# **ACDIS** Research Report

### **Modeling Global and Regional Energy Futures**

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#### MODELING GLOBAL AND REGIONAL ENERGY FUTURES

T.S. Gopi Rethinaraj, Ph.D. Department of Nuclear, Plasma, and Radiological Engineering University of Illinois at Urbana-Champaign, 2005 Clifford E. Singer, Adviser

A rigorous econometric calibration of a model of energy consumption is presented using a comprehensive time series database on energy consumption and other socioeconomic indicators. The future of nuclear power in the evolving distribution of various energy sources is also examined. An important consideration for the long-term future of nuclear power concerns the rate of decline of the fraction of energy that comes from coal, which has historically declined on a global basis about linearly as a function of the cumulative use of coal. The use of fluid fossil fuels is also expected to eventually decline as the more readily extractable deposits are depleted. The investigation here is restricted to examining a comparatively simple model of the dynamics of competition between nuclear and other competing energy sources.

Using a defined tropical/temperate disaggregation of the world, region-specific modeling results are presented for population growth, GDP growth, energy use, and carbon use compatible with a gradual transition to energy sustainability. Results for the fractions of energy use from various sources by grouping nine commercial primary energy sources into pairs of competing fuel categories are presented in combination with the idea of experiential learning and resource depletion. Analysis based on this division provides estimates for future evolution of the fractional shares, annual use rates, cumulative use of individual energy sources, and the economic attractiveness of spent nuclear fuel reprocessing. This unified approach helps to conceptualize and understand the dynamics of evolution of importance of various energy resources over time. © 2005 by T.S. Gopi Rethinaraj. All rights reserved.

#### MODELING GLOBAL AND REGIONAL ENERGY FUTURES

BY

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#### DISSERTATION

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### Abstract

A rigorous econometric calibration of a model of energy consumption is presented using a comprehensive time series database on energy consumption and other socioeconomic indicators. The future of nuclear power in the evolving distribution of various energy sources is also examined. An important consideration for the long-term future of nuclear power concerns the rate of decline of the fraction of energy that comes from coal, which has historically declined on a global basis about linearly as a function of the cumulative use of coal. The use of fluid fossil fuels is also expected to eventually decline as the more readily extractable deposits are depleted. The investigation here is restricted to examining a comparatively simple model of the dynamics of competition between nuclear and other competing energy sources.

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### List of Abbreviations

 ${\bf GDP}\,$  Gross Domestic Product.

**PPP** Purchasing Power Parity.

**IEA** International Energy Agency.

**USEIA** United States Energy Information Administration.

**CDIAC** Carbon Dioxide Information Analysis Center.

**PERL** Practical Extraction Report Language.

**IAEA** International Atomic Energy Agency.

**IIASA** International Institute for Applied Systems Analysis.

WEC World Energy Council.

**MERGE** Model for Evaluating the Regional and Global Effects of GHG Reduction Policies.

**USEPA** United States Environmental Protection Agency.

**IPCC** Intergovernmental Panel on Climate Change.

**UNSD** United Nations Statistics Division.

**UNESD** United Nations Energy Statistics Database.

### Preface

The global and regional environmental impact of increasing energy consumption presents a major challenge for national energy policies and international relations. Explosive growth in literature on quantifying uncertainties in environmental response to atmospheric carbon increases in various climate models is only beginning to be complemented by similar rigor in quantifying uncertainties in econometric projections of the future evolution of the emission sources themselves. In particular, the systematic use and calibration of various parameters based on longer historical times series data on population, gross domestic product (GDP), primary energy consumption from various sources and carbon use remains a major challenge. Based on our recently compiled database of various socio-economic indicators, this thesis provides a rigorous econometric calibration of a model of energy consumption in a form suitable for follow-on work on quantifying uncertainties.

It is not uncommon in many long-term energy use projections to assume exponential or linear energy use growth as a terminal boundary condition. The idea of exponential energy use growth captured considerable attention in the 1950s and 1960s when energy use was growing rapidly alongside population growth. Recent trends in population growth, fertility changes and energy efficiency have led to a moderation of such views, but the idea of a high energy growth scenario in the future remains deeply ingrained. A more realistic and self-consistent approach takes into account the evolution of various factors that influence the distribution and consumption of energy resources in a way that is consistent with an eventual transition to long-term sustainability. The work reported here builds an integrated "look-ahead" model of how energy production, fossil fuel use, and gross domestic production respond to evolution of labor supply (population) increases, increases in production efficiency, and utilization of depletable energy resources.

Another motivation for this research is to examine the future of nuclear power in the evolving distribution of various energy sources. An important consideration for the long-term future of nuclear power concerns the rate of decline of the fraction of energy that comes from coal. Historically, this fraction has declined on a global basis about linearly as a function of the cumulative use of coal. The use of fluid fossil fuels must also eventually decline as the more readily extractable deposits are depleted. So we will look at distributions of different alternatives for the future of the fraction of non-fossil energy sources that would come from nuclear sources and renewable energy. A relevant question in this context is the question of managing spent fuel from current and planned nuclear reactors, because this is one of the biggest as yet incompletely resolved challenges for nuclear expansion. Nuclear power expansion also faces other challenges like public resistance (particularly in many developed countries), deregulation pressures, proliferation concerns, and safety concerns. However, these issues are beyond the scope of this dissertation research. Our interest here is restricted to examining a comparatively simple model of the dynamics of competition between nuclear and other competing energy sources, and finding the storage time for spent nuclear fuel before reprocessing.

It is sometimes suggested that a steep price increase of uranium will be needed before reprocessing becomes economically competitive. Such an increase may occur as land based resources deplete, because extraction of uranium from the oceans would involve processing large quantities of sea water, and is likely to be cost-intensive and perhaps environmentally controversial. Permanent disposal of spent fuel, as current experience in the United States suggests, is so difficult to arrange that some think reprocessing of spent fuel from new nuclear reactors needs earlier planning. In either case the question still arises of how long spent nuclear fuel should be kept in convectively cooled storage before reprocessing occurs. A quantitative answer to this question requires more than a costing model of individual facilities of national programs. This is because the economic attractiveness of reprocessing for commercial purposes is very sensitive to uranium prices, with uranium increasingly being traded on a global market. A complete assessment thus requires an estimate of the impact of cumulative global use of uranium on its cost compared to that of the reprocessed material from spent nuclear fuel. Here we estimate the latest time at which reuse of spent fuel from these reactors would occur, on the assumption that reprocessing does eventually become competitive with use of fresh uranium.

The organization of the thesis is as follows. Chapter 1 provides an overview of technical, economic, social and political complexities underlying the evolution of various energy systems. This chapter ends with five qualitative propositions that guide the quantitative work to follow. Chapter 2 provides a literature survey of relevant energy studies and modeling efforts. Both the advantages and shortcomings of extant approaches are described in order to further motivate the detailed approach described in subsequent chapters. Chapter 3 introduces the concept of purchasing power parity and the databases used for population and gross domestic product (GDP) time series. This chapter also describes the methodology used for estimation of a set of four parameters that are taken to be universal constants in the subsequent macroeconomic modeling. Chapter 4 then describes methods used for constructing time series for consumption of nine different energy sources for 220 geographic entities back to 1700. Chapter 5 describes a disaggregation of the global economy into two regions: "tropical" /developing and "temperate" /developed. It then presents results for modeling evolution of population growth rate and total population, GDP growth rate and total GDP, carbon intensity of energy production, energy production, and carbon use. Chapter 6 provides a detailed analysis of the fractions of energy from various fuel sources and combines this with the results from Chapter 5 to give a projection of future use of various energy sources. Chapter 7 describes a simple model of atmospheric response to carbon emissions and show results for projecting atmospheric carbon loading and changes in global average temperature based on carbon emissions formulas whose calibration is outlined in Chapter 5. Chapter 8 gives an example of how the results from this thesis may be used to inform policy decisions and provides overall conclusions and suggestions for further work.

Six appendices with technical details are also provided. Appendix A summarizes energy units and conversion factors used. Appendix B describes methods used for interpolation of data for years not covered in data sources, and the aggregation and disaggregation of historical geographic entities into a standard set of reporting units. Appendix C outlines how the equations used for macroeconomic modeling are converted from integral maximizations to Euler-Lagrange differential equations and then approximated by expansion in small parameters to produce formulas that retain a suitable richness of complexity but are nevertheless analytically computable. Appendix D presents equations for a simple model of atmospheric response to carbon emissions and power series expansion and numerical integration methods for solving these equations. Appendix E describes the derivation of equations used in the fuel fractions model. Appendix F describes statistical methods used for maximum likelihood calibrations and methods that can be used for sampling probability distributions for the calibrated parameters. Appendix G includes a brief introductory description of the Practical Extraction Report Language (PERL) coding used for database assembly and lists the time series data used in this thesis, the entire contents of which are provided in the attached read-only electronic compact disc (CD-ROM).

# Chapter 1 Energy Perspectives

This chapter provides a broad overview of some of the technical, economic, social and political complexities underlying the evolution of various energy systems. An understanding of all four of these themes is needed to guide projections of future energy use. We begin this chapter with a description of the historical context of energy utilization and its contemporary relevance. This will be followed by a brief analysis of international military conflicts where control of energy resources played a central role, as these conflicts had a major impact on the evolution of socioeconomic indicators in the twentieth century. The question of energy resource depletion will then be examined as background for modeling the effect of depletion on energy use. Finally, a summary of regional and global environmental problems associated with energy production and utilization will be presented. Particularly for the case of coal, environmental impact has played a significant role in shaping energy use in the past and is expected to continue to be important in the future.

#### 1.1 Energy, Growth, and Wellbeing

Energy is indispensable for economic growth and human wellbeing. For a greater part of their evolutionary history human beings needed energy primarily to fight hunger and dangerous predators. The "hunter-gatherer" lifestyle constrained the growth of human population until about 12,000 years ago, but subsequent invention of agriculture and domestication of animals made it possible to support a larger population and facilitated the emergence of civilizations in different parts of the world (McEvedy and Jones, 1979; Diamond, 1997). While agriculture helped liberate the human race from prehistoric wretchedness, it was the knowledge of extracting and utilizing energy resources that paved the road to industrialization and rapid economic growth. During the preindustrial era human labor and draft animals were the main sources of energy for doing mechanical work, and wood remained the primary source of thermal energy. With the introduction of mineral energy resources since industrialization, the world has experienced unprecedented growth in human population and economic output as well as improving conditions of human life in many countries. It is now almost universally recognized that access to affordable and reliable energy resources is an important factor for promoting and sustaining economic growth and productivity.

Britain started the use of coal to fuel its early industrial growth and was soon followed by the rest of Europe and North America. From the adoption of coal-powered steam engines since early industrialization through the First World War, the subsequent transition to petroleum-based fuels for transport, and the use of coal, natural gas, hydro power, uranium, and renewable energy sources for electricity generation, there has been a continually evolving trend in global and regional energy consumption. This trend has mostly been in the direction of increasing efficiency of energy use and shift toward energy resources with lesser carbon intensity. However, the initial phase of industrial development in North America, Western Europe and Japan was highly energy intensive. Energy became comparatively inexpensive, and many of the countries that had fossil fuel resources were underdeveloped and their domestic energy needs were modest. With technological improvements in the transport of fluid fossil fuels, a global energy market developed that allowed intensive energy use in countries with modest mineral resources of their own. Japan in particular, with practically no domestic energy resources, was a world leader in energy use growth. The availability of cheap energy in the global market in the 1950s and 1960s did precious little to encourage energy efficiency in developed countries. So the pace of economic growth during this period was accompanied by rapid growth in energy consumption. In the 1960s per capita energy use growth exceeded per capita economic growth significantly.

Since industrial-era economic growth was mostly characterized by energy intensive societies, there evolved a widespread perception that economic growth invariably entailed growth in energy consumption. The policy implication of this perception is that countries should strive for continued energy growth to achieve higher levels of economic growth. This conventional wisdom was consistent with empirical evidence until the politically inspired oil supply disruption in 1973 following the fourth Arab-Israeli war, which triggered a widespread panic about an impending energy crisis and stagnation in economic growth. A reported estimate of direct and indirect costs to the U.S. economy due to the oil shocks between 1973 and 1981 was \$2 trillion in 1987 dollars (Copulos, 1989). Energy price fluctuations resulted in short-term economic disruptions in oil importing countries, but developed countries demonstrated continued economic growth in the ensuing period even while decreasing their energy intensity of production. This was made possible due to improvement in energy efficiencies across the board in various sectors of the economy. The increase in economic growth was largely fueled by conservation. Interestingly, the role of energy efficiency in moderating energy demand without impacting economic growth was not a historically unique result of post-1973 enlightenment. The U.S. per capita gross domestic product (GDP) grew sixfold while per capita energy use only doubled between 1870 and 1950 when the economy was moving from agriculture to energy intensive industry (Freeman et al., 1974). Lovins' (1976) prognosis that improvements in energy efficiency and other innovations supporting a "soft energy path" would decouple economic growth and energy use now seems even more plausible. The Ford Energy Study, which was conducted in the wake of the first oil shock, also came to a similar conclusion, that economic growth is not entirely tied to energy growth (Freeman et al., 1974).

This is not to suggest that economic growth and energy use are completely disconnected. There is definitely certain threshold level of energy consumption needed for any country to achieve even modest levels of economic and industrial growth. Once a country reaches that level, there exists a "wide latitude" in the amount of energy needed to ensure decent living standards for its population (Rosa et al., 1988). Pasternak (2000) examined the relationship between energy use and individual wellbeing for various countries using per capita electricity consumption as a factor in the Human Development Index (HDI), which is a quantitative measure of individual wellbeing developed by the United Nations. Pasternak showed that the wellbeing index plateaued for most countries for an annual per capita electricity consumption of 4000 kWhr. Although many developing countries are still behind this threshold, Pasternak's observations suggest that these countries can achieve substantial increases in standard of living for much lesser per capita energy consumption than most of the current developed countries.

#### **1.2** Energy Resources and Security

Energy security is generally articulated in terms of a country's ability to function in times of war and to protecting the economy in a way deemed "normal and politically acceptable" (Willrich, 1976). Historically energy resources have been considered vital to national security. The role of mineral energy resources during major international conflicts in post-industrial history has further strengthened the perception about them as "strategic" resources. Beginning from the settlement of the Franco-Prussian war through the French neocolonial policy in Africa during the Cold War and the U.S.-led military interventions in the Middle East, control of energy resources is often perceived as a major influence on the evolution of international military conflicts involving major powers. This view remains deeply ingrained, even though international trade arrangements have proved more effective for ensuring energy supply security since the end of the Second World War, and global collective bargaining has the potential to provide greater price stability (Singer et al., 2005).

Although it improved Germany's energy efficiency somewhat by shifting control over high quality iron ore resources from France to Germany, the 1871 settlement of Franco-Prussian war had only marginal relevance in the context of coal supply security (Eckel, 1920). Germany annexed the coal and steel producing regions in Belgium and Luxembourg at the outset of the First World War (WWI). The motivations that led the main European countries into this protracted and disastrous war have more to do with imperial prestige and idiosyncrasies of individual rulers than control of coal resources (Kohn, 1999). However, the result left an enduring dent in the population of Europe. After WWI Germany agreed to a humiliating armistice agreement and had to ship a substantial part of its domestic coal produce as part of war reparations (Saward, 1919). Resentment over the Armistice and an era of protectionism vainly trying to protect domestic industries set the stage for the Second World War (WWII), which again produced an enduring reduction in Europe's population. With the establishment of the European Coal and Steel Community after WWII, coal resources ceased to be a bone of contention among western European countries. Given the increasing trend to shift away from intensive coal use and the absence of any perceived supply security risks in international coal trade, it is unlikely that a transnational military conflict will erupt for control of coal resources. Moreover, major coal consuming countries like the United States, Russia, China, and India have adequate coal deposits for domestic use (USEIA, 2004b). After WWII even the economies most seriously affected by the war recapitalized and within about a decade had recovered to a previously established pattern of exponential growth in per capita GDP (Barro and Sala-i-Martin, 2004). This recovery of developed countries devastated by war is reflective of an underlying stock of "human and social capital," which allows high labor productivity once destroyed physical capital is replaced.

The Soviet Union exercised effective control over the uranium resources in the Eastern Block. A result of France's military intervention in Africa was to gain preferential access to uranium resources of another country (Pederson, 2000). With the collapse of the Soviet Union and demilitarization and liberalization of uranium mining operations worldwide after the Cold War and France's embrace of the comprehensive nuclear test ban mollifying the concerns of uranium-rich Australia, use or threat of military interventions to secure uranium supplies for France's civilian nuclear power program has been rendered irrelevant. Uranium supply security may nevertheless still be relevant to India because of its opposition to the Nuclear Non-proliferation Treaty (NPT). The protagonists of India's nuclear program have indeed viewed reprocessing as effective insurance against perpetual dependence on outside sources for uranium (Gopalakrishnan, 2002). However, if Indian demand for uranium imports becomes large enough, based on historical experience with accessing fuel for its two reactors requiring enriched uranium, the international market is then likely to find a way to accommodate India's needs. Long term security of nuclear fuel supply is also a significant factor underlying spent fuel reprocessing policies of some countries. One of the original motivations of the French breeder reactor development was its concerns about future fuel supplies, although subsequent economic realities persuaded the French authorities to shelve their ambitious breeder program. Reprocessing of spent nuclear fuel provides "recessed military nuclear deterrence" capability for Japan notwithstanding its stated rationale of securing long-term security of nuclear fuel supply. For these reasons a limited amount of reprocessing of spent nuclear fuel is likely to persist even under conditions where this is substantially more expensive than acquisition of fresh uranium from the international market.

There has never been any reported disruption of natural gas supplies between countries, and exporters and importers rely on extensive transnational infrastructure for this trade. However, expansion of international gas trade through regions already embroiled in ethnic conflict could potentially exacerbate the security of natural gas networks in those regions. It also remains to be seen whether large scale liquefied natural gas (LNG) trade would present significant supply security risks in future. This concern is heightened by the fact that nearly half of global natural gas reserves are concentrated in Russia and Iran.

