT	.0013	.0003	.0006	.0011	-.0030	.0014	-.0007	0
FC	.0004	8E-5	.0002	.0003	-.0009	.0004	-.0002	0
WT	-.0311	-.0273	-.1331	.0001	.0001	-.3394	-1.1416	-.0602
HT	.2635	.2347	.1497	.1282	.0906	.2750	.2954	.2028
CT	-.0868	-.0770	-.1540	-.0120	-.0084	-.0744	-1.0934	-.0962
VC	-.0135	-.0166	-.0180	-.0206	-.0184	-.0346	-.0483	-.0408
OT	-.0003	0	0	-.0141	-.0138	0	0	-.0040
OC	-4E-5	0	0	-.0003	-.0191	0	0	-.0009
DT	-.0144	-.0193	-.0054	-.0003	-2E-5	-.0054	-.0791	-.0109
MT	-.0042	-.0053	-.0015	-.0004	-.0002	-.0020	-.0207	-.0004
VT	-2E-6	-3E-6	-3E-5	-.0003	-1E-6	-4E-5	-5E-5	-.0028
ST	-.0084	-.0005	-.0006	-.0019	-.0041	-.0001	-.0003	-4E-5
KT	-.0073	-4E-5	-7E-5	-.0011	-.0003	-3E-5	-.0001	-3E-5

Table S8B. Sheet 8 Part B of % GDP Impacts in 2019

Type	CAM	SAM	SAS	SEA	CHI	NAF	SSA	SIS
AR	-.0012	-.0006	-.0010	-.0003	-.0016	-.0012	-.0014	-.0010
AL	.0886	.0085	.0642	.0191	.1462	.0707	.1190	.0397
AQ	-.0215	-.0057	-.0151	-.0057	-.0403	.-0202	-.0256	-.0069
AC	.1751	.1343	.1114	.1105	.3740	.0924	.1934	.1980
FT	.0005	.0006	.0008	.0013	.0012	0	.0003	0
FC	.0001	.0002	.0002	.0004	.0004	0	.0001	0
WT	-.0619	-.0669	-.0519	-.1301	.2016	-.4036	-.1651	-.0622
HT	.0670	.0805	.0377	.0076	1.6358	.0090	.0039	3E-5
CT	-.1013	-.1084	-.1030	-.2580	-.7938	-.7564	-.3467	-.0990
VC	-.0521	-.0557	-.1615	-.0641	-.0516	-.0806	-.5056	-.0696
OT	-.0044	-.0004	-.0026	-.0491	-.0003	-.0015	-.0014	-.4754
OC	-.0010	-.0003	-0004	-.0072	-5E-5	-.0010	-.0047	-.0648
DT	-.0550	-.0050	-.0022	-.0079	-.0005	-.1157	-1.3205	-.0607
MT	-.0013	-.0012	-.0007	-.0007	-.0004	-.0022	-.0118	-.0017
VT	-.0005	-.0005	-3E-5	-.0003	-1E-5	-.0425	-.3100	-.0117
ST	-.0108	-.0001	-.0043	-.0016	-.0040	-7E-7	-.0021	-.0361
KT	-.0001	-.0001	-.0002	- 3E-5	-2E-5	-2E-5	-.0001	-.0028

E-n is 10^{-n}. Tabels 8A and 8B contain c_r^{XY}, used in Eqs. 4.114 and 4.115.

6. Derivation and Calibration

6.1. **Prescribed Radiative Forcing.** Estimates of radiative forcing from changes in land use F_4, contrails and cirrus clouds F_5, halogens F_6, ozone plus black carbon on snow F_7, and tropospheric aerosols F_8 came from "Table A.III.3" of International Program on Climate Change Working Group I Sixth Assessment Report (Masson-Delmotte *et al.*, 2021). Least squares fits to constant plus logistic functions were used from 1850–2019 for changes in land use, for years from 1950–2019 for halogens, and from a more detailed time series from 2000–2018 for effects of global aviation (Lee *et al.*, 2020). Using radiative forcing estimates from Table A.III.3, a set of pollutants with regional environmental effects were modeled as having logistic rates of change, leading to fits with constants plus temporal derivatives of logistic functions. Radiative forcing from these pollutants, tropospheric ozone and black carbon on snow, and were combined for simplicity with anthropogenic radiative shielding from stratospheric ozone, leading to a net radiative forcing.

Uncertainty about the absolute value of radiative shielding from tropospheric aerosols (Masson-Delmotte *et al.*, 2021) was estimated as 1.62 times as large the square root of the sum of squares of a measure of all other contributions to radiative forcing. The least squares fit to the nominal radiative (negative) forcing from 1810–2019 contributions from tropospheric aerosols (Masson-Delmotte *et al.*, 2021) was thus multiplied by a factor with a value calibrated with other global heat balance parameters as described in the subsection on Sea Level and τ below.

6.2. **Nitrous Oxide and Methane.** Parts per billion by volume (ppb) concentrations were modeled as evolving according to equations of the form

$$G' = e_G - G/t_G \tag{6.1}$$

with G being increases in $<N_2O>$ or $<CH_4>$ over fits to Julian year 1750 concentrations G_{pre}. The inputs b_{Gn} for $G = N$ and M and $n = 1, 2, 3$ are parameters of logistic functions for anthropogenic emissions e_G. Solutions for the above analytic function solutions to $G' = e_G - G/t_G$ with initial conditions in 1750 of G_{pre} gave least squares fits to estimates of atmospheric concentrations of $<N_2O>$ or $<CH_4>$ from 1750–2019. Those concentrations came from a combination of ice core (MacFarling Meure *et al.*, 2006) and direct atmospheric measurements Butler & Montzka (2017). The ice core measurements were rescaled to match the direct atmospheric measurements by multiplying by the 1980 direct to ice core ratios of 0.998 for $<N_2O>$ and 1.005 for $<CH_4>$. Reference values for t_G of 116 yr for $<N_2O>$ and 9.1 yr for $<CH_4>$ came from Masson-Delmotte *et al.* (2021).