Development of hydropower resources by upper riparian (up-river) states continues to be a source of security concern for lower riparian states. This has little to do with the control of head waters as an energy source per se, but it may be relevant in the context of perceived threat to water and food security.

The only energy resource that continues to resonate prominently with supply security risks

and national security imperatives to this day is oil. The underlying reasons are geographical and historical: two thirds of proved global oil reserves are located in the Middle East, and access to petroleum resources influenced the outcome of crucial military campaigns during World War II. Access to petroleum resources was not only crucial to the victory of Allied forces in World War II, the initially primarily continental war evolved into a major maritime conflict in part due to Japan's lack of access to petroleum resources Yergin (1991). Japan's imperial ambitions were driven largely by the economic motivation to secure raw minerals and energy resources to support its industrial growth. Although capturing oil fields was not the sole war objective for Hitler, Germany's blitzkrieg campaigns were significantly driven by the need to secure oil supplies for fueling its war machine. Thus in the East and West control of oil became a central strategic focus of World War II.

The U.S. involvement in the Middle East began with President Roosevelt's deal with the Saudi royal family to protect its regime from internal and external threats as quid pro quo for privileged access to petroleum resources (Clare, 2002). This objective was later unambiguously expressed by President Carter's justifying military intervention to secure the "free flow" of oil from the region. The United States' position regarding oil requirements for military purposes remained secure through 1950s, but it became vulnerable in the 1970s due to rising imports of petroleum products. In 1971 nearly half of U.S. DoD's oil requirements came from imports, and the defense planners feared that imports could increase further (Gorton and Meador, 1977). While current U.S. oil imports are more diversified than those of European countries (UNSD, 2005), the United States is likely to import a greater fraction of its oil demand in the future from the Middle East. Lugar and Woolsey (1999) cite this "unwelcome dependence" on oil as the principal reason for continued U.S. military presence in the region. This historical experience continues to influence the political and military mindset of the generation that emerged from World War II (Singer, 2004). In addition, memories of the 1973 Arab oil embargo and the geographical concentration of these resources in a geopolitically unstable region continues to feed the perception that "unfriendly" states will use oil as a political weapon. There have been over fifteen significant oil supply disruptions lasting from one month to four years during the past 60 years (USEIA, 2005a). However, in the post-WWI era only the 1973 embargo and a subsequent additional increase in oil prices accompanying the 1980-86 Iran-Iraq War appear to have helped trigger a major global economic disruption. By comparison the impact on the global economy of the 1990 and 2003 military actions against Iraq had been comparatively modest at least through mid-2005.

#### **1.3** Depletion of Energy Resources

Concern about energy resource depletion is not a recent phenomenon. As early as 400 BC Plato had raised the prospect of losing the woods to ship-building and other activities in ancient Greece (Goldemberg, 1996). Similar concerns surfaced in eighteenth century England following large scale exploitation of the forests for firewood (Hatcher, 1984). A hundred years later when the Britons began using coal in substantial quantities, Jevons (1866) warned his countrymen that the seemingly inexhaustible coal reserves would be exhausted within a century at prevailing consumption rates. In a controversial assessment, Hubbert (1956) predicted, quite accurately, that U.S. petroleum production would peak in the early 1970s. Hubbert-type analysis was extended across the board to predict the depletion time horizons for various energy sources. The publication of *Limits to Growth*, popularized by the widespread use of "bucket models" (Meadows, 1972). Such energy resource assessments have traditionally focused on immediate to short-term availability of various fuels. Longer-term assessments are more useful in the context of resource depletion, penetration of newer energy sources, and climate change issues. Resource assessments for a time horizon of a hundred years or more is beyond the scope of traditional reserve analysis. Such questions have not been reflected in reserves exploration by energy companies because their interests in this context are mostly related to investment decisions, which does not extend beyond 10-15 years.

The current debate concerning energy resource depletion is mostly centered around oil. Here the terms "reserves" and "resources" are used loosely in different contexts, which is a major source of confusion. It is common a practice to use the ratio of reserves to annual production rate as an indicator for estimating the time for depletion. McCabe (1998) suggests this method is "seriously flawed" because such estimates have been proved incorrect many times in the past, and suggests a much larger resource base for petroleum. Nevertheless, there has been an increasing expectation among petroleum industry experts that global oil production will peak before 2010 (Deffeyes, 2001). Accounting for natural gas and coal, however, fossil energy resources are sufficient to fuel the world economy through the twenty-first century even in the case of significant growth in global energy demand. The global fossil energy resource base is large and is estimated at around 5000 Gtoe (gigatonne oil equivalent) with additional occurrences five times this number (Rogner, 1997). Hence overall fossil resource depletion per se should not greatly constrain the use of fossil energy resources for a century or more, even as depletion of fluid fossil fuels shifts the emphasis on what types of fossil fuel are used. However, environmental considerations may constrain fossil fuel use to below current rates long before resource scarcity becomes the limiting factor.

Resource depletion is not going to be relevant for uranium on a similar time scale of a century or more. With a crustal abundance growing exponentially with ore grade down to a crustal abundance of 2 ppmv (parts per million by volume), the uranium resource base in very large. Uranium can also be extracted from other than conventional ores, including phosphate production byproduct and even sea water (Edmonds and Reilly, 1985). Deffeyes and MacGregor (1980) argue that supply of uranium will not be a limiting factor for nuclear growth, implying that early concerns about shortage of uranium supplies as a justification for breeder reactors were misguided. This thesis will elucidate how conventional uranium reserves alone can fuel nuclear power growth through the twentieth century. Singer (1997) has estimated a global uranium availability of 19 Mtonne for extraction costs up to \$130 per kg at 1995 prices. Extraction of uranium from sea water and reprocessing can in prin-

ciple extend the time essentially indefinitely, provided the extraction rate is not so large that surface waters are depleted much faster than they are replenished by upwelling. There is also significant scope for expansion of hydro power. Current utilization of hydro power is less than ten percent of the "technically feasible" limits (UNSD, 2005), but such a large expansion globally is likely to be difficult due to environmental concerns. In practice conventional hydro power capacity has saturated well below the "technically feasible" limit in the most developed countries, although some expansion of smaller scale hydro power projects is possible under appropriate market conditions.

#### **1.4** Energy and the Environment

In a broader social and philosophical context, Hardin (1968) examined the problem of environmental pollution as an example of the "tragedy of the commons" and advocated strict laws to prevent earth's atmosphere from becoming a global "cesspool" as result of various human activities. Hardin's essay became a classic in the scientific environmental movement literature, and served as a "powerful metaphor" for those who were uncomfortable with the idea of sustaining economic growth at the cost of human health and environmental degradation. The changing composition of the earth's atmosphere as a result of human activities is an illustration of the problem of environmental global commons. Energy-related activities are a very significant source of all major classes of environmental pollutants, and there is no known energy system that is totally free from "environmental liabilities" (Holdren, 1987). Fossil fuel combustion releases carbon dioxide, sulfur dioxide, nitrogen oxides, lead, mercury and particulate matter into the atmosphere. The health and environmental effects of these pollutants are well known and documented (USEPA, 2005). Nuclear power plants produce their share of environmental problems. These are less significant compared to the long term effects of unconstrained burning of fossil fuels, provided of course that expanding availability of nuclear technology does not lead to large-scale nuclear war. Although nuclear wastes are highly radioactive and toxic, their volume is around six orders of magnitude less than fossil fuel wastes producing the same amount of energy (Izrael, 1987).

At the local level urban air pollution is a leading public health concern. This problem is largely due to inefficient burning of fossil fuels and absence of adequate environmental regulations. Concerns about air quality and acid rain in the 1960s and 1970s led the developed countries to enforce strict environmental standards for various energy systems (Rubin, 2001). These measures have resulted in a substantial decline of environmental pollutants and better urban air quality in the developed world. Urban air pollution is currently a major health and environmental problem for developing countries with rising energy consumption. China and India have introduced regulations targeting motor vehicle use and coal plants, but huge energy subsidies in these countries are impediments to effective regulation of environmental standards comparable to developed countries (USEIA, 2004b). Air pollution is also manifested in a much larger regional scale such as the Asian Brown Cloud. Composed of a mix of ash, soot, sulfates, nitrates and aerosols, this brown patch has formed in the atmosphere mostly as a result of current pattern of energy production in developing countries. An international consortium of scientists that investigated this problem in the Indian Ocean region reports that similar clouds exist over East Asia, South America and Africa. The observations of this study attributed to emissions from South and Southeast Asian countries the air quality degradation over an area of around 10 million square kilometers, which threatens to grow into a "global plume" in the absence of an international regulation (Lelieveld et al., 2001). Besides putting millions of people in Asia at risk for various respiratory diseases and other ailments, the brown haze phenomenon is expected to alter regional temperatures, rainfall and agricultural patterns (Ramanathan et al., 2001).

While the problem of local air pollution receives considerable attention from governments in different countries, addressing the global environmental problem due to energy consumption remains most problematic because of the long time scales involved, uncertainties in climate models, and the lack of a global political authority for restraining overall greenhouse gas emissions. This is indeed a problem of global magnitude. In fact, the Cophenhagen Consensus, a group addressing global issues, listed climate change as a top issue along with other pressing issues (Lomborg, 2004). However, assessing the magnitude of the global air pollution problem is also beset by uncertainties in science and economics. The foremost of these concern the effects of release into the atmosphere of carbon dioxide—the principal by-product of fossil fuel combustion. This is because increasing concentrations of carbon dioxide may combine with other greenhouse gases in the atmosphere to warm the earth and alter global climate in a significant way.

There is a general consensus that in the absence of any effort to reduce greenhouse gas emissions a doubling of atmospheric a content could increase the earth's surface temperature from between 1.4 degrees Celsius up to 5.8 degrees Celsius (Houghton et al., 2001). The atmospheric level of carbon dioxide rose from the preindustrial value of around 280 ppmv (of dry air) to around 375 ppmv in 2003 (Keeling and Whorf, 2004), following the release of an estimated 283 billion tons (Gtonne) of carbon into the earth's atmosphere since the industrial age due to fossil fuel combustion (Marland et al., 2003). Paleo-climate records obtained from analysis of ice cores indicate that the levels of carbon dioxide in earth's atmosphere have varied, albeit over very long time intervals, between glacial and inter-glacial periods from a minimum of around 180 ppmv to a maximum of 280 ppmv (Petit et al., 1999). However, those long term climate changes were driven largely as a result of earth's orbital geometry and the amount of solar radiation received by earth, and one can notice distinct trends cyclical trends ranging from 23,000 years, 42,000 years and 100,000 years over the 400,000 year climate record before industrialization (Hays et al., 1976; Shackleton, 2000). Climate models and paleoclimate data suggest that continuation of current emissions levels through this century will raise the atmospheric carbon concentration level to a sufficient level—550 ppmv or more—to potentially trigger "global warming" comparable in magnitude but opposite in sign to the "global cooling" of the last Ice Age (Hoffert and Covey, 1992). The current level of carbon dioxide in the atmosphere is significantly greater than the upper bound of past natural variability for hundreds of thousands of years. The difference between the changes in past earth climate and predicted global warming trends is stark: while changes in earth's climate correlated with carbon dioxide levels in the atmosphere in the past over long time scales, current anthropogenically induced changes in atmospheric concentration of carbon dioxide threatens to drive climate change in a much shorter time scale.

The accumulation of scientific evidence of increasing atmospheric concentration of greenhouse gases and its implications for earth's climate led to an international policy response in 1992 at the first Earth Summit held in Rio de Janerio with the adoption of the United Nations Framework Convention on Climate Change (UNFCC, 1992). The convention stressed the need to stabilize "greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system." Subsequent multilateral negotiations led to the signing of the Kyoto Protocol in 1997, which urged the industrialized countries to reduce their overall greenhouse gas emissions by at least five percent below 1990 levels over the 2008 to 2012 period (UNFCC, 1997). Despite nearly universal acceptance of the objectives outlined in the Kyoto Protocol, there is still considerable scientific uncertainty about what the appropriate level of atmospheric greenhouse gas levels should ultimately be. According to the World Energy Outlook 2004, in the absence of international efforts the energy-related carbon emissions will grow faster than energy use and will be sixty percent higher in the next twenty-five years than now (IEA, 2004). Although such high atmospheric concentrations of carbon dioxide are more likely to "noticeably" change the earth's climate in future, substantial uncertainties remain about the magnitude, timing and regional impacts of climate change, and the associated socio-economic consequences. However, scientific opinion remains unequivocal in its assessment about the existence of anthropogenic climate change and attributes the observed increases in earth's temperature over the past fifty years to elevated levels of greenhouse gases (Houghton et al., 2001). Representing over eighty per cent of all anthropogenic greenhouse gas emissions, the production and
use of fossil fuels remains the principal source of concern. Hence future carbon stabilization depends largely on the evolution of the global energy system.

Five postulates important for the remainder of the present work and its future elaboration can be drawn from this overview. First, there are no obvious prospects for a recurrence of the kind of devastating global international struggles over control of energy resources that punctuated the first half of the twentieth century. While such events cannot be precluded, it seems interesting to model the future evolution of the global economy and its energy use under the assumption that nuclear deterrence and trade globalization will lead to smoother evolution of population and economic production into the twenty-first century than occurred during the first half of the twentieth. Second, it appears reasonable to treat large economic aggregates (like developing versus developed countries) as gradually evolving in a state not drastically far from an equilibrium that is rapidly re-established (on a time scale of a decade or less) after major perturbations. Third, concerns about the environmental effects of burning coal and exporters' self-imposed restrictions on oil production are likely to continue for the readily foreseeable future. This suggests that recent reductions of the carbon intensity of energy production are likely to continue, at least until the regional effects of coal pollution have been more fully internalized into market decisions. Fourth, any pause in reductions in the carbon intensity of energy production resulting from market internalization of regional pollution costs is likely to be temporary, until the effects of global warming become serious enough to precipitate effective global cooperation on reduction of carbon emissions. Insight into when such a pause might occur and how long it might continue can be gained from a from technical analysis and careful modeling and extrapolation of trends observed in historical data. Fifth, there is reasonably likely to be a significant increase well into the twenty-first century in the global use of nuclear energy. Moreover, it is likely to be possible to fuel this increase for a considerable amount of time without uranium resource depletion driving a need for reprocessing the majority of spent nuclear fuel. This raises the question of how long spent fuel might stay in interim storage before reprocessing, if indeed reprocessing ever

becomes economically competitive with extraction of uranium from seawater. The challenge addressed in this thesis, necessarily only in part due to the complexity of the problem, is to develop an econometrically calibrated energy use model that provides insight into questions about the magnitude of anthropogenic climate change and the likely interim storage time for most spent fuel.

# Chapter 2 Review of Energy Modeling

This chapter provides a literature survey of energy studies and modeling relevant to the work reported in this thesis. The historical origins of energy modeling are reviewed, followed by a brief description of energy studies conducted in the wake of the 1973 energy "crisis." Integrated climate assessment models will also be reviewed because of their treatment of long time horizons. Energy models devoted exclusively for projecting nuclear energy futures will be reviewed at the end of this chapter.

# 2.1 Origins of Energy Modeling

The literature on energy modeling by national and international energy agencies, think tanks, and academic institutions has become enormous, especially since 1973 when the oil shock made energy a major focus of government as well as public attention. The original motivation for energy modeling initiatives was primarily to explore ways of reducing the dependence on imported oil and to assess the effect of various energy policies on the economy (Messner, 1997). Eventually energy modeling began to cover a wider range of regional and global environmental issues related to energy systems.

One of the well known energy studies conducted in the early 1970s with wide professional representation was sponsored by the Ford Foundation (Freeman et al., 1974). This study focused on three scenarios—historical growth, technical fix, and zero energy growth—with assumptions of steady growth of GDP. The aim was to project energy demand in the United States through the year 2000 in order to examine the consequences of various energy policy options. The historical growth scenario projected a demand of around 187 quads (197 EJ) in 2000, assuming that the historical growth rates of energy use would continue through the projected years. (Here a quad is  $10^{15}$  British Thermal Units and an Exa Joule or EJ is  $10^{18}$  Joules). The "technical fix" scenario produced a moderated demand for energy—124 quads—through technical means like improved efficiencies and conservation efforts. The "zero energy growth" scenario captured the energy demand under assumptions of increased energy efficiencies and shift away from energy intensive industrial growth. Of these three scenarios, the historical growth scenario is of particular interest because of its resemblance to typical government and industry projections. However, the report's suggestion that historic growth rates in annual energy use is not a necessary condition for maintaining a dynamic economy and its recommendation of significantly reducing the annual energy growth from its historical growth levels proved to be contentious.

The U.S. Department of Energy conducted the "Project Independence" study following the above-mentioned 1973 events in order to assess the country's energy problems (FEA, 1974). The study was based on interregional equilibrium approach using the Project Independence Energy System (PIES) computer simulation model. PIES examined the dynamics of future energy supplies and demand, and addressed the issue of oil imports and the impact of domestic price regulation. For running simulations to test various policy options provided as inputs externally this study divided the country into nine regions, which continues even now in U.S. EIA's disaggregation. The government was motivated to conduct this study as a wake up call in the context of a bleak energy situation: a declining trend in domestic oil production, absence of new gas finds, and increasing regulatory problems faced by the nuclear industry. The report did not provide any recommendations along the lines of the Ford study, but it did provide a factual factual basis and analytical framework for "focusing the debate on difficult choices and trade offs to select a national energy policy" (FEA, 1974). While the above two studies looked ahead 20 to 25 years into the future, there were other energy studies that looked at short and medium term prospects to address specific questions concerning the oil industry and electric utilities in a regulated environment (Greenberger et al., 1983). Despite superficial differences most of the studies addressed a fundamentally similar set of questions like those in the Project Independence that were common in the aftermath of the 1973 crisis. Moreover, a majority of the studies were specific to the U.S. energy problems. The MIT study is one of the few studies that addressed some energy issues in an international context (Wilson, 1977). In most analyses GDP growth is one of the primary determinants of energy demand, an assumption that works satisfactorily for the period between 1950 and 1970 but not for the post-1973 years. This reality has brought in the idea of price induced conservation in energy models, which can be examined from both an engineering and macroeconomic perspective.

It is a common engineering bias to meet a goal of conservation by engineering methods, while scenario building is a common approach used to test the consequence of various policies (Greenberger et al., 1983). The next section describes these two approaches from the energy modeling literature. A common feature of many of those models is the energy demand projected for 2000 rarely close to actual consumption in 2000. Assumptions of exponential energy use growth are a commonplace source of this discrepancy.

# 2.2 Classification of Energy Models

There are two broad classes of energy models in the literature: "bottom-up" and "topdown" models. The former is based on engineering or technology-oriented approach, while the latter is derived from macroeconomic models with energy as a sub-sector in the overall economy. There is also a third class of "combined" energy models that conjoin the features of the two above models. Pictorial representations of energy models based on these ideas are given in Figs 2.1, 2.2 and 2.3. The most popular and widely used bottom-up approach is the MARKAL (Market Allocation) family of energy models. Developed at the International Energy Agency (IEA), MARKAL is a comprehensive energy technology assessment

### MARKAL Model



Figure 2.1: MARKAL and ETA-MACRO models

model that simulates various components of any geographically defined region's energy system (Goldstein and Greening, 1999; IEA, 2005b). It can also be used for energy analysis and optimization on a smaller scale. MARKAL has evolved over the years to meet the evolving needs of the energy and environmental policy community worldwide. This framework is widely used for assessing environmental impacts like acid rain and climate change at regional and national levels. The basic components in a MARKAL model are specific types of energy and emission control technologies, from which the model finds the least expensive way to meet a given set of energy demands. Thus the model results do not produce any predictions of the future, but only ways to efficiently achieve a desired result or future (Rosen and Glasser, 1992). MARKAL models are implemented using a popular open architecture called General Algebraic Modeling System (GAMS), which is specifically designed for various types of optimization problems relevant to energy systems (GAMS, 2005). Bottom-up models generally adopt a disaggregated approach to modeling energy use, and focus on specific steps and processes that provide energy services rather than energy per se. Hence bottom-up analysis is commonly used by government agencies and electric utilities for projecting short-term energy use.

Top-down energy models were originally designed to describe the market responses to changing energy prices under various levels of macroeconomic growth. Following macroeconomic models such as general equilibrium models and neoclassical growth models, energy is treated in this thesis as a separate sector from the rest of the economy. The energy model used in this thesis, and derived in Appendix C, draws upon this feature of top-down models. A well known example of the top-down model is ETA-MACRO, which is a combination of energy technology assessment model and macroeconomic growth model providing for substitution between labor, capital, and energy. The ETA-MACRO model is particularly suited to assess energy economy interactions, conservation, inter-fuel substitution, and new energy technologies (Manne, 1992). Top-down models provide a macroeconomic approach to modeling energy-economy interactions and the costs of changing them, which provides an analytic framework for integrated climate assessment models such as MERGE and DICE (Manne et al., 1995; Nordhaus, 1992, 1993).

Models that combine bottom-up and top-down features are also common in the literature and are described as "judgmental-eclectic" models (Franssen, 1978). The MARKAL-MACRO family of models are of this type, combining the energy systems model MARKAL with the economy module of ETA-MACRO. Developed at the Brookhaven National Lab, the MARKAL-MACRO model provides a "flexible" modeling approach for assessing policies relating to environmental impacts of energy use. The model uses non-linear optimization techniques for solving the problem of maximized discounted utility of consumption to choose among alternative time paths for energy costs, consumption and investment (USDOE, 1996). The model provides a "bottom-up engineering" and "top-down macroeconomic" approach to estimate the least cost energy system solutions to support planning and policy decisions, especially the implications for various energy or environmental strategies and policies. One of the advantages of this model is the "transparency and traceability" of the results.

### 2.3 Short-Term Energy Projections

Projecting future energy use is one of the principal motivations underlying energy models. Energy projections made by various modeling groups and government agencies in the United States during the 1970s for the year 2000 were significantly off the actual consumption in 2000 (Babcok et al., 1978). Although energy agencies significantly revised their estimates in response to this observation, the trend to rely on linear growth rates for fairly long periods of time propagates through even in the current projections (USEIA, 2004b). Here we describe two popular annual publications containing projections used in the government and other agencies and also among modelers, both of which project continuing increases in energy use. The International Energy Agency (IEA) regularly publishes its projections using various in-house modeling tools similar to the ones described above. IEA has been using its World Energy Model (WEM) since the early 1990s to generate future energy projections that are published in its annual flagship publication, *World Energy Outlook* (WEO). IEA's projections are based on a "partial equilibrium model" that is updated every year in terms of regional and sectoral disaggregation (IEA, 2004). This publication provides projections through the year 2030 for the world disaggregated into nineteen regions, including detailed analyses of global energy prospects, environmental impact of energy use, and effects of policy actions and technological changes. IEA's projections are sensitive to demographics, economic growth and international fossil fuel prices that are provided exogenously. The underlying assumptions in IEA's recent projections published in 2004 include a worldwide annual GDP growth of 3.2% through the year 2030, population growth rate of 1%, and international oil prices assumed to return by 2030 back to the 2000 levels (\$22 per barrel adjusted for inflation to year 2000 prices).

The U.S. EIA also regularly publishes various projections for future national and international energy demand covering various time horizons. The Short-Term Integrated Forecasting System (STIFS) is used for "near-term" forecasts and assessment of price elasticities of crude oil demand (USEIA, 2004a). The U.S. EIA has also a National Energy Modeling System (NEMS) for projecting domestic energy balances under conditions of low, intermediate, and high world oil prices (USEIA, 2003). The World Energy Projection System (WEPS) was for a long time EIA's modeling tool for international energy use projections. More recently the EIA has introduced a new international energy modeling tool, System for the Analysis of Global Energy markets (SAGE), for projecting energy use (USEIA, 2004b). SAGE is an integrated set of models that provide the basis for estimating energy consumption for economic and demographic projections. Projections of energy consumption are computed at five year intervals through the year 2025 estimated on the basis of each region's existing energy use patterns, existing energy infrastructure, availability of new technologies, as well as new sources of primary energy supply (USEIA, 2004b). These projections generally indicate a constant linear increase in energy use through the projected years.

# 2.4 Long-Term Energy Projections

Earlier energy models and studies were developed mainly for short-medium term projections, typically 20 to 30 years. However, there has been a tremendous growth in energy economic models in response to the emergence of climate change as an important element in future energy policy. There is adequate motivation for developing longer term energy models to look sufficiently far into the future to assess the effects of potential climate change and resource depletion. The thermal inertia of oceans and the long residence time for carbon in the atmosphere required climate assessment models to look at least a hundred years into the future, although this presents significant challenges for model formulation and computational manageability. Energy resource depletion is also likely to become particularly noticeable in a comparable time frame. In this section we will review some prominent long-term energy studies and models in the literature.

#### 2.4.1 IIASA and IPCC Models

The energy optimization model and studies developed at the International Institute for Applied Systems Analysis (IIASA) have a significant influence in literature, given the authors' role in the community associated with the International Panel on Climate Change (IPCC). A 1981 IIASA study is one of the earliest energy studies to take a long-term view of the energy problem from an international perspective (Haefele et al., 1981). Disaggregating into seven convenient geographical regions and covering a 50-year period, the report explored various future alternatives of the global energy system based on "depletable" fossil fuels and "sustainable" nonfossil fuels. The conclusion of this study was that fifty years is "too short" to see significant shift from fossil fuels. The report also examined future scenarios characterized by high and low energy use growths, and three variants of these scenarios that assumed nuclear moratorium, increased nuclear growth, and increased energy efficiency aimed at lowering energy consumption. The main feature of the modeling approach adopted in this study is its use of exogenously supplied assumptions about population and economic growth, and varying the energy demand with respect to changes in energy prices and income levels. Even though the report was a pioneering attempt in setting up the broader social, economic and technological framework for addressing the question of long-term energy futures, the conclusions depend on assumptions about population and economic growth that were not systematically calibrated against historical time series. This is readily understandable, since the amount of useful time series data available in 1981 was much less than it is today. The current IIASA model consists of a energy-economy model and energy technology optimization called MESSAGE III (Nakicenovic and Swart, 2000). A more elaborate sequel to the 1981 IIASA study was published in 1998 to integrate "near-term" and "long-term" energy policies through the year 2100 (Nakicenovic et al., 1998). This report used state-of-the-art bottom-up and top-down modeling tools to examine alternative energy futures described by six different scenarios, and their implications for eleven geographical regions. This regional classification was later adopted in IPCC assessment reports. The 1998 IIASA projections for future energy demand and carbon emissions were based on the assumption of a ten to fifteen fold increase in GDP and 25-fold increase in energy use globally by the year 2100. These scenarios use population growth and per capita economic growth as "exogenous" variables, i.e., which are provided externally. The energy demand and its composition for each scenario come from a combination of historical data and a "scenarios generator" model, which then generates various future energy paths. The study underscored the dominance of fossil fuels in all its scenarios through the year 2020, after which six different scenarios emerge according to given specifications. Underlying this study is a dominant strain in the scientific environmental movement that future energy policy must build a "prosperous and equitable future" that places importance for environmental protection and international equity by developing energy resources that are "efficient, cleaner and less obtrusive." Hence it is not surprising that the future scenarios described in the study are driven by local ecological and global environmental concerns as well as the modelers' motivation to bridge the gap between



Figure 2.2: MESSAGE Model

poor and rich countries. The recent IIASA reports still do not have the models calibrated against historical data. This is apparently a reflection of the IIASA studies' primary interest in self-consistent scenario-based analysis rather than using historical data to help quantify the probability of various scenarios being realized. As such, these models provide a good description of the technological issues relevant to long term energy futures. The IIASA models and energy studies also provided significant input to the IPCC's Special Report on Emissions Scenarios discussed below.

The IPCC's year 2000 special report was the culmination of effort of six major modeling groups with extensive top-down and bottom-up analyses to identify the driving forces of global future greenhouse gas emission paths (Nakicenovic and Swart, 2000). This report produced forty different scenarios to span possible evolutions of demographic change, economic development, and technological change. In all these scenarios the world regions are projected to be substantially more affluent than under current economic conditions. Even the lowest range of global GDP projections for the year 2100 is ten times bigger than current GDP, while the highest estimate is twenty six times bigger (Nakicenovic and Swart, 2000). Income differences among different regions in the world is assumed to be much narrower than now in most of the scenarios. In fact two scenario families, A1 and B1, specifically project GDP levels so as to narrow the current income gap between the rich and poor countries. Consequently, many of the scenarios used in the report have substantially higher estimates for future carbon emissions than the econometrically calibrated transition to sustainability developed in this thesis below.

#### 2.4.2 Utility Optimization Models

MERGE is an influential general equilibrium long-term energy model, which came out of a collaborative effort of the EPRI-Stanford energy modeling group (Manne et al., 1995). This model has considerably evolved over the years, after originally being developed for estimating the costs of limiting carbon emissions through the year 2100 and evaluating various climate



Figure 2.3: MERGE/Global 2100 Model

change mitigation policies. Earlier versions of MERGE suggested that even moderate steps toward cutting carbon emissions would be expensive and thus supported a "wait and see" attitude, while the current version of MERGE concludes that a gradual shift away from fossil fuels would not be an expensive option (Manne and Richels, 2005). The current model has the world divided into nine geopolitical regions. MERGE provides a bottom-up view of the energy system and the carbon emissions, and a top-down view of the economy with inputs from labor, capital, electric and non-electric energy. As cheaper sources of fossil fuels are depleted, their prices increase to make non-fossil sources more competitive. Benchmarked against the year 1990, the model's original projections covered ten-year time intervals from 2000 to 2100 and included constraints in the form of resource scarcity and environmental restrictions on the use of fossil fuels. The current version of MERGE includes projections from 2000 through 2200.

Long-term economic growth in MERGE is treated using a model where savings and investment for each region are determined by maximizing the discounted utility of consumption. In MERGE savings and investment time paths are significantly influenced by the choice of social discount rate. The macroeconomic modeling approach used in this thesis follows a similar utility optimization method described in Appendix C. Unlike the model described in Appendix C, carbon emissions in the MERGE model are provided "exogenously," including details about how individual regions would trade these rights on international markets (Manne, 1992). A key feature of future energy demand in the MERGE model is the role of energy efficiency improvements and price-induced substitution between energy and other inputs that lower energy consumption. Assumptions made about population growth are consistent with the projections based on more sophisticated demographic models. However, assumptions of nine fold increases of global GDP from current level and convergence between the per capita incomes in the OECD countries and those in the rest of the world are incompatible with the projections based on the data-intensive approach outlined in this thesis. Nevertheless, the MERGE model provides a rigorous quantitative theoretical framework for analyzing long-term energy futures and climate impact assessment.

Nordhaus (1992, 1993) provides an alternative integrated assessment model called DICE based on a rigorous utility optimization approach. While most of the assessment models initially focused on estimating the costs of various greenhouse gas reduction policies, DICE provided analytical framework for working out estimates of various damages that can be averted as a result of climate mitigation policies. In other words, DICE estimates the benefits of greenhouse reduction policies. The model provides a dynamic optimization framework for estimating the "optimal path" for greenhouse reduction in order to slow potential climate change. The resulting optimal path is also interpreted as "competitive market equilibrium" that incorporates the social costs of carbon emissions (Nordhaus, 1994). In this model population growth and technological change are "exogenous," and capital accumulation is found by utility optimization. DICE treats the entire world as a single economic unit and examines the optimal path in effect for a collection of equally endowed and empowered individuals. The individual's choice whether to consume or invest is represented by maximization of an objective function, which is the discounted sum of utilities of per capita consumption. The maximization is subject to standard economic constraints as well as emissions-specific constraints. The model has also been extend to analyze a 300 year time horizon (Nordhaus and Boyer, 2000).

### 2.5 Nuclear Energy Models

Post-1973 energy modeling efforts devoted significant attention for projecting various scenarios for nuclear energy growth. Until the 1960s most nuclear growth projections were modest. However, projections made between mid 1960s and mid 1970s increased nuclear growth prospects enormously. The U.S. Atomic Energy Commission projected 1500 GWe of installed capacity in the United States by 2000 (Franssen, 1978). An MIT energy study, which was conducted at a time when most countries embarked on major expansion in nuclear power, projected significant nuclear growth in the United States and rest of the world (Wilson, 1977). The study's "maximum likely" global projection for the year 2000 was 1772 GWe of installed capacity, which is four times the current global capacity. Its "minimum likely" estimate was 913GWe, which is still over twice the current capacity. There were other studies that looked into the prospects of nuclear growth in the United States and rest of the world (Brooks et al., 1979). Many of the early studies that looked into the future of nuclear growth were motivated to examine the economic viability of breeder reactors, since the projections made by various government agencies and the nuclear industry suggested an early exhaustion of low-cost uranium resources.

Following widespread global warming concerns in the 1990s, the Global Nuclear Vision Project at Los Alamos looked at the tradeoff between fossil and nuclear energy sources (Krakowski, 1997a,b). The Los Alamos study used a scenario building approach to a examine a range of long-term nuclear energy futures from once cycle of reprocessing to large scale induction of breeder reactors. External drivers of the models include population growth, economic growth, trade policies, and technological advances, while the internal drivers rest on the front end and back end costs of the fuel cycle. The study projected strong growth scenarios using assumptions of high carbon tax and decrease in end use efficiency of fossil fuels (Krakowski, 1997a). These conditions obviously lead to favorable economics for nuclear growth.

A more recent study conducted at MIT looked at nuclear power as a "significant option" for reducing greenhouse gas emissions and meeting growing energy demands. To explore these objectives and attain a specific goal of reducing 1.8 billion metric tons (1.8 Gtonne) of carbon that would otherwise be emitted annually, the study projected a global growth scenario of threefold increase in installed nuclear capacity (Deutch et al., 2003). The question of nuclear power economics was with more of an eye on existing market conditions addressed by a University of Chicago study that did a comparison of the levelized cost of electricity (the cost for covering capital and operation costs) for power plants running on coal, gas and nuclear fuel (Tolley and Jones, 2004). Based on a detailed financial model, the report estimated that this cost would be in the range of \$47 to \$71 per MWhr for nuclear in the absence of government subsidies, and \$31 to \$46 per MWhr if early costs of restarting nuclear plant construction are given plausible subsidies. The cost range per MWhr without any financial subsidies for coal-fired plants was estimated by the study between \$33 to \$41, and \$35 to \$45 for gas-fired plants. However, policies to significantly reduce carbon emissions would give an "unquestioned cost advantage" to nuclear fuel based electricity over coal and gas plants. Bunn et al. (2003, 2005) examined the long-term prospects of spent fuel reprocessing by comparing the costs against various other fuel cycle options, results which suggest that reprocessing is unlikely to become economically competitive with other fuel cycle options for the readily foreseeable future. This "Fetter model" provides a thorough model of nuclear electricity production economics that is transparently documented in readily available literature. Formulas from this model are used in fuel fractions model described in Appendix E of this thesis.

# 2.6 Energy Modeling: The Road Ahead

This survey of energy studies and models provides a summary of current trends in literature and various uncertainties and biases. A common feature in early energy models and studies is the high growth projections in energy demand, which is a reflection of the post-World War II optimism about the future of the global economy. Underlying many of the long-term energy models is the assumption of exponential energy use growth. Indefinite energy use growth (exponential or linear) is physically impossible, but the idea of continuing exponential energy use growth nevertheless captured considerable attention in the 1950s and 1960s when energy use was growing rapidly alongside population growth. Recent trends in population growth, fertility changes and energy efficiency have led to a moderation of such views, but the idea of a high energy growth scenario in the future remains deeply ingrained in many long-term energy models. Another drawback of many energy studies is the absence of a systematic calibration of various parameters based on longer historical times series data on population, gross domestic product (GDP), and primary energy consumption from various sources. A more realistic and self-consistent approach would take into account the evolution of various factors that influence the future distribution and consumption of energy resources. The work reported in this thesis attempts to build an integrated "look-ahead" model that reduces to the quasi-stationary case in the limit of gradual evolution of labor supply (population) increases, increases in production efficiency, and utilization of depletable energy resources.