6.3. **$<CO_2>$.** Anthropogenic carbon emissions in the form of CO_2 are divided into industrial emissions and emissions associated with deforestation and other land use changes. Estimates of historical industrial emissions were fit with a constant plus logistic function. Estimates for historical emissions from land use changes were fit with a constant plus two time derivatives of logistic functions. The fitting parameters were least squares fits to estimates from 1850–2019 (Global Carbon Project, 2020). A small fraction associated with production of cement net of recarbonization is included but not treated differently than emissions from combustion of fossil fuels.

Historically, the effect on prices of depletion of fluid fossil fuels (oil and natural gas), has been complicated by an interaction between technological progress in resource extraction and varying degrees of non-competitiveness in domestic and international markets. Looking forward, however, it is anticipated here that inflation-adjusted extraction costs may increase with global fluid fossil fuel resource depletion, c.f. Rogner (1977), as coming closer to physical limits constrains increasing extraction efficiency. The fraction that coal contributes to emissions from fuel combustion was approximately constant at 0.41 from 1965–2019 (BP, 2021) and is approximated as continuing so. Using Mexico as an example (Huntington *et al.*, 2017) of a middle income country heavily reliant on fluid fossil fuels, a price elasticity coefficient of $\beta_f = -0.35$ was used to temper extrapolated "No Deals" industrial carbon emissions. In general, that approach would involve numerical integration of the balance equation for the rate U' of industrial carbon emissions, which is a function of cumulative emissions U as a result of the resource depletion effect. For the purposes of the manuscript, extrapolations for more than a few times the inverse of the social discount rate 0.023/yr were of little interest, so it sufficed to use only the first step in a successive approximation approach where an analytic result for $U(t)$ without resource depletion is used to estimate the depletion effect factor f_d. That is, the analytic function result for $U(t)$ was used when computing the depletion effect function f_d. That result for f_d was close enough 1 that its expected accuracy did not justify the additional complication of using it to recompute $U(t)$ numerically for use in obtaining a slightly different estimate of f_d even once.

Values of the parameters ν_c and r_{sa} in the carbon balance equations were estimated by a least squares fit to an exponential decline of part of the atmospheric CO_2 content from GFDL model result (MacDougall *et al.*, 2020) for an abrupt termination of anthropogenic atmospheric emissions. Using those results, the remaining parameters in the carbon balance model were from a least squares fit to direct atmospheric CO2 concentrations from 1979–2019, from a database described in Butler & Montzka (2017).

6.4. **Forcing from All but SRM.** Radiative forcing from stratospheric water vapor in a given year is taken to be proportional to the anthropogenic increase $<CH_4>$-$M_{\text{measured,pre}}$ at time t_H=2 years earlier (Miller *et al.*, 2014). The proportionality constant $a_H = 0.000048$ W/m^2 was estimated by a least squares fit to a_H times that anthropogenic increase in radiative forcing, rounded to the nearest 0.01 W/m^2 to match the rounding in the (Masson-Delmotte *et al.*, 2021) list. $M_{\text{measured,pre}}$ is taken to be a measure of an effectively constant $<CH_4>$ in over the two years prior to 1750, since the measured values in those two year are respectively 0.1 ppb higher and lower.

Values for Julian year 1750 of $<CO_2>$, $<N_2O>$, and $<CH_4>$, taken from the above-mentioned ice core measurements and rescaled slightly to match direct atmospheric measurements, are not exactly equal to the parameters C_0, N_0, and M_0 used in radiative forcing formulas. For consistency with radiative forcing numbers used in calibrating parameters in the global heat balance equation, the small radiative forcing values obtained from inserting those measured concentrations are subtracted here from the results from the radiative forcing formulas so that the net radiative forcing is zero in 1750. The somewhat different values of a_{pre}/c_1, b_{Npre} and b_{Mpre} per Table S3 are used only for extrapolations from year 2019 of $<CO_2>$, $<N_2O>$, and $<CH_4>$. For that purpose, the parameters used in those extrapolations are calibrated against data from ranges of times that put more emphasis on capturing recent trends for extrapolation rather than precisely matching mid eighteenth century measurements.

6.5. **Sea Level and τ.** A linear global heat balance was used, in light of a zero coefficient of the quadratic term in a nonlinear model lying at or close to the middle of a range of uncertainties for that term per Rohrschneider *et al.* (2019) and references therein. A constraint on possible future use of the model described here is that paleoclimate analysis suggests nonlinearity at global average temperatures below the minimum values reached in examples described in the manuscript (Friedrich *et al.*, 2016). Nor, as noted in the manuscript, is the model meant to used for cases where a "tipping point" with a significantly nonlinear-response temperature response to radiative forcing is reached without having been anticipated and avoided, using SRM if necessary to do so.

Probability distributions and maximum likelihood estimates were made for four a priori uncertain parameters relevant to differences between data and global heat balance equation. Those parameters are the thermal inertia parameter c_{th}, λ, the multiplier c_a of the above-mentioned Table AIII.3 Masson-Delmotte *et al.* (2021) radiative forcing from tropospheric aerosols, and the difference $\Delta\tau_0$ from an estimate (Hawkins *et al.*, 2017) of 0.82°C of how much the 1951–1980 average of GISTEMP Team (2021) estimates exceed a temperature in equilibrium with zero radiative forcing. That is after accounting for a -0.02 °C correction or removal from the GISTEM numbers the effects of volcanic eruptions and Schwabe cycle variations that Hawkins *et al.* did not correct for. Like in Hawkins *et al.* the approach used here removed El Niño Southern Oscillation (ENSO) variations from the GISTEMP numbers.

The data used for calibration of parameters in the global heat balance equation were from annual global average temperatures from 1946–2019 and the 28 estimates of annual change in ocean stored energy computed from data from years 1991–2018. Earlier data were not used to avoid years with limited global and ocean depth coverage and a known global average temperature measurement anomaly of uncertain size during World War II. This limitation on the data used also avoided the complication of parameter estimation bias with a statistically significant temporal autocorrelation of type AR1 in the temperature data and the computationally troubling moving average (MA1) temporal autocorrelation in the annual changes in ocean stored energy data from von Schuckmann *et al.* (2020).

Statistical tests for temporal autocorrelation, skewness and kurtosis deviations from normal distributions, and for heteroskedasticity led to inclusion only of a statistically significant outlier in the 2001 to 2002 growth in ocean stored energy. That inclusion was accounted for with a different standard deviation for that year.

The maximum likelihood value $c_a = 0.37$ is within a 5–95% confidence range reported with figure 7.6 of the IPCC AR6 Working Group I report Masson-Delmotte *et al.* (2021). The accompanying maximum likelihood parameter estimate for λ in the global balance equation $c_{th}\tau' = F - \tau/\lambda$ was 0.5175 $\mathrm{W/m^2}$. That estimate was constrained by inclusion of ocean stored energy data from von Schuckmann *et al.* (2020) in the calibration exercise, which precluded matching of the data used by a combination of higher estimates of both the thermal inertia parameter c_{th} and λ.

The temporal evolution of tropospheric aerosols shielding from the above-mentioned Table AIII.3 was fit with the time-derivative of a logistic function, with a peak in 1995 as indicated in Table S2 and a subsequent decrease in that shielding. Growth in total radiative forcing after that year is reinforced with a continuation beyond 2019 of that decrease, but only much more weakly so than, for example, if $c_a = 1$ were prescribed with a resulting larger probability maximizing value of λ. That accounts for the observation that the temperature evolution through 2080 of the No Deals case with the present model is about the same as for the 32-model mean of the CIMP SP2-4.5 scenario McBride *et al.* (2021), even though SP2-4.5 has lower global carbon emissions than the No Deals case per Hausfather (2020). However, extrapolations using global balance equations with imposition by fiat of an estimate of one parameter inferred using a different parameter estimation methodology would be inconsistent. If it were desired to explore the ramifications for computed welfare results of global heat balance extrapolations with higher values of λ, then a more consistent approach would be to sample quadrivalent probability distributions for all four of the parameters used for the extrapolations.

The procedure for removing short term variations associated with five volcanic eruptions, ENSO, and the Schwabe solar cycle, was a modified form of that used by Foster & Rahmstorf (2011). That modification was to use their same values for temporal lag months between measures of those three transient effects on τ but to allow for different temperature response multipliers of those three effects while making estimates of four other parameters as described above.

For volcanic eruptions, effects of variations in atmospheric optical depth (NASA Goddard Institute for Space Studies, 2016) below a threshold were treated as a contribution to other sources of statistical deviations from a solution of the global heat balance equation. That AOD threshold was set at 0.012. Rounded to the nearest 0.001, that was twice the average of the below-threshold AOD variations from 1906–2018, a data-range chosen during studies of temporal autocorrelation effects in GISTEMP results before 1946. The variation of below-threshold AOD by ± 0.006 around a pre-industrial average, and of associated radiative forcing and global average temperature impact, was approximated as a minor component of substantially larger random departures of globally and annually averaged temperature from an underlying trend. The over-threshold volcanic AOD values were lagged by seven months and multiplied by -2.36°/AOD (Foster & Rahmstorf, 2011), and that result was subtracted from the GISTEMP numbers.

Multivariant ENSO index (NOAA Physical Sciences Laboratory, 2021) numbers were lagged by four months and multiplied by 0.08°/AOD, and the result was subtracted from the GISTEMP numbers. Differences between the above-mentioned three-cosine fit were converted to total solar irradiance LASP (2021), multiplied by 0.084°/(W/m^2) of total solar irradiance), lagged by one month, and subtracted from the GISTEMP numbers.

In the process of using principal component analysis to find a quadrivalent normal approximation to the joint probability distribution for the four a priori uncertain parameters, maximizing parameters of $c_{th} = 28.49$ (W/m^2)/°C, $\lambda = 0.5175$°C/(W/m^2), $\hat{c}_a = 0.3707$ and $\Delta\tau_0 = -0.02°$ were found. The value of $\Delta\tau_0 = -0.02°$ is not seen in the global heat balance equation, but it does (very slightly) affect the initial condition for τ in 2019 that is used as a starting point for extrapolations from that time on. The estimate $\Delta\tau_0 = -0.02°$ is within the range of uncertainty from Hawkins *et al.* (2017). Thus, the results for the operational definition of τ using the approach to estimating radiative forcing described above are not inconsistent with estimates from those authors. For the global heat balance model parameter calibration exercise, annually averaged estimates of radiative forcing were available, and it was computationally convenient to use a sum of analytic solutions for the increase in future years of annual averaged temperature that year and the preceding years. For extrapolations, smooth functions of extrapolated radiative forcing were available, so it was more computationally convenient to find solutions in the form of continuous functions of time.

Computed radiative forcing changes for annual stratospheric sulfur injection rates S_{SRM} up to 100 MtonneS/yr (Laasko *et al.*, 2022) were fit with functions of the form $\Delta F = F_{\mathrm{type}} e^{-S_{\mathrm{SRM}}/S_{\mathrm{type}}}$ for sectional and modal type models of the relevant atmospheric processes. Since the resulting values of S_{type} were the same to well within the accuracy of the procedure, those values were averaged for simplicity. The input value maximum value $S_{\max}$ used in the manuscript was set equal to the upper limit of the sulfur injection rates investigated by Laasko *et al.* (2022). A subsequent preprint from Laasko *et al.* (2023) notes that radiative shield estimates depend both on which earth systems model is is used for examining stratospheric sulfur injection and details of the injection scheme and concurrent evolution of $<\mathrm{CO}_2>$ in addition to where a sectional or model microphysics model is chosen, detailed modeling of which was avoiding here for simplicity. The value of the parameter F_w used for the manuscript to limit radiative shielding as S_{SRM} approaches and transiently exceeds $S_{\max}$ was chosen to limit that overshoot to one TtonneS/yr. With the choice of $F_{\mathrm{type}} = 15.545$ W/m^2 choice from Table S0, this limitation is insignificant through the twenty-second century even in the face of strong SRM cooling starting in the twenty-first century. However, with the "costlier SRM" choice $F_{\mathrm{type}} = 6.2693$ W/m^2, a limitation can be encountered early in the twenty-second century with strong SRM cooling starting in the twenty-first century, so users should take care to check if the default choice of $S_{\mathrm{SRM}} = 100$ MtonneS/yr in Table S1 is appropriate for their purposes if exploring such cases.