# Chapter 3 Socioeconomic Indicators

Demographics and economic development are significant factors in the future evolution of energy demand and composition of fuel supplies. Historical trends in human population growth and economic development since early industrialization are reviewed in this chapter. The sources and methods for constructing annual time series data for population and gross domestic product (GDP) for 220 countries and geographical regions from 1820 to 2001 are outlined. Concepts and methods underlying purchasing power parity (PPP) converters used for cross country comparisons of economic output are reviewed.

Time series data are used in this thesis for calibrating a model that adjusts the evolution of capital  $\tilde{K}$  so that the split of total production  $\tilde{Y}_t = \tilde{C} + \tilde{I}$  into consumption  $\tilde{C}$  and investment  $\tilde{I}$  optimizes the total time-integrated discounted per capita utility of production,  $\int_{\tilde{t}}^{\infty} d\tilde{t} \, \tilde{P} e^{-\rho \tilde{t}} (\tilde{C}/\tilde{P})^{1-\theta}/(1-\theta)$ , where  $\tilde{P}$  is the population and the investment  $\tilde{I} = \bar{r}\tilde{K} + \dot{\tilde{K}}$  is the sum of depreciation rate  $\bar{r}\tilde{K}$  and the net time rate of change of capital stock  $\dot{\tilde{K}}$ . Here to a first approximation when the fraction of capital and labor applied to energy production is small, total production is a constant times  $a^{\eta}\tilde{K}^{\alpha}\tilde{L}^{\omega}$  where a(t) is time-dependent measure of economic development and the labor supply  $\tilde{L}$  is taken for simplicity to be proportional to population. Production is assumed to have constant returns to scale with respect to capital and labor, meaning that multiplying both by a constant multiplies production by the same constant, so that  $\alpha = 1 - \omega$ . For the work in this thesis four parameters are assumed constant and independent of the group of production units whose economic growth is being modeled, and thus these parameters are calibrated against global data. These parameters are the "labor fraction of production"  $\omega$ , the "depreciation rate"  $\bar{r}$ , the "social discount rate"  $\bar{\rho}$ , and a parameter  $\theta$  which is called the "inverse of the intertemporal substitutability of consumption" for reasons discussed in this chapter. (Here parameters that are not dimensionless numbers generally carry an overbar when they are taken to be universal constants and an overtilde when they are not universal constants.) In addition to outlining global demographic and economic development trends, this chapter also discusses the data sources used for calibrating these four global constants.

## 3.1 Global Demographic Trends

Global population grew nearly tenfold over the past three centuries. Europe and the New World experienced the most rapid growth in population during most of this period, while Asia and Africa picked up momentum in population growth only in the second half of the twentieth century. The twentieth century also witnessed the largest ever increase in human population, rising from an estimated 1.6 billion in 1900 to around 6.1 billion at the end of 2000 (Maddison, 2003). Increased population growth became possible due to improving conditions of human life. Until a thousand years ago average human life expectancy was around twenty five years. One third of the population would die in their first year and the remaining survivors lived fighting hunger and epidemic diseases for the rest of their lives. Human life then, in Thomas Hobbes words, was "nasty, brutish, and short" (Wikipedia, 2005). For most of human history death rates fluctuated highly and birth rates remained fairly constant. Compensating for high death rates, fertility rates of six to seven children per woman were common. This prompted Malthus to write famously about the dangers of overpopulation in the absence of high mortality rates, suggesting that "misery and vice" could check uncontrolled population growth in the absence of more enlightened approaches. However, Malthus was unaware of a significant demographic development—transition from high to low fertility—that was underway in Europe when he first wrote his essay on population dynamics. This trend started in France and gradually spread to other parts of Europe

between 1870 and 1914 (Weir, 1991). There are many factors underlying the transition toward low fertility rates. Declining infant mortality rates, better living conditions, and greater control of infectious diseases resulted in social and behavioral change that drove down fertility levels. In France the fertility transition occurred due to widespread use of contraceptives for birth control, which was first practiced among the aristocratic class but later percolated to the rest of the society. It is also a general observation that economic growth and literacy among women help lower fertility levels.

Demographers have described these historical developments in human population growth in terms of "demographic transition" model, which provides a useful framework for understanding the current dynamics of population growth in different parts of the world. About a hundred years after the beginnings of fertility transition in the West, a similar process began in some Asian countries. Replacement level fertility—around two births per woman—has been achieved in some Asian countries in the 1980s and 1990s, and the United Nations' "medium variant" prediction suggests that almost all developing countries will reach replacement level fertility by the middle of this century (UN, 2005a). Despite these significant shifts in human reproduction the total population increased enormously during the previous century and is expected to continue increasing through much of this century because of the lag between the decline in mortality rates and fertility rates. Bongaarts and Bulatao (2000) view the transition from high to low fertility as close to a general principle and state that the dynamics of population stabilization for countries currently undergoing fertility transition will be similar to those observed in countries with low fertility rates.

Demographic predictions using sophisticated models provide plausible estimates of future population growth. Many estimates of population trends using probability distributions from these models predict stabilization in population levels before the end of this century. Lutz et al. (2001) predict that global population will stabilize before the end of this century and even give a sixty percent chance for a stable global population at less than ten billion. The United Nations Population Division regularly presents its demographic projections for various assumptions (constant, high, medium, and low) about future evolution of fertility levels. The United Nations' recent estimates are substantially revised from its previous estimates, and according to the medium variant estimate world population would reach around nine billion by 2050 (UN, 2005b). Even the high and constant fertility variants project a population significantly lower than twelve billion by 2050. The projections reflect a global trend, barring some exceptions in Asia and sub-Saharan Africa, that is clearly in the direction of declining birth rates. The idea of demographic transition is central to the work presented in this thesis because the historical rate of growth of population is used as a proxy for calibrating the development index used in the energy econometric models.

### **3.2** Contours of Economic Growth

Economic growth was a "slow crawl" for most of the history of the recent millennium. Since 1820, however, growth has been more "dynamic" with global per capita GDP rising around eightfold (Maddison, 2001). The surge in economic growth and productivity in such a short duration was result of industrialization, which was significantly fueled by the utilization of mineral energy resources. Until recently, however, the surge in growth has largely been a western phenomenon. Per capita GDP was not nearly as noticeably different across countries until two hundred years ago as it is now (Maddison, 2003). Current cross-country disparities in living standards are largely a reflection of the differences in the rates of economic growth. The reason for per capita income differences among countries is one of the central questions in economics. The answer to this question is complex and problematic, but a combination of factors such as advances in science and technology, and nurturing of political values, economic and judicial institutions are believed to be reasons behind the global dominance of Western countries in economic and military affairs (Economist, 1999).

Economic liberalization and globalization has lifted a substantial fraction of the human population out of perpetual poverty. Although indiscriminate implementation of such policies have at times brought disastrous consequences for some countries, global economic integration has helped achieve sustained levels of growth during the post-World War II period. The world economy has grown on an average of 3.5 percent per year between 1960 and 2000 (Dollar and Kraay, 2002). If such rates of growth are sustained over long time it can make significant difference in countries that have not adequately benefited from economic growth. However, the question of whether there will eventually be a convergence of per capita GDP remains controversial. Solow (1956, 1970) analyzed long run convergence in per capita GDP on the basis of a neoclassical growth model. A growth model that gives this "absolute" convergence has all countries eventually attaining a steady state growth path that will be determined only by population growth and convergent growth in technological development. One of the fundamental questions that remains unresolved is whether the current trend in economic growth will continue indefinitely till the developing countries close their current income gap with the developed countries, or whether countries would converge "conditionally" on similar development paths but keeping something like the current income gap a permanent feature. Calibration of the models used here for a simple two-region global disaggregation into "developed" and "developing" countries suggests that convergence of per capita GDP may be conditional rather than absolute.

# 3.3 Comparison of Economic Growth

#### 3.3.1 Purchasing Power Parity

One of the difficulties in comparing GDP across countries is the choice of a suitable metric that reflects the real value of goods and services produced in each country. The use of market based exchange rates of local currencies is a standard practice in international trade and financial transactions. However, using market exchange rates to compare the value of GDP across countries can lead to erroneous conclusions about the actual size of different economies when it comes to the utility of consumption. Since exchange rates are sensitive to the vagaries of capital flows in the international market, economists have developed alternative methods for comparing GDP across countries using purchasing power parity (PPP). The idea of using PPP has evolved considerably from its conceptual origins in the work of sixteenth century Spanish economists through the work of classical economists in the nineteenth century. The Swedish economist Gustav Cassel did pioneering work during the First World War to establish PPP as an important issue in mainstream economics (Eatwell et al., 1991). In the 1950s the PPP system was used to handle the problem of computing a world index for agricultural output using weighting schemes (Geary, 1958). A wave of interest in cross-country comparison of GDP emerged after the United Nations began its pioneering work in the late 1960s in collaboration with the University of Pennsylvania through the International Comparison Program (UN, 1992).

Even though PPP was originally developed for understanding the dynamics of exchange rates, it is now widely used for comparing living standards and GDP across countries. For this purpose PPP exchange rates are calculated by comparing the prices for a large number of goods and services. These rates are then used to convert different currencies into a common currency—usually the U.S. dollar—to measure the purchasing power in different countries. The PPP exchange rate does not have any bearing on the market exchange rate although both adjust and become identical in the long run (Lafrance and Schembri, 2002). An illustration of purchasing parity concept in a lighter vein is the well-known currency converter devised by *Economist* using a single good, the Big Mac sandwich, to convert various currencies into PPP dollar, with prices based on official exchange rates varying from country to country for this widely available standardized product. On the basis of comparisons of prices and expenditures for several hundred goods and services in a number of countries, the relative values of local currencies have been adjusted to reflect purchasing power parity (Heston et al., 2002). PPP currencies produce lower GDP figures for richer countries and higher GDP figures for poorer countries than market exchange rates. Since many goods and services are not traded internationally, market exchange rates can significantly overstate or

understate the size of an economy measured in terms of local purchasing power. Although market exchange rates play a crucial role in the economic system, the use of PPP is appropriate under certain circumstances. Castles and Henderson (2003) have highlighted the appropriateness of using PPP method for comparing economic output across countries in the context of climate policy instead of using market exchange rates as done by the IPCC. Since a utility optimization approach is used in this thesis, a PPP measure of GDP is appropriate here.

#### 3.3.2 Geary-Khamis Aggregation

There are many aggregation methods that produce PPP currencies using specific weighting techniques. The time series data for GDP used in this thesis is given in terms of a particular type of PPP currency called Geary-Khamis (G-K) dollars. The Geary-Khamis dollar is most suited here because this method was specifically devised to enable multilateral comparisons of economic output. The G-K method of aggregation produces additive results that can be compared for various goods. The following discussion briefly describes the purchasing power parity converter derived from the Geary-Khamis method, and is provided in the United Nations Handbook of the International Comparison Program (UN, 1992).

The purchasing power parity is generally given as

$$PPP_j = \frac{GDP_j}{rGDP_j}$$

Where  $GDP_j$  is the GDP in local currency and  $rGDP_j$  is GDP in international prices

$$GDP_j = \sum_{i=1}^m E_{ij}$$
$$rGDP_j = \sum_{i=1}^m \pi_i q_{ij}$$

$$q_{ij} = E_{ij}/pp_{ij}$$

In the above  $E_{ij}$  is the expenditure in local currency on a item (also known as basic heading) of price  $pp_{ij}$  in that currency *i* by a country *j* and  $q_{ij}$  is the corresponding value in international prices. In the Geary-Khamis system of aggregation the international price for a given heading *i* is given as

$$\pi_{j} = \frac{\sum_{j=1}^{n} (pp_{ij}/PPP_{j})q_{ij}}{\sum_{j=1}^{n} q_{ij}}$$

or equivalently

$$\sum_{j=1}^{n} q_{ij}\pi_j = \sum_{j=1}^{n} \left( pp_{ij}/PPP_j \right) q_{ij}$$

In final equation above  $q_{ij}$  is known as "notional quantity," which is the expenditure of a country for a basic heading converted to the currency of the reference country. Each side in the above equation is a measure of the contribution of output of a basic heading to GDP. In Geary-Khamis method countries are given weight according to the size of their total GDP in the aggregation. Despite some of these disadvantages the G-K system produces GDP figures closer to the values measured in standard currencies than other methods. The GDP time series used in this thesis is given in 1990 PPP G-K dollars as provided in Maddison (2003).

# **3.4** Population and GDP Estimates

Systematic estimation of population and other demographic indicators for all countries has been done only since 1950, following the establishment of the United Nations. There are independent population estimates for earlier periods based on available records at statistical offices and census bureaus in different countries. McEvedy and Jones (1979) present rough estimates of population in different countries from as early as 400 BC through the later half of the twentieth century using various historical and archaeological records. While these type of estimates may be of broader historical interest, calibration of population data should rest on more reliable estimates. The United Nations Population Division provides detailed population estimates and projections for every country in the world from 1950 (UN, 2005a). The U.S. Census Bureau publishes similar demographic statistics for all countries regularly (USCB, 2002). For the work reported in this thesis population data is derived from the seminal works of the Dutch economist Angus Maddison, with one apparent anomaly corrected for China in 1982 using U.S. Census Bureau data. Maddison (2003, 2001) provides estimates for population and GDP covering 220 countries and geographical regions from a "millennial perspective."

Measuring economic output for cross-country comparison is also primarily a post-1950 effort. Before World War II only a handful of countries had official estimates of national income, which was put together without any guidelines for international comparability. Several area scholars have done pioneering work in estimating the economic output in different countries before 1950. Since 1950 many international organizations such as the World Bank, International Monetary Fund, and others have made estimates of GDP in different countries. The United Nations' National Accounts Statistics provides detailed estimates of GDP for all countries by different sectors. Maddison (2003, 2001) also provides a unique GDP time series with annual estimates using purchasing power parity from 1820 through 2001 using different sources. Maddison has systematically estimated GDP for various countries using most recent benchmark estimates of PPP converters and actual data, and captures the broad contours of economic development against the backdrop of various historical events during the past two millenniums.

Maddison (2003) provides continuous population and GDP data only for major countries from 1950 to 2001. For 20 small Asian countries, 24 small Caribbean countries, and 12 small West European countries, data is provided only for 1950, 1973, 1990, 2001. Separate data for reforming economies—fifteen former Soviet Union republics and six former Yugoslav republics—are available only from 1990-1991. Continuous data from 1820 to 1949 is available for fewer countries, and data for others are given only for select years. Using Maddison's annual aggregate estimates for smaller countries and reasonable assumptions about population and GDP growth as described in detail in Appendix B a more complete time series can be constructed using various manipulations (interpolation and disaggregation) of the primary data. The interpolated data are consistent with the annual aggregate data of the larger groups to which the smaller countries belong. Thus, starting from Maddison (2003) and with the aid of various manipulations of the primary data, a more complete time series for population and GDP for 220 countries has been constructed for use in this thesis. We use data only from 1950 to 2001 for time series calibration, but the earlier data can be used for checking back extrapolation of model results and are used for getting sensible preindustrial base values for population and GDP. In what follows from now we describe some of the parameters used in the energy econometric model.

# 3.5 Labor Share, Depreciation, and Interest Rates

The exponents in the product  $\tilde{K}^{\alpha}\tilde{L}^{\omega}$  in the above-mentioned production function approximation are assume to add to one, so estimation of one of them gives an indirect estimate of the other under the assumption of constant returns to scale. Gollin (2002) provides empirical estimates for  $\omega$  in different countries using the employee compensation shares of national income available in the United Nations National Accounts Statistics. Capital share is then computed from these estimates since  $\alpha = 1 - \omega$ . Using reported employee compensation alone to estimate  $\omega$  can be misleading for some countries, where compensation received through self employment and small enterprises is usually unreported in the national accounts statistics. This problem is prominent in poor countries where unreported labor incomes are treated as part of capital share. Poor countries tend to have slightly lower share of employee compensation than the rich countries as percentage of GDP, which is true in a capital intensive economy having a low-skilled work force that is not adequately compensated. Gollin (2002) corrects for the inconsistencies arising due to underreported and unreported labor compensation in poor countries. The corrected data suggests that income shares can be treated as being approximately constant across countries. The method we use for calibrating maximum likelihood estimates and global probability distribution for  $\omega$  and  $\alpha$  from the country-by-country estimates given by Gollin is described in Appendix F. Capital depreciation statistics for cross country comparison is scantly available in literature, but data is available for the United States from various studies. The studies reported in the U.S. context suggest that capital depreciation rates tend to remain approximately constant in time. These rates essentially reflect investment decisions between expansion and replacement and modernization. Bischoff and Kokkelenberg (1987) estimate capital depreciation rate based on detailed empirical and theoretical analysis. This study relates depreciation rate to capacity utilization by using the Federal Reserve Board's capacity utilization index to model the capital depreciation rate. Consistent data is not available for other countries and so we use the U.S. rates as proxy for other countries in our list. A least squares estimator and Student's t-distribution for the depreciation rate  $\bar{r}$  are derived from the Bischoff and Kokkenlenberg estimates, as described in Appendix F.

# 3.6 Individual and Societal Utility

The term "utility" is used by economists in objective and subjective contexts. In the former it is used to ascribe a value to a good or service to satisfy a need, while in the latter it used to describe the degree of individual or societal wellbeing and happiness. Most classical economists used utility in a broad sense to describe "desiredness" (Black, 1991). The concept of utility was later developed into a rigorous theory of economic behavior (Ramsey, 1928). From an individual point of view utility theory explains how people seek to satisfy their "subjectively felt needs" most efficiently, and how individuals trade off immediate consumption for later consumption. On a much broader level utility theory is used to describe how the present society values it own interests in comparison to that of the succeeding generations. This provides a useful framework for understanding the dynamics of long-term policy decisions involving economic tradeoffs and intergenerational transfers of wealth or economic burden.