The parameters in the equation for sea level were calibrated against data for the difference of sea level from a fit to its height in 1990. The increase $H_0 = 0.26$ m of sea level from 1750 to 1990 is from Grinsted *et al.* (2010). (Estimates for differences from 1990 were as an average for estimates from 1980–1999.) This model provides a good fit to post-World-War-II estimates of changes in global mean sea level, but it does not attempt to separately model several processes that affect that level. In particular, it is not designed for long-term modeling of situations where effects of ocean steric expansion and melting of land ice combine to have a different response to warming than over the 1946–2015 time span (Dangendorf *et al.*, 2019) used for calibration of the present model's parameters.

6.6. **Background Economy.** Economic impacts of climate change are treated as a perturbation on a background economy for each of the sixteen regions described in the manuscript. Per capita

GDP in the background economy is fit with a region-dependent constant plus a function that depends on logistic growth of productivity. Managers of the division of production between investment and consumption are assumed to not attempt to guide that division in a way that would take account of the value of that that region-dependent constant. The differences between a region-dependent base level of per capital production given by the numbers in Table S5 and a bare subsistence level y_s are wasted, in the sense that they does not contribute to welfare. The population fixed in each region is approximated as the constants B_0 in Table S5 plus a logistic function. Managers of the division of production between investment and consumption are also assumed to not account for the labor supplied by that part of the population.

The approach used here was adopted primarily to simplify the resulting equations. However, it does recognize the existence of unequal distribution of wealth, influence, and effective participation in the labor force. It also recognizes that not all parts of consumption increase welfare (e.g. some expenditures on items deleterious to human health). While this is a highly idealized way of dealing with complicated issues, so is a simple underlying model that counts average per capita income as enhancing welfare for all segments of a population using a formula depending only on consumption averaged over the size of the whole population.

The pure time rate of preference $\bar{\rho} = 0.023\ \text{yr}^{-1}$ was estimated form data on real interest rates from Bank (2005) less θ time per capita GDP growth rates (with down weighting of data with higher nominal interest rates and corresponding variability of associated real interest rates). $\theta = 1.345$ is the inverse of the inter-temporal substitutability of consumption, estimated from a least squares fit for a regression of ln[((100-(self-reported wellbeing))/100] on per capita income for 41 countries from Myers and Diener (1995). The capital depreciation rate $\bar{r}_{\text{dep}} = 0.0106/\text{yr}$ was computed from the value of $\bar{\rho}$ and the value of $\bar{t}$ that was rounded to three significant figures. This result with that rounding is very close to an estimate from data from the United States of 0.0107/yr from Bischoff and Kokklenberg (1987). The labor fraction of production $1 - \alpha = \omega = 0.675$ is the mean of estimates for 31 countries from Gollin (2002).

Both the underlying framework for how managers of capital arrange the division of production and consumption and the simplicity and vintage of the above-mentioned parameters taken to be constant both globally and in time are somewhat different from other models. For example, in table 1 of Gazotti (2022) describing RICE50+, in the notation used here global constants are $\{\theta, \alpha, \bar{\rho}, \bar{r}_{\text{dep}}\} = \{1.45, 0.3, 0.015/\text{yr}, 0.1/\text{yr}\}$ compared to $\{1.345, 0.325, 0.023/\text{yr}, 0.106/\text{yr}\}$. (The parameter here denoted by θ is referred to by Gazotti as elasticity over the marginal rate of consumption.) In both of these two models, the underlying assumption about how managers of capital make decisions, and the use of these regionally and temporally constants parameters are chosen as much for simplicity as they are as a reflection of how investment decisions are actually made. Some commonalities in the conceptual framework can nevertheless be helpful by providing a framework for understanding what distinguishes one mode from the other.

The evolution of capital stock $\bar{K}_0 = \bar{K}K_0$ is determined by maximizing welfare

$$\int_{t_b}^{t_m} P\frac{1 - ((\tilde{C}_0/P)/y_S)^{1-\theta}}{\theta - 1} e^{-\bar{\rho}(t-t_0)} dt \tag{6.2}$$

Here $P = B_1 L = B_1/(1 + e^{-(t-B_2)/B_3})$ for each region is increase in population since 1820, $y_S = 0.601$ k\$ 1990 ppp (purchasing power parity) is an estimate of subsistence level per capita GCP based on fits to the CHI region, and $\tilde{C}_0 = C_0/(\bar{K}/\bar{t})$ is per capita GDP. Converting time to the dimensionless variable $s = (t - t_b)/\bar{t}$,

$$\mathcal{L} = e^{-\rho s} L^{\theta} C_0^{1-\theta} \tag{6.3}$$

$$\beta = ((\bar{K}/\bar{t})/B_1 y_S))^{1-\theta} \tag{6.4}$$

$$W_0 = 1000 B_1 \bar{t} e^{-\rho s_2} \int_{s_b}^{s_m} ((e^{-\rho s} L - \beta\mathcal{L})/(\theta - 1))\, ds \tag{6.5}$$

with $s = (t - t_b)/\bar{t}$, and $s_m = (t - t_m)/\bar{t}$. The factor of 1000 converts from Gperson-yr to M-personyr.