Since utility maximization underlies the energy econometric models developed in this thesis, the essential elements of utility theory will be presented here. The following discussion is based on the analysis of Ramsey (1928), Blanchard and Fischer (1992), and Barro and Salai-Martin (2004) about savings behavior and optimal allocation of economic resources. The preferences of an individual's consumption over time is represented by the utility or welfare integral given as

$$U_s = \int_{\tilde{s}}^{\infty} u(c_t) e^{-\bar{\rho}(\tilde{t}-\tilde{s})} d\tilde{t}$$

where  $c_t$  is per capita consumption divided by any convenient normalization constant. A commonly used utility function in intertemporal optimization models is

$$u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}, \text{ for } \theta > 0, \theta \neq 1$$

which reduces in the limit  $\theta \to 1$  to

$$u(c_t) = Log[u(c_t)], \text{ for } \theta \to 1$$

The individual's utility or welfare at a time  $\tilde{s}$ ,  $U_s$ , is the discounted sum of instantaneous utilities  $u(c_t)$  at various times. The parameter  $\bar{\rho}$  is the personal rate of time preference, or the subjective discount rate, which is assumed to be positive. If instead of an individual we have a social planner as the "optimizing agent" taking decisions on behalf of the rest of the society, then the utility integral represents the social utility or social welfare. In this case  $\bar{\rho}$ represents the social rate of time preference, which is also known as the social discount rate. In both the cases the objective is to maximize utility or welfare—either of the individual or the society.

The mathematical procedure to find the optimal path for this utility function yields the

Euler equation with the following condition

$$\tilde{\Re} = \bar{\rho} - \left(\frac{u(c_t)''}{u(c_t)'}c_t\right) \left(\frac{dc_t/d\tilde{t}}{c_t}\right)$$

where  $\prime$  denotes derivative with respect to  $c_t$ . Equivalently

$$\hat{\Re} = \bar{\rho} + \theta \tilde{g}$$

where

$$\left(\frac{u\left(c_{t}\right)''}{u\left(c_{t}\right)'}\times c_{t}\right) = -\theta; \left(\frac{dc_{t}/d\tilde{t}}{c_{t}}\right) = \tilde{g}$$

The utility functions described here have a constant elasticity of substitution, which implies that the elasticity of substitution between consumption at two different points in time is a constant  $1/\theta$ . Here  $\theta$  is the magnitude of the elasticity of marginal utility given as  $-\theta$ , or the inverse of the intertemporal elasticity of substitution. The term  $\theta$  also has alternative interpretation as the "coefficient of risk aversion" when used in the context of describing attitudes toward risk. In this thesis we use  $\theta$  for computing the social discount rate and the departure from quasistationary equilibrium, which is proportional to it as detailed in Appendix C. Extensive research has been devoted to estimating  $\theta$  by looking into how people in general are willing to shift consumption in response to changing interest rates (Attanasio and Weber, 1995). Attanasio and Weber (1989) provide a value of 0.514 for  $\theta$  based on average consumption data of married couples in the United States. Mankiw et al. (1985) suggest a value between 1 and 1.5 for  $\theta$  using data on consumer preferences for durable and nondurable items. Estimates for  $\theta$  based on British savings data range from 0.2 to 2, but mostly concentrated around 1.5 (OXERA, 2002). The value 1.5 for  $\theta$  implies that a marginal increment in consumption to a generation that has twice the consumption of the current generation will reduce the utility or welfare by half.

While most of the estimates for  $\theta$  in literature are derived from financial data we follow

an alternative approach to estimate this parameter by using results recently available from surveys done by psychologists and social scientists. This has the dual advantage of relating directly to utility and allowing access to a quantitative multi-country database that allows a direct measure of  $\theta$  and its uncertainty. There has been a recent interest expressed in literature in more directly measuring things such as wellbeing, happiness and satisfaction with life Myers and Diener (1995). Although these surveys are obviously subjective measures, peoples' self-reported wellbeing and happiness converges with other measures for which more reliable statistics are available. Inglehart and Klingemann (2000) provide one such survey of peoples' welfare across different countries. The method for estimation of  $\theta$  using this survey and the relevant statistical tests for calibrating the parameter is provided in Appendix F. The term  $\tilde{g}$  is the annual growth rate of per capita consumption. Here per capita GDP growth rate derived from the GDP time series is used as a proxy by assuming it is equal to the per capita consumption growth rate, since it is not unreasonable to assume that the latter moves in parallel to the former.

# **3.7** Social Rate of Time Preference

Discounting provides a quantitative measure for comparing economic effects occurring at different times. There is a lack of consensus, however, in choosing an appropriate discount rate for analyzing the costs and benefits of a policy decision spread out over many decades or few centuries because of the uncertainties of the distant future (Cline, 1992; Lind, 1995). This dilemma emerges in the context of climate policy decisions, nuclear waste disposal, investment in alternative energy technologies, and other long-term projects. In climate policy analysis there are two broad categories of discounting: "prescriptive" and "descriptive" methods. The prescriptive approach favors substantial investments for climate mitigation policies and thus tends to generate low discount rates, while the descriptive approach generally adopts a "wait and see" policy and favors spending on immediate social needs and justifies high discount rates (Arrow et al., 1996). The latter approach is consistent with market realities and short-term investment priorities of international funding institutions (Birdstall and Steer, 1993). Although the prescriptive approach has significant moral appeal, it can potentially result in inefficiencies of economic institutions (Nordhaus, 1997). Moreover, many discussions ignore the difference between discounting goods and utility. WDI (2005) succinctly summarizes the reasons for the deep division in the estimation of social discount rate using the identity  $\tilde{\Re} = \bar{\rho} + \theta \tilde{g}$ . "The descriptive approach reasons from the left hand side to the right hand side and asks what combination of parameters can rationalize existing rates of returns. The second approach reasons from right hand side to left hand side on the basis of first philosophical principles. It begins with a view about time preference and inequality aversion and from this concludes what the appropriate discount rate is." As with the rest of this modeling exercise, the approach to estimation of the social discount rate used in this thesis is descriptive.

Real interest rates  $\hat{\Re}$  are the difference between nominal interest rates and inflation rates. Since there are large uncertainties in the results when nominal lending (and inflation) rates are high,  $\hat{\Re}$  value contributions are weighted using the inverse of the lending rate. The lending interest rate and real interest rate are derived from the World Bank Development Indicators (WDI, 2005). Detailed statistical tests and calibration of capital share in production, capital depreciation rate, and real interest rates are included in the Appendix F.

The formula for determining the social discount rate is essentially based on the social time preference rate derived from Ramsey's savings model, where  $\bar{\rho}$  is the rate at which individuals discount future utility. The *Green Book* used by the British Treasury to evaluate and manage long-term projects and risks provides a useful description of social discount rate (Treasury, 2003). According to this guidebook,  $\bar{\rho}$  can be further decomposed into sum of two terms: "catastrophic risk" and "pure time preference." The former is the likelihood of major events like wars and natural disasters that eliminates all returns from an investment, while the denotes "myopia" or "shortsightedness," which reflect individuals' preference for immediate consumption rather than savings or later consumption. Widespread differences over discounting methods notwithstanding the *Green Book* considers a lower discount rate more appropriate for the longer term (beyond 30 years) policy decisions, and provides the following schedule for evaluating costs and benefits occurring more than 30 years into the future: 3.5% for up to 30 years, 3% for 31 to 75 years, 2.5% for 76 to 125 years, 2% for 126 to 200 years, 1.5% for 201 to 300 years, and 1% for a time horizon extending over 300 years. Attanasio and Weber (1989) arrive at a social discount rate of 2.8% using an alternative approach. This value is reasonably close to the value obtained in this thesis as result of the calibration described in Appendix F.

# Chapter 4 Historical Energy Statistics

This chapter provides a description of sources and methods underlying the development of a comprehensive set of time series data on various forms of primary energy consumption and carbon use. As background for this, literature on energy statistics and alternative sources of information is first reviewed. Sources and methods for constructing the times series data for nine different types of commercial energy sources are then explained. The procedure for estimating the carbon use from the amount of fossil fuel consumed is also described.

# 4.1 Review of Energy Statistics

There are two widely used time series data on energy production and consumption, sourced respectively to the United States Energy Information Administration and the International Energy Agency (USEIA, 2005b; IEA, 2005a). These two sources provide comprehensive and consistent energy statistics for purposes of cross-country comparison and inter-fuel analysis. However, their coverage is restricted to recent decades. The U.S. EIA time series data on production and consumption of various commercial energy forms contain comprehensive energy statistics for most countries in the world, but begin only from the year 1980. The IEA's energy data series, which extends back to 1972, is mostly specific to the Organization of Economic Cooperation and Development (OECD) countries for the years before 1980. British Petroleum is another source providing consumption and production data from a global perspective since 1965, although its coverage of countries is limited compared to the U.S. EIA and IEA sources (BP, 2005). While these sources provide authoritative statistics
for recent trends in energy consumption, they do not adequately cover the energy transition precipitated by some crucial geopolitical events since the end of World War II, most notably transition from before the 1973 Arab-Israeli war through the 1973–86 Organization of Petroleum Exporting Countries' (OPEC) first effective cartel pricing period. International energy statistics have been a focus of attention only since the beginning of the 1973-86 cartel period when countries initiated efforts to systematically measure energy data produced and consumed locally.

Energy statistics before 1950 are even more scattered in the literature. There are many sources of primary and secondary information in different countries containing energy data before 1950. However they are often incomplete and inconsistent, requiring considerable care in their use. There have nevertheless been a few attempts to cut through the pre-1950 statistical maze and provide consistent time series data that reflect the various phases of energy history since industrialization. Mitchell (2003) provides time series data for production of coal, crude oil, and natural gas from 1800 as well as estimates for select years in the eighteenth century for some countries. Etemad and Luciani (1991) have made systematic estimates of energy production data for coal, oil, natural gas, hydroelectricity, and geothermal sources covering the period between 1800 and 1985. This source is well known in the literature because of its widespread use for estimating global carbon emissions due to fossil fuel production. Darmstadter (1971) provides estimates for consumption of coal, oil, gas and hydroelectricity in different countries covering the period between 1925 and 1965. This is one of the few known attempts to estimate energy consumption using trade statistics and production data for an earlier period. To date, however, these resources have not been pulled together in a convenient and consistent electronically accessible format. In particular, before the work in this thesis Darmstadter's useful data had not evidently been available in digitized form.

For this thesis a self-consistent time series data for consumption of nine different commercial energy forms was assembled for 220 geographic entities using various primary and secondary sources of information. These energy sources include coal, oil, and natural gas fossil fuels and also nuclear, hydroelectric, geothermal, tidal, wind, and solar thermal sources of electricity. Photovoltaic electricity production is not included because much of it is produced and consumed without transmission on the power grid and is therefore hard to quantify. In the context of this thesis photovoltaic energy, biomass, and non-electric solar heating are taken to be operationally equivalent to conservation measures that can also mitigate the demand for the sources cataloged here, which are operationally defined as "primary" energy sources. The following sections provide a brief description of the sources and methods used.

## 4.2 Data Sources and Methods

The United Nations Statistics Division (UNSD) through its annual United Nations Energy Statistics Yearbook provides comprehensive international energy statistics on long-term trends in the supply and demand of various commercial primary and secondary forms of energy. The information found in these issues represents the officially reported data, as opposed to independent estimates made by energy agencies and private companies. Using the back issues of this publication from 1950, a continuous time series data on various forms of energy consumption and production can be constructed. The time series can be more conveniently and reliably constructed using the United Nations Energy Database, a large electronic repository containing all energy-related information available with the UN Energy Division, which is updated every year. To facilitate updating of the more complete data base, it is important that a well automated procedure be available for the complex process of extracting an reformatting the required data from this comprehensive but somewhat opaquely structured database.

For constructing various energy consumption time series from 1950 to 2002, the most recent version of the electronic files released by UN was used (UNSD, 2005). The electronic database is not supplied in a ready-to-use format, but it does contain comprehensive energy statistics for more than 220 countries or regions for production, imports, exports, bunkers and stock changes of primary and secondary commercial and non-commercial sources of energy. The electronic energy database has the following structure In the UN energy database

Field Number	Byte Position	Length	Description
1	1	3	Country Code
2	5	9	Energy Code
3	4	4	Year
4	17	3	Unit Code
5	36	2	Official data/UN Estimate
6	-	-	Reported Value

Table 4.1: Source: Energy Statistics Database Documentation (UNSD, 2005)

countries are represented by a three digit numeric code, which is also used by the International Standards Organization (ISO). Each energy code is represented by a combination of a two digit commodity code and a 2 to 5 digit transaction code, and a unit code which represents the original units in which the data has been reported. The UN provides its own estimates in some places where official country data is absent. We do not distinguish between the officially reported country data and UN estimates in the extracted data. The database contains 57 different types of primary and secondary fuels and 72 different types of transactions. The country codes, commodity codes and transaction codes for various primary and secondary energy forms are taken from the accompanying documentation and various energy publications published by the United Nations (UN, 1982, 1987, 1991). As described in detail in Appendix G, a computer package written in PERL was developed for this thesis to extract consumption data for nine different types of commercial energy sources covering the period between 1950 and 2002 from the UN source files.

Energy data before 1950 lacks the completeness and continuity of the UN energy data. Using the best available estimates, Darmstadter (1971) captured the energy consumption from 1925 to 1965 for most countries. We use this source for the period between 1925 and 1949 for the consumption of coal, oil, and gas. Consumption data for hydroelectricity and geothermal electricity for the years before 1950 are assumed to be equivalent to production data taken from Etemad and Luciani (1991), since generally electricity was in that period consumed in the same region in which it was produced. Pre-1925 consumption data for coal, oil, and gas are derived from Goldewijk (2004) and Mitchell (2003).

In order to have any meaningful cross-country comparison of energy use and inter-fuel analysis, energy data should be converted to standard units. In this thesis we use ExaJoule (EJ) as the preferred unit of measure for energy consumption. Conversion of coal data to standard units is most problematic because of the variations in heat content by type, quality, and source. In order to avoid complications, just two different standard conversion factors are used across the board for hard coal and brown coal, irrespective of the country of origin. Energy production and consumption from all non-fossil sources—nuclear, hydro, geothermal, tidal, wind, and solar thermal electric—are provided as primary electricity (in terms of millions of kWhr) in all the sources used here. These are converted into equivalent heat units by assuming that they displace energy consumption from fossil-fueled electricity sources operating at a fixed electric/thermal efficiency of 0.38. Natural gas data is provided in heat units in the UN files and volumetric units in other sources. Data given in volumetric units are converted to energy units using standard conversion factors. The conversion factors used for various fuels are provided in Appendix A. In addition to standardizing the data from various sources into common units of measure, the other challenge is to aggregate and disaggregate geographic entities to reflect the current political landscape and match the list of countries in the population and GDP time series provided in Maddison (2003). Aggregation is done by simple summation, while data reported earlier only in the aggregate are disaggregated in proportion to the values when the first disaggregated data are reported. In addition, missing data are also interpolated using geometric growth rates between pairs of points for which data is available. These manipulations are also done in PERL according to the procedures detailed in Appendix G.

## 4.3 Energy Consumption Data

Consumption of various primary and secondary energy sources listed in the UN database released in March 2005, which covers the period between 1950 and 2002, are estimated using the following relationship

Consumption = Production + Imports - Exports - Bunkers - Stock Changes

Bunker fuel is used largely for open ocean transport and is thus not readily assignable to a particular country. Because bunker fuel use does contribute to global carbon emissions, a separate file on bunker fuel has been constructed, but its contribution to global carbon emissions is sufficiently modest that it is not further considered in this thesis. Coal consumption reported in this thesis is the sum of consumption of hard coal and brown coal calculated individually using this expression plus the net imports of secondary fuels made from these two fuels: namely brown coal briquettes, hard coal briquettes, brown coal coke, coke oven coke, and gas coke. Oil consumption for this period is the sum of eleven different energy products derived from crude petroleum and natural gas liquids. The derived products are aviation gasoline, feed stocks, gas-diesel oil, jet fuel, kerosene, liquefied petroleum gas, motor gasoline, naphtha, natural gasoline, plant condensate and residual fuel oil. Natural gas consumption excluded flaring, re-injected gas and extraction losses. Flaring is excluded as a contribution to energy input to GDP production, but it has contributed to carbon emissions on a global basis. Thus a separate file has been constructed for natural gas flaring history, though the overall contribution is sufficiently modest that it is not further considered in this thesis. Primary electricity consumption is estimated from production and net imports as the terms bunkers and stock changes are irrelevant. Thus from the UN source files (UNSD, 2005) nine different time series data for energy consumption by sources has been extracted. Direct estimates of energy consumption for most countries are available for select years between 1925 and 1950 in Darmstadter (1971). Consumption time series for coal, oil, and gas from 1925 to 1949 are derived from this source and patched with the files derived from UN sources after conversion to standard energy units and rescaling to match UN data where both data sources overlap. As noted above, inland production of hydroelectricity and geothermal electricity before 1950 are assumed to have been consumed within the country. Similarly, all inland production of natural gas is assumed to have been consumed within the producing country before 1925. Although the volume of international natural gas trade is currently growing rapidly, it is safe to assume that trade was negligible before 1925. In particular the United States, the largest consumer of natural gas, from the beginning imported less than a tenth of a percentage point of its natural gas demand in the 1950s (Schurr et al., 1979). Also, major transnational natural gas networks were not present before that time. Using similar assumptions for coal and oil even before 1925 would be misleading because of the substantial volume of these items traded internationally. A rigorous estimation of consumption of coal and oil before 1925 using detailed production and trade statistics is beset with difficulties due to the scattered and inconsistent data on commercial trade of these fuels during earlier time. We thus use a simple approximation to estimate consumption time series for coal and oil before 1925. Using the percentage shares of consumption of coal and oil for 220 countries in 1925, global production data was disaggregated in this proportion for earlier years.

For the purposes of this thesis only cumulative energy uses within each very large aggregate region before the 1960s affects the result. Thus for the present purposes the details of pre-1925 energy use are not important, but for detailed historical studies users of the database are advised to check with original sources if the disaggregation method used might have a significant affect on their conclusions. The procedures for aggregation, disaggregation, unit conversion, interpolation and other manipulation are provided in detail in Appendix G.

## 4.4 Carbon Content in Fuels

We here describe the procedure for calculating the carbon content of energy use from the energy time series data derived from different sources. The purpose is to provide a more transparent and readily reproducible estimate for carbon use due to fossil fuel use than available in the literature. The advantage of this approach is one can now have a self consistent country by country time series data for carbon use. This is needed for the modeling done in this thesis, where the cumulative carbon use by each region affects its overall energy production efficiency.

In the format described here and detailed in Appendix G, estimates of carbon use can be readily aggregated for any desired set of groups of countries. This flexibility is needed for the energy production model used here, and it may also be helpful for future work based on game theory that needs to be able to attribute responsibility for atmospheric carbon accumulation to fluid assemblies of negotiating groups.

Other sources such as the U.S. EIA, IEA, and the Carbon Dioxide Information and Analysis Center (CDIAC) follow a more detailed but less transparent "bottom-up" approach to estimate carbon emissions due to fossil fuel use, making but not necessarily completely publicly documenting individual assumptions about carbon intensity of energy production for various countries and fuel sources. It is thus useful to compare the results from both approaches in the aggregate, but inconvenient to use these other sources for the present purposes. The formula for estimating carbon content in energy use sourced to different types of fuel is from Marland and Rotty (1984) and given as:  $CO_2 = P_k \times FO_k \times C_k$ . The first term represents the quantity of fuel, the second represents the fraction of fuel that is oxidized, and the third is the carbon fraction of each fuel category k. Using this and appropriate conversion factors from Haefele et al. (1981), the formula for estimation of carbon use in million metric tons of carbon can be written as

 $CO_{2Coal} = Coal Consumption(EJ) \times 0.7461 \times 34.14$  $CO_{2Oil} = Oil Consumption(EJ) \times 0.851 \times 22.34$  $CO_{2Gas} = Gas Consumption(EJ) \times 13.7$  The above formula from does not recognize the difference in calorific values of various fossil fuels by quality and geographical origin. This is not feasible in the present context, because systematic tabulation of data on the carbon content of fuel produced and consumed all over the world is not available. Even though the UN energy database includes a separate conversion factor file for select countries from 1970, using these would complicate the procedure and make it very difficult to provide continuity before and after 1970. Such continuity and consistency is particularly important for avoiding discontinuous jumps in residuals between data and model fits which would then subsequently need to be resolved by estimating recalibration coefficients on an ad hoc case by case basis. The simplest and most tractable approach for the present purposes is to use standard conversion factors for all fuels and retain consistency throughout. When using carbon consumption data for providing input to a model of atmospheric carbon accumulation, the work reported in this thesis does allow for carbon use as estimated to be multiplied by a common factor to obtain emissions into the atmosphere. This allows for incomplete combustion, which releases into the atmosphere about ninety-eight percent of the carbon for coal, natural gas, and the refined petroleum products included here. Adjustment of this multiplier also allows the carbon use estimates provided here to be rescaled to reproduce the average value from any other estimate in the literature, the absolute accuracy of which has been estimated as being uncertain to within around ten percent (Marland and Rotty, 1984).

# Chapter 5 Energy Econometric Model: Results

This chapter provides a summary of the main results of the energy econometric model used in this thesis. We first review the reasons for using a two-region disaggregation of the world and then proceed to present the model results for population, GDP, energy use and carbon use. The mathematical and statistical details underlying the model results presented in this chapter can be found in Appendices C and F respectively.

## 5.1 Tropical and Temperate Region

With the availability of a continuous and consistent time series data for population, GDP, and energy use for 220 geographic entities, any number of conveniently aggregated regional or other groupings can be formed for energy econometric modeling. In this thesis, however, we shall confine the model runs and discussion to a two-region aggregation of all countries for convenience. Modeling other regional groupings has been done by C.E. Singer and the present author for classroom instructional purposes using this database on a preliminary basis for the eleven geographic regions customarily used by the International Panel on Climate Change. While such finer disaggregation is tractable, it comes at the cost of greater computational burden and variability in the aggregated data, and detailed discussion of the minor methodological adjustments thus needed lies outside the scope of this thesis.

The two-region aggregation used here divides the world into "tropical" and "temperate" regions specified as follows. Entities are designated as temperate if any part of their territory lies poleward of 40 degrees latitude, and the remaining entities are defined as tropical. Exceptions to the stated rule in the temperate region include South Korea, Malta and Cyprus (as part of the European Union), Puerto Rico (with the United States), and Macao, Hong Kong and Taiwan (with China). These entities are included in the temperate region for socioeconomic contiguity with their northern neighbor. Most of the entities in the data base are sovereign states, but for historical reasons the data base also contains entries for entities such that are not universally recognized as sovereign. The two groupings are represented pictorially in Figs 5.1 and 5.2. The division of the world into tropical and temperate regions



Figure 5.1: Geographical entities designated as temperate region if any part of territory lies poleward of 40 degree latitude



Figure 5.2: Geographical entities designated as tropical region if any part of territory does not lies poleward of 40 degree latitude

as defined above has two useful features. Looking toward future more detailed studies of

the impact of climate change, the "tropical" region is more likely to be adversely affected by the earlier effects of increased atmospheric  $CO_2$  than the "temperate" region. However, the main immediate appeal of this division is that it conveniently divides the world into two regions having distinct demographic profiles. The countries in the temperate region are mostly those that have already largely undergone the demographic transition from higher fertility to lower fertility, and the countries in the defined tropical region comprises mostly those that are in the process of fertility transition at different stages. Since population growth rate is used here as a proxy for a development index that will be used to quantify increases in economic production efficiency, this division is particularly useful. It also allows to an adequate approximation the division of the world into developed and developing regions.

# 5.2 Demographics and Development

#### 5.2.1 Fitting Population Growth Rate

As discussed earlier, a simple demographic model is an integrated part of the approach to energy econometric modeling in this thesis. More specifically, the rate of population growth is used as a proxy for the "need for development." If population growth rate can be used as a measure of the shortfall of a development index, a, from the maximum value of 1 obtained in the limit of zero population growth, then the population growth rate can be used to calibrate a development index which evolves logistically. The population growth rates for the two regions are derived from the population time series data provided in Maddison (2003).

The population growth rates are given as  $dLog\tilde{P}/d\tilde{t} = \tilde{\nu}z$ , where z = 1-a. The growth rates of population increment over its preindustrial value are then fit to the following function

$$z = 1 - a = 1 - \frac{1}{1 + e^{-\tilde{\nu}(\tilde{t} - \tilde{t_0})}}$$

For the results shown here the increments over a "preindustrial value" for population and GDP are obtained by subtracting values for the "base" year 1820, before which annual fractional growth of population and GDP were generally much slower than thereafter. The data base also contains values for later years and for 1700, 1600, and 1500 in case it is desired to use a different base value.

In the above equation  $1/\tilde{\nu}$  is the development time scale and  $\tilde{t}_0$  is the inflection time for each region. Near z = a = 1/2 the complement z of the logistic function declines nearly linearly with time, with a deviation from a linear fit only of order  $(z-1/2)^3/6$ . Thus a linear fit is used as a starting point for the nonlinear minimization. The relation between the linear fit slope and intercept  $\{\tilde{\theta_1}, \tilde{\theta_2}\}$  and the logistic function parameters is  $\tilde{\nu} = \sqrt{\tilde{\theta_2}}$  and  $\tilde{t_0} = (\tilde{\theta_1} - \sqrt{\tilde{\theta_2}})/\tilde{\theta_2}$ . These expressions are used for  $\{\tilde{\nu}, \tilde{t_0}\}$  and the nonlinear but nearly linear minimization is done vs.  $\{\tilde{\theta_1}, \tilde{\theta_2}\}$ . The population growth rate fits with and without periodic correction for the tropical and temperate regions are shown in Fig 5.3. (The periodic corrections included in the solid curves in Fig 5.3 include the two largest amplitude components of the discrete Fourier spectrum of the residuals between the data and fit without periodic corrections, as described in detail in Appendix F.) The data ranges chosen for this fit avoid the immediate post-WWII economic readjustments in the "developed" temperate region (which here includes China). The data ranges chosen also start the "developing" region at about the same development index as at the start of the data calibration period for the developed/temperate region. This keeps the tropical region population growth rate data within the approximately linear region and thus makes the linear approximation particularly useful as a starting point both for the full nonlinear fit (and for eventual sampling of the resulting probability distribution for the fitting parameters using the bivariate Student's t-distribution as described in Appendix F). Fig 5.3 illustrates how the developing countries show a similar pattern in population growth rate decline to the developed countries, with a lag of a little more than one human generation (which is about twenty-five years based on the average age difference between children and their mothers). This lag is a measure of the time required for a complex process of social and technological diffusion to spread a transition to lower fertility rates through most of the globe. In the present work this lag time is estimated empirically as approximately thirty-two years from the data and fits shown in Fig 5.3.

The fits shown in Fig 5.3 for the tropical and temperate regions are consistent with wellknown findings of declining birth rates globally, which are at the lowest level on a global basis in known history. Fertility levels in developing countries have fallen from around six children per woman in the 1970s to 2.9 at present (UN, 2005b). Accounting for mortality before reaching childbearing age, the steady state replacement level for fertility is about 2.1, and with continuation of present trends the global fertility rate is expected to further decline to around this level before the end of the present century.



Figure 5.3: Fits for rate of growth of increment over 1820 base populations versus data points. Dashed and solid curves are respectively fits with and without two periodic corrections

#### 5.2.2 Calibration of Development Index

The development index a provides a measure of the accumulation of "social capital," which reflects the level of social organization that results in lower fertility rates and increased efficiency of labor and capital use. Growth of a is exponential in the initial phase, but then saturates as advanced institutional skills are developed as required to maintain higher levels of social complexity. In other words, developed societies are characterized by saturated or near-saturating values of the development index. Hence this "social capital" is assumed to increase at a rate proportional to the product of the level of development, a, and the need for development, 1 - a. This assumption results in a logistic function of the form given above for a. For small values of the independent variable (time in this case), the increasing logistic function is well approximated by an increasing exponential function. However, for large values of time, the function behaves differently. The difference between this function and its limit value at large time decays exponentially at large time. At the inflection point the function has zero second derivative vs. time and hence is approximately linear there. For the logistic function the inflection time is the point at which it has half of its limit value. The development time scale and inflection time completely determine the shape of the unit logistic functions shown in Fig 5.4. For the calibrations shown here the inflection point for the tropical region was in the year 2002 and for temperate region in 1969.

#### 5.2.3 Fits to Total Population

For diagnostic purposes it is convenient also to fit the evolution of total population increment over the preindustrial base year. This evolves according to the logistic function  $\tilde{P} = \bar{P}a$ , where  $\bar{P}$  is the limit population increment, i.e. the population increment (over the base value) at equilibrium. (The fits in Fig 5.5 are done by fitting  $\tilde{\nu}$  and  $\tilde{t}_0$  to the differenced population growth data and then fixing these parameters in a subsequent least squares fit for  $\bar{P}$  to the population data. The base populations are also added back in for the graphical



Figure 5.4: Development index  $a=1/1 + e^{-\tilde{\nu}(\tilde{t}-\tilde{t_o})}$  for tropical and temperate region extrapolated (dashed curves) beyond the time over which calibrated (solid curves)

presentation.) The motivation for the procedure used here is that only the set  $\{\tilde{\nu}, \tilde{t}_0\}$  is needed in subsequent calculations, and this more "parsimonious" set can be fit to the annual fractional population growth rates without the need to simultaneously estimate the limit populations. The maximum likelihood fits for the total population are extrapolated through 2040 in Fig 5.5. This model projects a limit population of around 5.5 billion for the tropical region and 2.6 billion for the temperate region. According to recent UN projections the world population is likely to reach 9.1 billion by 2050 if the expected decreases in fertility rates occur, while an unexpected sudden freezing of current fertility levels would lead to global population then of around 10.7 billion (UN, 2005b).



Figure 5.5: Population extrapolations without periodic corrections, compared with calibration data, and for the whole world, all including addition of base year values

## 5.3 GDP and Economic Development

#### 5.3.1 Per Capita GDP Growth Rates

The GDP increases over the preindustrial base values are fit by maximizing utility through first order in the departure from quasistationary equilibrium, as described in more detail in Appendix C. The formula used for this fit is

$$dLog[G_{DP}/a]/da = \xi + (\alpha/\omega)dLog[F_1]/da$$

where the "capitalization delay" factor is

$$F_1 = (1 + \epsilon_1 a)/(1 + \epsilon_1)$$

Here  $\epsilon_1 = \nu \theta \xi$  gives a measure of the initial departure from quasistationary equilibrium, which is the result to which the solution for gross domestic product  $\tilde{G}_{DP}$  would relax on the capitalization timescale  $\bar{t} = 1/(\bar{r} + \bar{\rho})$  if the evolution of productivity and labor supply were frozen. The ratio  $\nu = \tilde{\nu}\bar{t}$  of the capitalization timescale to the development timescale has  $\tilde{\nu}$  available from the population growth calibration and  $\bar{t}$  from the estimates of depreciation rate  $\bar{r}$  and social discount rate  $\bar{\rho}$  described in Sections 2.3 and 2.4 of Appendix F. The exponent  $\theta$  in the relation between utility and per capita consumption is available from the calibration described in Section 2.2 of Appendix F, and the labor compensation fraction of production  $\omega$  and its complement  $\alpha = 1 - \omega$  are available from the calibration described in Section 2.1 of Appendix F. Thus, the only parameter that needs to be estimated from the GDP growth rate data is the scaling exponent  $\xi$ . (The parameter  $\xi$  is referred to as the GDP scaling exponent, since the expression for the GDP itself is  $\tilde{G}_{DP} = \bar{G}_{DP} a^{1+\xi} F_1^{\alpha/\omega}$  with  $\bar{G}_{DP}$  a constant for each region.)

Since the capitalization timescale t is typically small compared to the development timescales  $1/\tilde{\nu}$ , their ratio  $\nu = \tilde{\nu}\bar{t}$  is small and hence so is the departure  $\epsilon_1$  from quasistationary equilibrium. Since  $dLog[F_1]/da \sim \epsilon_1$ , to a first approximation  $dLog[\tilde{G}_{DP}/a]/da \approx \xi$ . This result is then used as a starting point for the nonlinear fit through first order in departure from quasistationary equilibrium (c.f. Appendix C). The fits for the tropical and temperate regions to annual fractional GDP growth rate data are shown in Fig 5.6 and 5.7 with (solid curve) and without (dashed curve) periodicity corrections. Periodicity corrections are found by adjusting the amplitudes of the Sin and Cosin functions with the two dominant frequencies found from Fourier spectrum analysis of the residuals between the uncorrected fit and the data, as described in Appendix F. The effect of various business cycles ranging from the shorter term (7-10 years) to the much longer term (30-40 years) on macroeconomic output are noticeable in the graphs. Although the exact timing of the cycle is by nature unpredictable, the oscillations show statistically significant recurring patterns.

Business cycles result from the inventory and production and consumption behavior of individual firms and and consumers, with an aggregate impact on the larger economy. Economists often cite four phases of economic dynamics—prosperity, liquidation, depression (or recession) and recovery as recurring features of economic growth and development cycles. Many explanations have been offered—ranging from climatic, psychological, monetary and others as the underlying causes of these cycles. Backus and Kehoe (1992) report, based on studies of economic data of various countries spread over a century, a great regularity in the cyclical behavior of rate of economic output. In addition to the higher frequency cycle particularly apparent for the tropical region, here low frequency cycles are evident in both the tropical and temperate regions. The dip in the fit for the tropical region around 1997 roughly coincides with the impact of the so-called economic Asian Flu, a down-turn in the region's economic output that was driven largely by monetary factors. The minimum of these cycles corresponds approximately to the end of the period of high global oil prices in the early 1980s. For the present purposes only the GDP scaling exponent  $\xi$  is needed for



Figure 5.6: Tropical region rate of fractional increase with increasing development of GDP increment over its base per capita of incremental population over its base with (solid curve) and without (dashed curve) two periodicity corrections, compared with data

subsequent calculations. This is because the desired overall scales of energy production and carbon use, denoted respectively below as  $\bar{w}$  and  $\bar{E}$ , will be determined directly by calibration against energy production and carbon use data. The periodic behavior of GDP cycles



Figure 5.7: Temperate region rate of fractional increase with increasing development of GDP increment over its base per capita of incremental population over its base with (solid curve) and without (dashed curve) two periodicity corrections, compared with data

is thus only of peripheral interest because it has a slight effect on the procedure for calibrating the GDP scaling exponent. However, visually and by the statistical tests described in Appendix F the inclusion of the periodicity corrections makes the residuals between the data and the fitting curves indistinguishable from being independently and identically normally distributed. This will be important for future work when probability distributions for the scaling exponent will be sampled, to support probabilistic assessments of the impact of carbon emissions on the evolution of global average temperature.

#### 5.3.2 Evolution of GDP

As just noted, the evolution of GDP itself is also only of diagnostic interest since the scales of energy and carbon use of primary interest here will be calibrated directly against their data sets. However, for diagnostic purposes results from calibration of the absolute scales for evolution of GDP are also shown here. In particular, Fig 5.8 shows fits without periodicity corrections for the total GDP for the tropical and the temperate countries and projections through the year 2040. As above for population, the limit GDP is not used in the remainder of the calculation and is also estimated only for diagnostic purposes. The maximum likelihood values for the limit GDP for the tropical and temperate regions are 20 trillion and 41 trillion (both in terms of 1990 PPP dollars) respectively. These numbers are substantially lower than those projected in many long-term energy models, which sometimes have strong economic growth as a terminal result even with finite energy intensity of production (a combination which does not extrapolate to a sensible long-term limit). It is to be emphasized that here it is only energy-dependent economic production that is of interest, and the assumption is that historical economic production increments above preindustrial levels are dependent on primary energy use. (Primary energy use as operationally defined here includes fossil fuels and the thermal energy equivalent of electricity production from other sources.) If GDP is to grow indefinitely, then it is physically necessary that this be in the form of production that is not dependent on a minimal level of primary terrestrial energy intensity of production and is assumed here to have been a negligible portion of economic production during the historical period covered by the data used.

The use of PPP dollars for multilateral comparison and aggregation of economic output is increasingly gaining ground as the method for cross-country valuations of economic output. The use of market based exchange rates for comparing GDP across countries, as done in many long-term energy models, has recently been criticized for its lack of measuring the real quantity differences associated with economic activity, and the method chosen here addresses this objection per force.