The times t_b at the beginning of the data ranges noted in Table 1 of the manuscript and the maximum time t_m are chosen long enough before and after the times for which the result is used that their precise values are not significant. The comparative simplicity of the expression for W_0 results from the observations that the Euler-Lagrange equations for dimensionless capital stock $K_0 = \tilde{K}_0 \bar{K}$ are independent of multiplicative constants and of any functions of time that do not depend on the control variable $K(s)$. Here, for each region, as above for but for brevity without the subscript r,

$$C_0 = Y_0/\alpha - r_{\rm dep} K_0 - \dot{K}_0 \tag{6.6}$$

and

$$Y_0 = a K_0^{\alpha} L^{\omega} \tag{6.7}$$

The productivity coefficient is $a = 1/(1 + e^{-(s - b_2 \bar{t})/(b_3 \bar{t})})$, and an overdot indicates differentiation with respect to s. The function $K_0(s)$ must satisfy the Euler-Lagrange equation

$$\delta \mathcal{L}/\delta K_0 = d(\delta \mathcal{L}/\delta \dot{K}_0/ds) \tag{6.8}$$

where $\delta/\delta K_0$ and $\delta/\delta \dot{K}_0$ represent derivatives with K_0 and $\dot{K}_0$ considered as independent variables and then d/ds is considered to be differentiation with respect to s with $\delta \mathcal{L}/\delta \dot{K}_0$ considered to be a function of one variable, s.

Multiplying both sides of the Euler-Lagrange equation by $e^{\rho s} L^{-\theta} C_0^{\theta}$. Note that $\bar{t}$ is defined so that $\rho + r_{\rm dep} = 1$ with $\rho = \bar{\rho}\bar{t}$ and $r_{\rm dep} = \bar{r}_{\rm dep}\bar{t}$. Also, $\dot{L}/L = \mu M$ with $\mu = \bar{t}/B_3$ for each region, and $M = 1 - L$. The Euler-Lagrange equation becomes

$$R_0 = \dot{C}_0/C_0 \tag{6.9}$$

where $R_0 = (F - 1 + \mu\theta M)/\theta$ (Eq. 4.83 above) and $F = Y_0/K_0$ (Eq. 4.81 above). Setting $G_0 = (C_0/K_0)/\gamma$ with $\gamma = \alpha^{-1} - r$, using the zeroth order monetary balance equation $C_0 = Y_0/\alpha - rK_0 - \dot{K}_0$ from Eq. (4.84) above, computing $\dot{C}_0$, and rearranging terms gives

$$F_0 = 1 + \delta z - (\theta/\omega)\dot{F}_0/F_0 + \theta \dot{G}_0/G_0 \tag{6.10}$$

with $z = 1 - a$. Expanding this equation in powers of δ yields an asymptotically convergent series solution for F_0, the first two terms of which add to $1 + \delta z$. Solving $1 + \delta z = Y_0/K_0 = a(L/K_0)^{\omega}$ for K_0 then gives $K_0 = (a/(1 + \delta z))^{1/\omega} L$, which is Eq. (4.77) above with Y_0 as in Eq. (4.79) and the subscript r again omitted for brevity.

The approximation for K_0 does not depend on boundary conditions at s_b and s_m for the second order differential equation $\delta \mathcal{L}/\delta K_0 = d(\delta \mathcal{L}/\delta \dot{K}_0/ds)$. Numerical integration confirms that the influence of the initial condition decays exponentially on a timescale of $\bar{t}$. The influence of the terminal boundary condition similarly decays exponentially on a timescale of $\bar{t}$ moving towards times earlier than the terminal boundary time. That is, the solution "forgets" about the initial boundary condition at much earlier times and "does not anticipate" a terminal boundary condition set sufficiently far in the future.

The formulas used to model the rate growth of the capital stock in each region is approximated as being small compared to the inverse of the capitalization time, $\bar{t} = 1/(\bar{\rho} + \bar{r}_{\rm dep})$. That the rate of growth of capital stock is small compared to $1/\bar{t}$ is more appropriate for the USA region, for example, than for the CHI region. The background economy model used here should thus be viewed as semi-empirical. That is, while it has a foundation in an underlying theory, the circumstances in which that theory describes decision processes leading to the rate of change of capital stock are limited. The primary purpose of the treatment of the background economy used here is to fit general historical data trends and extrapolate those trends over the shorter term while approaching a longer-term limit consistent with physical limitations on the evolution of total carbon emissions.

The range of historical data from 1820 (Maddison, 2003, 2010) and more recent data and short-term International Monetary Fund (2019) extrapolations used for most of the model's sixteen regions includes years 1820–2024. (Maddison's results were mapped onto the year 2019 boundaries of countries and other UN reporting regions with partitioned units having earlier GDP numbers divided in proportion to fractions at the time of partition, e.g. for the Baltic countries. Estimates for missing years were geometrically interpolated between years for which estimates were available. Maddison's GDP numbers, accumulated for each of the sixteen regions, were then multiplied by the ratio of their IMF data sums from 1995–1999 by those sums from Maddison.)

In view of economic reforms in India, the range of years used for parameter calibration starts in 1991 for the SAS region. Setting aside years of recovery from the collapse of the Soviet block, the years 1990–1990 are omitted for the CEE region and years 1990–2006 are omitted for the FSU region. In view of a major economic perturbation in Southeast Asia, the years used for the SEA region are 1820 and 1998–2024. Other economic perturbations, e.g. due to COVID, and even China's Great Leap and the twentieth century World Wars, are treated as events that economies recovered from quickly enough that including data from those years does not obscure underlying trends in economic growth.

Derivation of the rest of the expressions in the background economics module is summarized in the comments below on the Welfare module. Total computed welfare for the background economy for each region that comes from evolving K_0 to maximize welfare (in formula numbered 6.1 above) is given by Eqs. (4.146–4.148). The values of parameters in Table S4 determine the values of per capita GDP corresponding, for example, to 1 and 2 person-years of computed welfare.

6.7. **Impacts of Climate Change on Economic Productivity.** This subsection summarizes some of the differences between the model used in the manuscript and the FUND 3.9 model. Note that the relationship between incremental GDP and productivity a is $d\ln(y - b_0)/d\ln a = (1/\omega)(1 + (\delta\alpha/(1 + \delta z))$, with $\alpha = 0.325$. Neglecting the factor $(\delta\alpha/(1 + \delta z))$, as being small compared to uncertainties in the FUND 3.9 estimates of impacts of climate change on GDP, the small fractional changes in incremental GDP can be approximated as $(1/\omega)$ times small fractional changes in a.

6.7.1. *Agriculture and Impacts Elasticities with Respect to per Capita Consumption.* The model used here has the impact of the rate of change of τ proportional to $(\tau')^2 - (\tau'_0)^2$. This avoids solution of a FUND 3.9 finite difference equation by, in effect, neglecting a contribution of order the ratio FUND 3.9 timescale of 10 years for agricultural adaptation to the timescale for changes in τ'.

FUND 3.9 has the elasticity of the ratio of gross agricultural product to GDP with respect to per capita GDP the same for all sixteen regions. Here, that elasticity depends varies from region to region, based on regressions using (FAO, 2023) data from 1961–2016.

6.7.2. *Heating.* Here, the reduction in impact on economic productivity with increasing τ is proportional to $\text{ArcTan}(\tau - \tau_0)$, where τ_0 is the year 1990 temperature. FUND 3.9 uses the same function of temperature, but with a coefficient for the MDE region that is 4.8 times as large as for the USA. For the regions where it reduced the FUND 3.9 impacts (MDE, CAM, SAM, SAS, SEA, and SIS), the coefficients for the $\text{ArcTan}(\tau - \tau_0)$ function were here set equal to the USA region values times ratios of years 1997–2013 averages of U.S. National Weather Service heat index heating degree days (Atalla *et al.*, 2017) to that for the USA region. Those numbers were weighted by year 2005 population for all countries in each region for which heating degree days entries were listed.

Other than for agriculture and the new VC and OC models, numbers for elasticity of impacts with respect to per capita GDP are the same here as in FUND 3.9. That is after accounting for how FUND 3.9 variously uses GDP and per capita GDP in its formulas and using here the increment of per capita GDP over the constants listed in Table S2. The effects of <CO_2> on ventilation or human exposure and ocean coral have elasticities $\epsilon_{VC} = -1$ and $\epsilon_{OC} = 0$ respectively. This difference is because the VC effect is computed on a per capita basis, while coral reef damage is computed as a fraction of each affected region's total economic production.

6.7.3. *Cooling.* FUND 3.9 models the economic impact of space cooling as proportional to the 3/2 power of the difference between global average temperature and its year 1990 value. To avoid imaginary numbers for temperatures lower than the 1990 value, Eq. 4.110 above has that effect proportional to $\tau^2 - \tau_0^2$, where τ_0 is the year 1990 temperature. This model matches the FUND 3.9 model for temperatures in 1990, 2019, and when $\tau = 4.19$ °C.

6.7.4. *Ventilation.* Studies of the effects of carbon dioxide exposure on human mental performance (Satish *et al.*, 2015; Snow *et al.*, 2019) motivated inclusion of a model of the cost of improving ventilation (or of failing to do so) as a function of <CO_2>. The values of c_r^{VC} listed in Tables S8A and S8B are $\alpha_{VC}/(1000 y_{1r})$. Here $\alpha_{VC} = C_{VC} V_{VC} \rho_{VC}$. The year 2009 dollar cost per tonne of CO_2 scrubbing in USD2019 of $C_{VC} = 600(225.7/229.4)$ is an estimate of the cost (APS, 2011) times a 2019 to 2009 U.S. consumer price index ratio. The ventilation rate in liters per year for each person is $V_{VC} = 11.7 * 0.3 * (3600 * 24 * 365.25)$, with occupancy space (O'Keefe, 2017) of 11.7 m^2/person and ventilation rate of 0.3(liters/s)/m^2 (ANSI/ASHRAE, 2019) converted to (liters/yr)/m^2 by multiplying by the number of seconds in a year. The number of tonnes of CO_2 per ppm CO_2 is $\rho_{VC} = 10^{-6}(44 * 10^{-6})/22.4$. The factor of 1000 multiplying the tear 2019 incremental per capita GDP converts from k\$2019ppp/yr to \$2019ppp/yr.

6.7.5. *Sea Level.* Here, for relative simplicity, only impacts of dry land loss are included from a substantially more complicated FUND 3.9 model that includes coastal protection options and effects on wetlands. The results are close enough to those of the FUND 3.9 model to highlight regions (e.g. SIS) where impacts of sea level change are not very small compared to other impacts of climate change on productivity.

Fractional impacts on GDP from sea level change are estimated as $(\mathrm{km})^2$/yr of dry land loss times value per km^2, divided by GDP. The formula for this is

$$f_r^{OT} = -R_{OT}(\delta_r^{OT}/(10^6 A_r^{OT}))dH_m^{1+\sigma_r}/dt \tag{6.11}$$

where $H_m = H/(1\,m)$. The expansion of the time derivative $dH^{1+\sigma_r}/dt = (1+\sigma_r)H^{\sigma_r} dH/dt$ with $dH/dt = dS/dt = a_S(\tau - \tau_S)$ is used to get the results for c_r^{OT} in Tables S8A and S8B.