### 5.4 Energy Model

In the model used here, for each region the formula for the energy production rate is

$$\tilde{w} = \bar{w} p a^{\psi} F_1^{\alpha/\omega}$$



Figure 5.8: Annual GDP extrapolations without periodic corrections, compared with calibration data, and for the whole world, all including addition of base year values, in terms of 1990 purchasing power parity in trillions of dollars

Here the factor p = h(1 - bu) with b = (h - 1)/h describes the evolution, normalized to a long-term limit value of 1, of the dependence of production efficiency on the ratio u of fossil carbon used to date to the asymptotic limit amount of fossil carbon that will ever be used. As described in detail in Appendix C, the evolution of the fractional carbon depletion u follows from balancing carbon depletion with carbon use under the assumption of a piecewise linear evolution of the carbon intensity of energy production as a function of integrated fossil carbon use. Fig 5.9 shows piecewise linear fits of the carbon intensity of energy production by each region as a function of cumulative carbon used in each region.

The formulas that result from these assumptions are the most complicated of the set used here. They are nevertheless readily computable in terms of the Lerch transcendent function  $\Phi = \sum_{j=0}^{\infty} a^j / (\psi + j)$ , which is simply related to the incomplete Beta function as described in Appendix C. The results expanded through first order in the departure  $\epsilon_1$  from quasistationary equilibrium for each part k of the piecewise linear fit shown in Fig 5.9 are

$$u = (x-1)/(s_k x - 1)$$

where

$$x = x_k e^{hf_k(s_k - b)(R - R_k)}$$

and

$$R = (\epsilon_0/\nu) \int_0^a da \ a^{\psi} F_1^{\alpha/\omega}/(za) \approx (\epsilon_0/\nu) a^{\psi} (\Phi - (\epsilon_1/\psi)\alpha/\omega)$$

is evaluated at the boundary values  $a_k$  between the piecewise linear fitting regions shown in Fig 5.9. Here  $f_k$  and  $s_k$  are respectively the slopes and and vertical axis intercepts of the piecewise linear fits shown in Fig 5.9. The parameter  $\epsilon_0$  is the ratio of the capitalization time to the carbon depletion time. The carbon depletion time is formally defined as the inverse of the fractional annual carbon depletion rate with energy use with the maximum possible carbon intensity of production and the energy use rate set at its long-term-limit value. Since the ratio  $\epsilon_0/\nu$  of the development timescale to the carbon depletion timescale is also typically small, for the time over which the model is calibrated against data we have  $R \approx 0$ ,  $x \approx 1$ , a small fraction u of ultimately used carbon depleted to date, and the production efficiency factor remaining approximately equal to its initial value  $p \approx h$ . For this reason, the parameter h and the limit energy use rate  $\bar{w}$  cannot readily simultaneously be calibrated against the available time series data, and the parameter h must instead be estimated from a technology assessment. Physically, the parameter h corresponds to the ratio of the cost of energy from renewable resources to that with little fossil carbon depletion and mature technological development of both types of energy production. By examining, for example, the range of costs of electricity production at various locations where inexpensive fossil fuels are and are not available, one can infer that  $h - 1 \sim 1$ , so for a reference model here we chose h = 2. For the results shown here, the logarithm of annual energy usage for the two



Figure 5.9: Carbon use intensity of energy use in Gtonne/EJ from piecewise linear fits

regions is fit to a two-sector model with total time-integrated discounted utility of per capita consumption optimized through first order in departure from quasistationary equilibrium. The result is shown in Fig 5.10 without (dashed curves) and with periodic corrections derived as described above for fits to population and GDP growth rates (solid curves). The formulas used for these fits are given above and derived from utility maximization of energy use as detailed in Appendix C. In the temperate region fit in Fig 5.10 one can see the gradual saturation of annual energy use. The growth of annual energy use for the tropical region and the corresponding fits can be seen more clearly on a linear scale in Fig 5.10. Compared to the background trend without periodic corrections (dashed curve in Fig 5.10), the rate of growth of energy in the better fit (solid curve) is lowest toward the end of period high oil prices that peaked in 1980 and lasted until collapse of effective OPEC cartel cooperation in 1986.



Figure 5.10: Fits for increment over 1820 base annual energy use versus data points. Dashed and solid curves are respectively fits with and without two periodic corrections



Figure 5.11: Fits for tropical region total annual energy use vs.data points. The fit with periodic corrections (solid curve) dips below the background trend (dashed curve) before the 1986 end of the first OPEC cartel period and crosses back over it in the mid-1990s during the 1986-98 cartel interregnum



Figure 5.12: Annual energy use extrapolations without periodic corrections, compared with calibration data, and for the whole world, all including addition of small base year values

## 5.5 Carbon Growth Model

Data for the carbon use model is derived from the consumption of coal, oil, and gas from the database (c.f. Appendix G, which is provided in the attached CD-ROM). It is customary in the literature to report carbon emissions rather than carbon use, but because we report use of petroleum products rather than of crude oil the fraction of carbon used that is emitted into the atmosphere is approximately constant across fuel types and at a nominal value of 0.98 is nearly equal to 1. Given the difficulties in associating emissions corresponding to individual fossil fuel use we have thus preferred to report the carbon content in the consumed fossil fuels.

For use in climate impact studies, what is really desired is a historical fit and projection of carbon emissions, which in the approximation used here is a constant factor times carbon use. To this end, it is convenient to directly calibrate the model against historical data on carbon use. The formula for this in dimensionless units is simply

$$F = f w$$

where f is the ratio of carbon intensity of production shown in Fig 5.9 to its initial value (which before 1860 is based on burning of coal alone as a primary energy source as operationally defined here). In dimensional units of Gtonne/yr, we have

$$\tilde{E} = \bar{E} f w$$

where as noted above for the historical calibration period we have  $w = \tilde{w}/\bar{w} \approx ha^{\psi}$ . Thus, as the normalized carbon intensity of production f is not far from 1 (c.f. Fig 5.9),  $Log[\tilde{E}] \approx Log[\bar{E}] + Log[f] + \psi Log[a] + Log[h]$  is also approximately a linear function of development index a. Since a itself is a nearly linear function of time near its inflection point, then the logarithm of the carbon use rate,  $Log[\tilde{E}]$ , is also nearly a linear function of time over the data range used here, as illustrated in Fig.5.13. In Fig 5.14 the actual data is plotted on a linear scale against the fitting function and the projections through the period considered. The tropical region's carbon use rate has continued to rise, given the large reliance on coal use particularly in India. However, carbon use rates for temperate region shows saturation and projects to peak early in the present century. The carbon use rates shown here are slightly higher than the carbon emission values reported by Marland et al. (2003). When computing carbon emissions, as noted above this difference can be accounted for to a good approximation by multiplying our carbon use rate values by the ratio of more detailed country-by-country studies of emission levels to our use rate values for a characteristic reference year. The slight dip below the curve for the temperate region at the end of the twentieth century is largely due to a large reduction in reported coal use in China, which however recovered smartly in view of rapid growth in demand for electricity in the early part



Figure 5.13: Fits for increment over 1820 base annual carbon use versus data points. Dashed and solid curves are respectively fits with and without two periodic corrections

of the following decade. For purposes of further illustration, Figs 5.15, 5.16, and 5.17 are provided to show the trends of evolution of per capita GDP, per capita annual energy use, and energy intensity of production for the two regions described.

## 5.6 Utility Optimization

To be optimized by varying the fractions  $\beta k$ , and  $\beta l$  of capital and labor to energy production and the normalized carbon intensity of production f is

$$w = pa^{\psi}(kK)^{\alpha}(la)^{\omega} = \tilde{w}/\bar{w}$$

Here  $\tilde{w}$  is the energy production rate,  $\bar{w}$  its steady state limit. The normalized carbon intensity of energy production  $f = \tilde{f}/\bar{f}$ , where  $\tilde{f}$  is the carbon intensity of production and



Figure 5.14: Annual carbon use extrapolations without periodic corrections, compared with calibration data, and for the whole world, all including addition of small base year values



Figure 5.15: Per capita GDP (thousands of 1990 PPP dollars) extrapolations for the world, tropical, and temperate regions



Figure 5.16: Per capita annual energy use (giga Joule per person) for the world, tropical, and temperate regions



Figure 5.17: Energy intensity of GDP (giga Joule thousands of 1990 PPP dollars) extrapolations for the world, tropical, and temperate regions

 $\bar{f}$ =0.0255 Gtonne/EJ is the value for coal—only for primary energy. Also

$$p = (h-1)q + 1$$

Where technology assessment suggests  $h \approx 2$ . Here

$$q = \left(1 - \left((f/g) - 1\right)^2\right)(1 - u)$$

 $u = \tilde{u}/\bar{u}$  is integrated carbon use  $\tilde{u}$  normalized to its limit value  $\bar{u}$ . Also,

$$g = f_k(1 - \tilde{s}_k \tilde{u})$$
 for  $\tilde{u}_{k-1} \le \tilde{u} \le \tilde{u}_k$ 

where  $\tilde{u}_0 = 0$ ,  $\tilde{u}_1$  is the value at the first use of oil (so  $\tilde{u}_1/\bar{u} \ll 1$ ),  $\tilde{u}_2$  is the break point in the piecewise linear fit of carbon intensity of energy production vs. integrated carbon use,  $\tilde{u}_3$  is larger than the amount of carbon used so far.  $\bar{u}$  is the amount of carbon used in the asymptotic limit when  $f \to 0$ . To lowest order in  $\epsilon_0 \sim 0.05$ , where

$$\epsilon_0 = \bar{t}/\bar{t}_C$$

where  $\bar{t}=1/(\bar{r}+\bar{\rho})$  is the capitalization time and

$$\bar{t}_C = \bar{u} / \left( \bar{f} \bar{w} \right)$$

is the timescale of for carbon depletion, we have g = f as outlined in Appendix E. In this approximation q = 1 - u and

$$p = (h-1)(1-u) + 1$$

Note that for h = 2 initially p = 2, and p declines linearly with carbon depletion u to reach an asymptotic limit  $p \to 1$ . Also, the optimization gives

$$\tilde{w} = p a^{\psi} \bar{w}$$

For the results shown in this thesis, h is set equal to 2, and  $\psi$ , and  $\bar{w}$  are calibrated against energy time series data, and u and f are computed by integrating the dimensionless form of the carbon balance equation  $\dot{u} = \epsilon_0 f w$ , using the piecewise linear approximation for f(u)calibrated against time series data on the carbon intensity of energy production.

# Chapter 6 Fuel Fractions Model: Results

This chapter provides a summary of the main results of a model of the distribution of total energy use amongst various energy sources. It examines models of technology substitution in general and their particular manifestation for the energy substitution models used in this thesis.

## 6.1 Modeling Energy Substitution

Energy substitution operates in a similar fashion to many technological advances through a series of competitive processes. In the context of general technological substitution, Fisher and Pry (1971) observed that the fractional rate of substitution of a new product or technology is proportional to the remaining amount to be substituted. Historical evidence of various substitutions in the industry is inclined toward a logistic growth pattern and compatible with this general idea. When a promising new technology is developed, it generally has a greater potential for improvement and cost reduction in the early stages of development than do more mature competing technologies. This gives the new technology potential to capture a greater share of the market until saturation, followed by subsequent decline in market share when a better substitute finds its way into the market. Marchetti (1977) provides a direct application of this idea for describing the historical trends in primary energy substitution. An alternative approach to energy substitution is based on the idea of "constant elasticity of substitution" used in integrated climate assessment models such as MERGE (Manne et al., 1995). One manifestation of this alternative uses an incentives-based

approach for fuel substitution aimed at fixing allowable limits of greenhouse gas emissions per unit of energy produced. As an emphasis of this thesis is on careful calibration of models using historical time series, it is appropriate that the fuel fractions models developed in this thesis are based on the market penetration concept. Based on this analysis, the evolution of total energy production developed in the previous chapter is divided into contributions from various energy sources based on market penetration models.

For results shown in this chapter a market penetration approach is particularly useful for conceptualizing the evolution of the global energy system as a series of substitution processes. The market penetration models used here are based on the idea that a new energy technology has an exponential growth phase in capturing increasing market share during which much of the "learning by doing" occurs. Then it is followed by saturation of the market share, and for depletable resources an eventual decline in response to depletion or rising costs of the relevant fuel supply. The reason for market share saturation for mature technologies is that capturing larger fractions of the total market with a given energy source past the "learning by doing" phase leads to increasing costs. For example, shipping coal over longer distances to capture increasing market share requires increasing allocations of capital and labor per unit of delivered energy. This is also true in the case of renewable energy resources, which can efficiently capture greater market share in areas closer to production facilities compared to longer distances where other cheaper sources are available. Here for fossil fuels we do not specifically look into how costs evolve with rising market concentration of a given energy resource, but rather use empirical evidence on how the market share evolves through initial market penetration through to a period of large cumulative use of fossil fuel sources. For nuclear energy, however, so far the impact of uranium resource depletion has not been sufficient to have a noticeable affect on market share, so in this case a separate costing model is needed to extrapolate the impact of future uranium resource depletion. The complications involved in applying these simple ideas in practice are described in detail in Appendix E.

# 6.2 Formulation of Fuel Fractions



Figure 6.1: Division of different sources of primary energy into groups competing pairwise

Technological substitution models are most simply applied to analyze the competition between two products. In the case of energy, however, generally more than two sources compete for market share. To avoid the mathematical difficulties of treating competition between multiple sources of energy simultaneously, we here divide different forms of primary energy sources into successive pairs and analyze the competition between each pair. This is done following the tree depicted in Fig 6.1. This division makes the problem easily tractable and offers analytic convenience. At the level of analysis completed so far, this is done on a region by region basis. When aggregated in the two large groups of countries analyzed here each region is assumed to follow a structurally similar evolution of energy source substitution, albeit with different particular formula parameters calibrated against historical data specific to the region.

Following Chapter 5 we use the "tropical" and "temperate" division of the world to for the results presented in this chapter. Total energy use in each of these regions is the sum of the nine different primary energy sources. For developing the fuel fractions models the nine energy sources are taken to compete with each other grouped in the following pairs: fluid fossil vs. not-fluid-fossil; coal vs. non-fossil; water vs. non-water non-fossil; new renewables vs. nuclear; uranium ore vs. other nuclear sources; and byproduct and seawater uranium vs. reprocessing. We first develop a model for the fraction of total energy use rate that comes from fluid fossil fuels as a function of the integrated use of fluid fossil fuels. Integrating the result gives an expression for integrated fluid fossil fuel use as a function of integrated energy use, which can then be inverted to give the total fluid fossil fuel depletion as a function of integrated energy use. Using the same method we can find the fractional shares and integrated use of various energy sources, working all the way down the tree depicted in Fig 6.1.

### 6.3 Fluid Fossil Fraction Model

In this model we allow fluid fossil energy sources (oil and gas) to compete with the rest of energy sources. Despite the differences between them, oil and gas have comparable natural resource endowments and offer convenience of comparatively easy fuel transport useful for many applications. It is thus appropriate to treat them together. The evolution of the

fractional share of energy from fluid fossil fuels includes an early period of increasing competitiveness due to experiential learning and then slowly declines as the more inexpensive to extract resources are depleted and the environmental impacts of their use are internalized in market decisions. This description captures the essence of the problems faced by the oil and gas industries through history. In the case of oil, lack of spare production capacity within OPEC (even discounting for Iraq's recent underproduction) itself is a manifestation of the early declining phase of oil's share in the global energy mix (Salameh, 2001). Natural gas use is likely to moderate the decline in the fractional share of fluid fossil use through the middle of this century, but eventual continuing decline is inevitable due to depletion of cheaper oil and gas resources. Thus the fractional share of oil and gas can be fit to a combination of a saturating "learning by doing" model and then patched to a fuel depletion model, in both cases as a function of integrated fluid-fossil use  $x_1$ . We here use the function  $f_1 = c_{13}(1 - e^{-c_{12}x_1}) - c_{11}x_1$  for fitting the fluid fossil fuel fraction of the total energy use obtained from the time series data. The term  $e^{-c_{12}x_1}$  in this function reflects the experiential learning, while the term  $-c_{11}x_1$  models the decline of the fluid fossil fuel fraction with increasing cumulative fluid fossil fuel use. Fitting this function to data shows that the period of early rapid growth of the fossil fuel fraction as a function of integrated fluid fossil fuel use is well separated from later gradual decline with increasing depletion, as shown in Fig 6.2. For this reason it proves adequate as well as convenient to find the cumulative use of fluid fossil fuels by piecewise integration, as described in more detail in Appendix E. This uses the approximation  $f_1 = c_{13}(1 - e^{-c_{12}x_1})$  up to the point where the impact of remaining experiential learning is equal to the impact of cumulative resource use (i.e., where  $c_{13}e^{-c_{12}x_1} = c_{11}x_1$ ). Thereafter the mature technology approximation  $f_1 = c_{13} - c_{11}x_1$  is used for integration to find  $x_1$ . This procedure is convenient because it gives a simple formula for extrapolating farther into the future where the resource depletion effect becomes increasingly important while the technology can be treated to an increasingly excellent approximation as mature. The solution to the fitting function also provides the integrated fluid fossil energy use that
can be extrapolated further in time to see the effects of technology maturation and resource depletion of fluid fossil fuels. This allows us to compute the integrated not-fluid-fossil energy use  $u_2 = u_1 - x_1$ . Fig 6.2 show the data and least squares fits for the fluid fossil fuels fraction. Clearly extrapolations so far into the future as shown in Figs 6.3 and 6.4 are highly



Figure 6.2: Data and fits for fraction of energy from fluid fossil fuels as a function of cumulative use of fluid fossil fuels in Zeta Joule (ZJ) for each region. The top curve is for tropical region and the bottom curve is for the temperate region

speculative. Nevertheless, the results shown are not completely unreasonable. In particular, the cumulative global fluid fossil fuel use over the period shown is about six times the use so far. If extraction costs were linear with cumulative extraction, then this would imply about a sixfold increase in extraction costs. Very late twentieth century delivered oil prices and North American well-head natural gas prices of around \$10/barrel and \$1.5/kft<sup>3</sup> respectively (USEIA, 2005c) put a rough upper limit on what extraction costs were then, and current prices that consumers are willing to pay suggest that some market demand may well persist even with a 5-6 fold increase in extraction costs. The results for cumulative fluid fossil fuel resource use  $x_1$  shown in Fig 6.3 allow estimation of cumulative use  $u_2 = u_1 - x_1$  of energy from other than fluid fossil fuels, and thus set the stage for the next step in evaluation of the



Figure 6.3: Extrapolation of annual use rates of fluid fossil fuels for the tropical and temperate regions far into the future, depicting the effects of resource depletion and thus reduced importance of these fuels in the energy mix



Figure 6.4: Extrapolation of cumulative annual use of fluid fossil fuels for the tropical and temperate regions far into the future

market competition tree shown in Fig 6.1. This analysis will be followed all the way down to examine the near and long-term future of spent nuclear fuel reprocessing, as described in Chapter 8 of this thesis. Because amounts of uranium ore that are estimated to be economically competitive with spent fuel reprocessing are so large, to estimate what spent fuel storage times might be on an economically competitive basis requires tracing energy use far into the distant future. This requires at least some reasonable assessment of the use of competing energy sources on a similarly long time frame.