The value of $R_{OT} = 6.3$ is the ratio 4/0.635, rounded from 6.299, of parameters denoted in the Fund 3.9 documentation as ϕ=4 M\$/$(\mathrm{km})^2$ and YA_0=0.635 M\$/$(\mathrm{km})^2$. The parameters δ_r^{OT} (from FUND 3.9 table SLR column 2), are $(\mathrm{km})^2$ of cumulative land area losses per unit of $(S/(1\,m))^{\sigma_r}$, where $S/(1\,m)$ denotes sea level increase since 1750 (with 1 m in the denominator to make $S_r/(1\,m)$ dimensionless). The exponents σ_r listed above in Table S6 iare computed from FUND 3.9 table SLR column 3 values of $(1+\sigma_r)$. Total summed regional land areas (UN, 2017), A_r^{OT}, are in $(\mathrm{Mm})^2$. The factor of 10^6 in front of A_r^{OT} in the above equation for f_r^{OT} converts these numbers to $(\mathrm{km})^2$.

6.7.6. *Coral Reef Loss.* The entries for c_r^{OC} in Tables S8A and S8B are equal to $-V_{OC} A_r^{OC}/Y_{1r}$ Here $V_{OC} = (255.7/177.2)0.177$ in T\$2019 per $(\mathrm{Mm})^2$ of preindustrial coral reef areas is inflation-adjusted from year 2000 to year 2019. A_r^{OC} are preindustrial coral reef areas (Brander *et al.*, 2009) summed by region. $Y_{1r} = y_{1r} P_{1r}$ are regional GDP values in T\$2019. Inclusion of impacts of coral reef loss is meant to highlight environmental impacts on productivity in a way taken to be more readily quantifiable way by managers of capital investment, not to fully substitute for the broader scope of such impacts covered in the FUND 3.9 model.

6.7.7. *Diseases.* Here, and in the following description of damage from storms, parameter values from tables in Anthoff & Tol (2014a) are followed by table numbers in parentheses from that reference. The fractional impacts on GDP from mortality and morbidity caused by diarrhea are respectively

$$f_r^{DT} = -(y_r/y_{0r})^{\epsilon_{DT}} 2^{-\nu_{DT}(t-2000)} V_K(\mu_r^{DT}/1000)\eta_{DT} R_r^T T/C_T \tag{6.12}$$

$$f_r^{MT} = -(y_r/y_{0r})^{\epsilon_{MT}} 2^{-\nu_{MT}(t-2000)} V_M(\mu_r^{MT}/1000)\eta_{MT} R_r^T T/C_T \tag{6.13}$$

The value of $C_T = 13.6$ American Institute of Physics (2022) is an estimate of the preindustrial global average in °C (i.e. absolute temperature in Kelvin less 273.13). The expression $(1 + \delta_{DT}\tau/C_T)^{\eta_{DT}} - (1 + \delta_{DT}\tau_0 C_T)^{\eta_{DT}}$ that would follow from using the formula in the FUND 3.9 documentation has been replaced by expansions through first order in τ/C_T and τ_0/C_T for simplicity, and similarly for f_r^{MT}.

The parameters $V_K = 200$ and $V_M = 0.8$, from the FUND 3.9 documentation section 5.12 are ratios. Those ratios are cost per person of death, or the onset of morbidity form diarrhea, divided by per capita GDP. The parameters μ_r^{DT} (HD 3) and μ_r^{MT} (HD 4) are respectively the corresponding increases of mortality and morbidity per °C of regional temperature increase. Values for η_{DT} and η_{MT} are listed in Table B2. Values for ϵ_{DT}, ϵ_{MT} and $\nu_{DT} = \nu_{MT}$ are listed above in Table S7

The fractional impacts on GDP from mortality due to vector-borne disease are

$$f_r^{VT} = -(y_r/y_{0r})^{\epsilon_{VT}} 2^{-\nu_{VT}(t-2000)} V_K 10^{-6}(\mu_r^{MD}\alpha_{MD} + \mu_r^{DF}\alpha_{DF} + \mu_r^{SM}\alpha_{SM}) R_r^T T \tag{6.14}$$

Here μ_r^{MD} (HV 2), μ_r^{DF}, (HV 4), and μ_r^{SM} (HV 6) are respectively base level annual mortality rates (per million people, hence the factor of 10^{-6}) respectively from malaria, dengue fever, and schistosomiasis. Corresponding changes in those mortalities per °C of regional temperature increase, α_{MD}, α_{DF}, and α_{SM} from Table HV of Anthoff & Tol (2014b) are listed in Table B2. Values of ϵ_{VT} and ν_{VT} are listed in Table 1 of Section 2.

6.7.8. *Storms.* Fractional GDP impacts due to property damage from storms use parameter values from (Anthoff & Tol, 2014a).

$$f_r^{ST} = f_r^{PS} + f_r^{PE} \tag{6.15}$$

where

$$f_r^{PS} = -(y_r/y_{0r})^{\epsilon_{ST}} \alpha_r^{PS} \delta_{ST} \gamma_{ST} T \tag{6.16}$$

and

$$f_r^{PE} = -(y_{r/}y_{0r})^{\epsilon_{ST}} \alpha_r^{PE} \delta_r^{PE} R_{TC} T \tag{6.17}$$

with

$$R_{TC} = \frac{((< \mathrm{CO_2} >_1 / < \mathrm{CO_2} >_{\mathrm{pre}}) - 1)}{T_1} \tag{6.18}$$

with $T_1 = \tau_1 - tau_0$ Here, α_r^{PS} and α_r^{PE} are background property damage rates from tropical and extratropical storms respectively. The parameters δ_r^{PE} from Table ETS of (Anthoff & Tol, 2014a) are regional sensitivities to extratropical storms from climate change. Conversion from sensitivities to $< CO_2 >$ to sensitivities to global average temperature is done using the ratio R_{TC}.

Fractional GDP impacts due to people being killed by storms are

$$f_r^{KT} = f_r^{KS} + f_r^{KE} \tag{6.19}$$

where

$$f_r^{KS} = -(y_r/y_{0r})^{\epsilon_{KT}} V_K \alpha_r^{KS} \delta_{ST} \gamma_{ST} T \tag{6.20}$$

and

$$f_r^{KE} = -(y_{r/}y_{0r})^{\epsilon_{KT}} V_K 10^{-6} \beta_r^{KE} \delta_r^{PE} R_{TC} T \tag{6.21}$$

Here α_r^{KS} (TS 3) and β_r^{KE} (ETS 4) are background mortality rates from tropical and extratropical storms respectively. (FSU and CAM FUND 3.9 table entries for β_r^{KE} (ETS 4) are identical to five digits but were left as is in view of their resulting very small values of c_r^{KE} for both.) Including the factor of 10^{-6} in front of β_r^{KE} assumes that the entries for β_r^{KE} (ETS 4) are per million people, for consistency with other entries in the FUND 3.9 parameter tables. Here $\gamma_{ST} = 3$ and the values of δ_{ST} are from (ETS 3). The value of ϵ_{KT} is listed in Table S7 above.

6.8. **Computed Welfare.** The Euler-Lagrange equation with climate change impacts on productivity included as $a(1+\epsilon D)$ is the same as described above, but with C_0 replaced by $C_0 + \epsilon C_1$ Multiplying that Euler-Lagrange equation $F - 1 + \mu\theta M = \theta\dot{C}/C$ by θ gives

$$(F - 1 + \mu\theta M)C/\theta = \dot{C} \tag{6.22}$$

Expanding $Y = (a + \epsilon aD)(K_0 + \epsilon K_1)^\alpha L^\omega = a(1+\epsilon D)(1+\epsilon K_1/K_0)^\alpha K_0^\alpha L^\omega$ through first order in ϵ gives $Y = Y_0 + \epsilon Y_1$ where $Y_0 = aK_0^\alpha L^\omega$ and $Y_1 = Y_0(D + \alpha K_1/K_0)$. Expanding $C = Y/\alpha - rK_0 - rK_1 - \dot{K}_0 - \dot{K}_1 = C_0 + \epsilon C_1$ through first order in ϵ gives $C_0 = Y_0/\alpha - rK_0 - \dot{K}_0$ and, using $Y_0/K_0 = F_0$,

$$C_1 = Y_0 D/\alpha + F_0 K_1 - rK_1 - \dot{K}_1 \tag{6.23}$$

Thus,

$$\dot{C}_1 = \dot{Y}_0 D/\alpha + Y_0\dot{D}/\alpha + \dot{F}_0 K_1 + F_0\dot{K}_1 - r\dot{K}_1 - \ddot{K}_1 \tag{6.24}$$

On the right-hand side (rhs) of the Euler-Lagrange equation, expanding $F = Y/K = bK^{-\omega}L^\omega$ through first order in ϵ gives $F = F_0 + \epsilon F_1$, with

$$F_1 = F_0(D - \omega K_1/K_0) \tag{6.25}$$

The multiple $(F - 1 + \mu\theta M)$ of C/θ on the rhs of the Euler Lagrange equation, expanded through first order in ϵ, can be more compactly written as $(\theta R_0 + \epsilon F_1)$ where

$$R_0 = (F_0 - 1 + \theta\mu M)/\theta \tag{6.26}$$

Multiplying $(\theta R_0 + \epsilon F_1)$ by $(C_0 + \epsilon C_1)/\theta$ and again expanding through first order in θ, cancelling $R_0 C_0$ on the left-hand side and $\dot{C}_0$ on the rhs, and dividing by ϵ gives

$$R_0 C_1 + C_0 F_1/\theta = \dot{C}_1 \tag{6.27}$$

Inserting the above expressions for C_1, F_1, and $\dot{C}_1$ gives

$$\begin{aligned} &R_0(Y_0 D/\alpha + F_0 K_1 - rK_1 - \dot{K}_1) + C_0 F_0(D - \omega K_1/K_0)/\theta \\ &\quad = \dot{Y}_0 D/\alpha + Y_0\dot{D}/\alpha + \dot{F}_0 K_1 + F_0\dot{K}_1 - r\dot{K}_1 - \ddot{K}_1 \end{aligned} \tag{6.28}$$

Putting only the inhomogeneous terms on the rhs and collecting coefficients of $\dot{K}_1$, K_1, and D gives

$$\ddot{K}_1 + c_d\dot{K}_1 + c_k K_1 = c_p D + (Y_0/\alpha)\dot{D} \tag{6.29}$$

where $c_d = r(1 - R_0) - F_0$,

$$c_k = R_0(F_0 - r) - \dot{F}_0 - (\omega/\theta)F_0 C_0/K_0 \tag{6.30}$$

and

$$c_p = \dot{Y}_0/\alpha - R_0 Y_0/\alpha - F_0 C_0/\theta \tag{6.31}$$

For examples where the timescale for changes in the inhomogeneous terms on the rhs of this equation are long compared to the capitalization time $\bar{t}$, as allowed by the Green Deal and SRM policies parameters used for exploration of such examples, solution for K_1 is approximated as $K_1 = K_{10} + K_{11}$ where

$$K_{10} = (c_p D + (Y_0/\alpha)\dot{D})/c_k \tag{6.32}$$

and

$$K_{11} = -(\ddot{K}_{10} + c_d\dot{K}_{10})/c_k \tag{6.33}$$

Numerical solution of the full equation confirms that, like for the equation for K_0 as described above, the solutions "forget" about initial conditions and "do not anticipate" terminal boundary conditions except for exponential decay of such effects on a $\bar{t}$ timescale.

The reference empathy factor $f_{\mathrm{emp,ref}} = 0.05$ is based on non-military U.S. foreign aid to Africa in 2020 (Haines, 2023) as a fraction of U.S. GDP.

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Supplemental Information

This report serves as supplemental information for a companion report entitled “Regional Welfare Impacts from Options for Limiting Global Average Temperature” https://acdis.illinois.edu/sites/default/files/2025-03/Regional%20Welfare%20Supplement%202025%20%282%29.pdf by Chenghao Ding, Seungmi Kim, Clifford Singer, and Ryan Sriver. Other than inclusion of the present paragraph and a correction of Equation 6.4, this report is as written in January of 2024.