#### 6.4 Coal Use Fraction

We use a different fitting function for coal because the technology of coal-fired power plants is treated as mature, obviating the need for a learning by doing term. Learning by doing using carbon sequestration and other technologies not developed commercially lies if at all in the future, and is not included in the model calibrated against historical data. The coal fraction is fit to a piecewise linearly declining functions of the form  $f_2 = c_{21} - c_{22}x_2$ , where  $x_2$  is the integrated coal use. The results are shown in Fig 6.5 for the tropical region and in Fig 6.6 for the temperate region. For tropical countries, there is a clear break in the pattern of declining coal fraction of energy production from other than fluid fossil fuels, which is the primary motivation for the use of a piecewise linear fitting function. The recent slow rate of decline in fractional share of coal is a reflection the continuing predominant role of coal in non-fluid-fossil energy mix in tropical region, especially in India. The driving term behind the declining fractional share of coal in the temperate region is market internalization due to significant regional environmental protection requirements and to some extent by concerns primarily in Europe over long term impacts posed by increasing concentrations of carbon in the atmosphere. How far these trends will persist into the future is of course uncertain. So far the decline in the coal fraction in temperate countries has been driven in part by market internalization of the costs of mitigating local and regional impacts of coal mining and of



Figure 6.5: A two-piece linear fit for the declining coal use share of non-fluid-fossil energy sources for the tropical region



Figure 6.6: A two-piece linear fit for the declining coal use share of non-fluid-fossil energy sources for the temperate region

power plant ash and effluents production and emissions of particulates, sulfur dioxide, and nitrogen oxides. At some point the effect of the market internalization of these local and regional effects will saturate. Thereafter economic pressure to continue to reduce the coal use rate, e.g. along the lines shown in Fig 6.7 for the temperate region, will likely need to come from market internalization of commitments to moderate atmospheric carbon loading and concomitant global warming. (To make this more concrete, results for projections of atmospheric response to carbon emissions are given in Chapter 7.) Because they are based on the assumption of continuation of historically observed trends, the results in this thesis implicitly assume that effective action to reduce carbon emissions in general and use of coal in particular will be taken on the part of developed countries after local and regional pollution controls become insufficient to continue these trends. Should this not occur, then nuclear power should be less economically attractive compared to coal. This should lead to a lower rate of depletion of uranium ore resources and may produce an even greater delay until spent fuel reprocessing becomes economically attractive on a market competition basis. Fig 6.8



Figure 6.7: Extrapolation of annual coal use in Exa Joule (EJ)

shows projected cumulative energy production from coal. Note that cumulative coal use for

the tropical region approaches but does not surpass that for the temperate region over this long time period. These results are again driven by the assumption of a long continuation of the trends on decreasing coal fraction of non-fluid-fossil energy sources shown in Fig 6.6. Note also that the energy content of cumulative coal use in these projections is considerably smaller than that for fluid fossil fuels shown in Fig 6.3. Since estimated coal resources extractable at costs similar to today's are much larger than those for fluid fossil fuels, the type of integrated coal use shown in Fig 6.8 is unlikely to have nearly as large an impact on resource extraction costs in the case of coal. Moreover, the projected cumulative use of coal by the tropical/developing region also remains less than that for the temperate region even over this long time period, despite the much larger population that accumulates in the tropical/developing region by the end of the present century. Thus, the argument that on ethical grounds developing countries should restrain coal use to curtail their atmospheric carbon emissions below that shown in Figs 6.7 and 6.8 is likely to fall on deaf ears in much of the developing world, should the developed countries also pursue the path illustrated in these figures. Thus the only way for the developed countries to impose a lower coal use on the developing ones is likely to be either by persuasion (such as subsidizing development of renewable energy) or by collusion (e.g. by major coal resource countries like Australia, China, Russia, and the United States cooperating to restrict exports of coal). Otherwise, an evolution along the lines shown in Fig 6.7, at least for much of the present century, is not unlikely even if temperate region countries do find a effective mechanism for cooperation on restricting carbon emissions.

#### 6.5 Water Use and New Renewables

The fractional share of energy use from water-driven electricity as a function of nonfossil energy use is fit using the function  $f_3 = 1 - c_{13}(1 - e^{-c_{12}x_3})$ , where  $x_3$  is the cumulative energy use from water driven electricity (hydro, geothermal and tidal). Fig 6.9 shows for



Figure 6.8: Extrapolation of cumulative coal in zetaoules (ZJ) for tropical and temperate regions

the temperate region the complement  $1 - f_3$ , which is the non-water (nuclear, wind, and solar thermal electric) fraction of non-water nonfossil energy. The result for the tropical region is similar but saturates and a much lower non-water fraction of 0.054, as opposed to 0.542 for the temperate region. This low market penetration in the tropical/developing region for a category so far dominated by nuclear power is not surprising in view of the high capital cost of nuclear technology and the political difficulties that some countries have had in getting full effective access to the international nuclear technology base (notably India, as a non-signatory of the Nuclear Nonproliferation Treaty). Were it possible for water-driven electricity production to be expanded without limit at constant returns to scale (with power output scaling linearly as capital and labor input are scaled up), then the saturated market fractions of (1-0.054) $\approx$ 0.95 and (1-0.542) $\approx$ 0.45 for the water-driven fraction of non-fossil energy would allow a larger water-driven energy output than eventually is likely to be the case given limitations on practically usable sites for large scale dams and for economic geothermal electric and tidal electric installations. In Appendix E a rationale is given for assuming limits of about 18 EJ for the Tropical region and 38 EJ for the Temperate



Figure 6.9: Market penetration: Temperate region nonfossil energy use rate fraction of nonwater energy (nuclear+wind-electric+solar-electric) vs. cumulative nonwater use (in Ze-taJoules)

region for annual water-driven electricity production at constant returns to scale. Beyond this, for the projections shown here half the additional market penetration levels (0.47 and 0.22 respectively for the tropical and temperate regions) for the water-driven fractions of incremental non-fossils energy are assumed. This accounts for the considerable potential of smaller scale hydropower projects as well as some additional siting of conventional dams and other water-driven electricity projects in more difficult sites. Results for extrapolation of the thermal energy equivalent of annual water-driven electrical energy production on this basis are shown in Fig 6.9. That these almost exactly coincide for the two regions at the end of the period plotted is largely coincidental. The new renewable fraction (solar and wind electricity) share of energy use as a function of nonwater nonfossil energy use is modeled using the formula  $f_4 = c_{43}(1 - e^{-c_{42}x_4}) + \epsilon_3\chi$ . The fitting function contains both the "learning by doing" term  $e^{-c_{42}x_4}$  and uranium resource depletion term denoted by  $\epsilon_3\chi$  as described in more detail below. Uranium depletion (from conventional mines) had a negligible impact on the costs for the over the period for which historical data is available. Moreover, new



Figure 6.10: The above figure shows the extrapolation of cumulative water-driven electric production for the tropical and temperate region

renewable energy source installations in the form of windmills are rapidly growing and are nowhere near saturation as a fraction of non-water-non-fossil energy production. For this reason the most that one can hope to obtain from data fitting is a rough approximation to the initial slope  $c_{43}c_{42}$  of the function  $f_4(x_4)$  in the approximation  $f_4 \approx c_{43}(1-e^{-c_{42}x_4})$ . That only a rough approximation to initial market penetration rates can in general be obtained by this method is particular apparent for the tropical region portion of the results shown in Fig 6.10. In practice the market penetration rate may slow as the cost of government subsidies or unfunded mandates for higher rates of new renewable energy rates becomes more appreciable with greater market penetration. Assuming, however, that initial market penetration rates are accurate to within a factor of three and the above formula continues to be appropriate, then the result by the end of the present century would be similar even for market penetration rates differing by as much as a factor of three from those estimated here. What that market penetration will be, on the other hand can not be assessed from historical data and must instead by evaluated on the basis of technology assessment. In Appendix E the rationale for choosing values of  $c_{42} = 0.3$  and for estimating the value of  $\epsilon_3 \approx 0.15$  are given. Here the sum  $c_{42} + \epsilon_3 \approx 0.45$  is the market fraction for new renewables of non-water-non-fossil energy in the very long term (leaving the remaining fraction of around 0.55 for nuclear power) since as described below the long term limit for  $\chi$  is  $\chi \to 1$ .



Figure 6.11: Linear fits, giving only a very rough estimate of the initial market penetration rate for "new renewables" / "nuclear+new renewables" (vertical axis) as a function of cumulative use of "new renewables" (wind electric and solar thermal electric), particularly given the limited tropical region experience with electricity generation from these "new renewable" sources

#### 6.6 Nuclear Energy Fractions

In the previous section we discussed the implications of using the functional form for fitting the new renewable fraction using  $f_4 = c_{43}(1 - e^{-c_{42}x_4}) + \epsilon_3\chi$ . For the purpose of projection nuclear energy use forward in time, a formula is needed for the function  $\chi$  describing the impact of uranium resource depletion. To model the effect of uranium resource depletion let  $x_5$  be energy obtained from cumulative mined uranium. As discussed in more detail in Appendix E, the formula used here is

$$\chi = y_5^{\gamma - 1}$$

where  $\chi$  is the ratio of the inflation adjusted cost of mined uranium to its long term limit value and  $y_5 = x_5/x_m$  is the cumulative nuclear energy from conventional ore normalized to the energy content of the limit amount from conventional uranium mining. The function  $y_5$ evolves according to the equation

$$dy_5/dv_4 = (1 - c_{43})(1 - y_5^{1 + \epsilon_2})(1 - \epsilon_5 y_5^{\gamma - 1})$$

where the ratio  $v_4 = x_4/x_m$  is known from the integrated use  $u_4$  of non-water-nonfossil energy, which is the difference between cumulative nonfossil energy and cumulative water-driven energy calculated as discussed above. These equations are integrated analytically for the case  $\epsilon_2 = 0$  by expanding through first order in the small parameter  $\epsilon_3$ , giving the result shown in Fig 6.12. For the results shown in Fig 6.12 the value chosen for  $\gamma$  was  $1+1/m_5 = 1.435$  for the



Figure 6.12: Annual thermal energy equivalent of production from nuclear energy and "new renewables"

reference value of the scaling exponent of conventional uranium resource availability versus mining cost of  $m_5 = 2.3$ . The ultimately economically recoverable conventional uranium resource at a nominal extraction cost of up to US\$360/kg of natural uranium metal (at 1995 prices) was divided into 108 ZJ of thermal energy equivalent for the tropical region and 248 ZJ for the temperate region, in proportion to their long term limit nuclear energy use rates, the approach to which is illustrated in Fig 6.13. The results described here follow from a



Figure 6.13: Annual thermal energy equivalent of production from nuclear energy

calculation of both the overall use of nuclear energy and the amount of that which comes from conventional uranium resources. The remainder comes from unconventional sources of fissile material, which are operationally defined here as including spent fuel reprocessing, uranium byproduct from mining of phosphates and other minerals, re-enrichment of old uranium enrichment tails discarded at the near the optimal tails assay described by the equations in Appendix E, and uranium recovered from brine and seawater. Of these unconventional sources, here 1/4 is assumed to come from reprocessing and the remaining 3/4 from other unconventional uranium resources. This is consistent with a long-term equilibrium with a conversion ration of 1/3. This is the ratio of recoverable energy in spent fuel from fresh uranium providing  $(1/3) \times (3/4) = 1/4$  of nuclear energy, with fresh unconventional uranium providing the remaining 3/4 in the long term.

Using the method described in Appendix E, with the results given here these assumptions allow a calculation of the allowable delay between burning of what was fresh uraniumonly nuclear fuel when loaded and the time of burning of fissile material recovered from the discharge of what was fresh nuclear fuel. The results of this calculation will be given in Chapter 8 as an illustration of the potential policy-relevant applications of the models developed as described here.

### Chapter 7

### Atmospheric Response Model: Results

This chapter provides the results of a simple model of the response of atmospheric carbon loading and global average temperature to anthropogenic carbon emissions. The basic elements of the global carbon cycle are described, including its relevance to current anthropogenic emissions and implications for future atmospheric composition. The atmospheric increase of carbon dioxide obtained from reported observations in Mauna Loa is fit to the results from integrating a linear differential equation. The impact of increasing concentration of carbon dioxide on earth's surface temperature is described using a simple model.

#### 7.1 Carbon and the Atmosphere

The earth's atmosphere is mostly made up of nitrogen, oxygen, and trace amounts of noble gases. Several of the remaining constituents such as carbon dioxide, methane, water vapor and others are collectively known as "greenhouse gases" because of their importance in regulating earth's climate. The presence of these gases renders the earth's surface temperature substantially warmer than it would otherwise be (Rubin, 2001). However, the total amount of carbon in the atmosphere—natural and anthropogenic—is very small compared to size of other reservoirs where the bulk of earth's carbon is stored (See Table 7.1). Paleoclimate records suggest that the global carbon cycle has been in a state of near-equilibrium for long periods of time with the atmospheric concentration of carbon dioxide remaining at preindus-trial level of around 280 ppmv throughout recent geologic (Holocene) times. Nevertheless, atmospheric carbon dioxide levels have fluctuated between a maximum of around 280 ppmv during interglacial periods and a minimum of 180 ppmv during glacial epochs in response to various biogeochemical cycles (Petit et al., 1999). These changes are effected by a combi-

Pools	Quantity (GtC)
Atmosphere	720
Oceans	
Total inorganic	$37,\!400$
Surface layer	670
Deep layer	36,730
Total organic	1,000
Lithosphere	
Sedimentary carbonates	60,000,000
Kerogens	$15,\!000,\!000$
Terrestrial biosphere	
Living biomass	600-1,000
Dead biomass	1,200
Aquatic biosphere	2
Fossil fuels	
Coal	3,510
Oil	230
Gas	140
Other (peat)	250

Table 7.1: Source: Earth's carbon reservoir (Falkowski et al., 2000)

nation of earth's orbital fluctuations and geophysical effects, including changes in mixing of the deep ocean with the surface ocean waters (Ruddiman, 2001). The resulting exchanges of carbon between the smaller atmospheric and surface ocean reservoirs and larger ground and deep open reservoirs happen on a much longer time scale. It is in this larger context that anthropogenic carbon loading of the atmosphere, albeit transient on a longer geological time scale, appear significant. Falkowski et al. (2000) say that moving further away from the atmospheric carbon loading domain that characterized the recent preindustrial global carbon cycle introduces significant uncertainties in the understanding of various climatic feedbacks that respond to changes in atmospheric concentration of carbon dioxide. There is broad scientific consensus that the current changes in earth's atmosphere, especially the increasing concentration of greenhouse gases, is a result of human activities dominated by combustion of fossil fuels and land use changes (Houghton et al., 2001). Although the surface ocean and biosphere can potentially slow the rate of atmospheric increases of carbon dioxide, there is no "natural savior" to rapidly absorb all future emissions resulting from unconstrained use of fossil fuels without a significant increase in atmospheric carbon loading. Even if anthropogenic carbon emission were to cease overnight, it would take several hundred years for the atmospheric carbon dioxide concentration to return to preindustrial levels. This is because of the long residence time for carbon in the atmosphere resulting from the varying rates at which the atmosphere exchanges carbon with other carbon sinks (Sarmiento and Gruber, 2002). Although all excess carbon in the atmosphere would be dissolved in the surface ocean and eventually reside in the deep ocean as bicarbonate ions, the removal of atmospheric carbon to the ocean is an extremely slow process. On the other hand, the atmospheric increases of carbon dioxide due to anthropogenic sources is happening at a much faster rate.

#### 7.2 Anthropogenic Carbon Emissions

For future projections of atmospheric response, the analytic approximation to the evolution of anthropogenic carbon emissions developed here has the convenience of making it comparatively easy to integrate with atmospheric response models. Here for emissions before the period over which the analytic model was calibrated we use a piecewise linear interpolation of annual estimates of carbon emissions based on country by country time series data for carbon use since 1700. All of the emissions data is rescaled from carbon use projections shown in Fig.7.2 using the emissions factor 0.95. This is the average of an assumed combustion release efficiency of 0.98 and the ratio of the emissions rate provided by Marland and Rotty (1984) for the year 2000 to the value from the data base described above.



Figure 7.1: Projections of global carbon emissions through 2200

#### 7.3 Atmospheric Carbon Levels

Several additional lines of evidence confirm that the continuing increase of atmospheric carbon dioxide levels is due to anthropogenic emissions—mostly fossil fuel burning. The decline in atmospheric oxygen levels at a rate comparable to fossil fuel emissions and the characteristic isotopic signatures of fossil fuel leave their imprint in the atmosphere (Ruddiman, 2001). However, atmospheric carbon dioxide increases only at about half the rate of fossil fuel emissions. The rest of  $CO_2$  emissions are taken up by various sinks in the global carbon cycle, which is already stressed as result of perturbations caused by various human activities. Keeling and Whorf (2004) report the measured increase in the mean annual concentration of atmospheric carbon dioxide from around 316 ppmv in 1959 to around 377 ppmv in 2004. The annual measurements of atmospheric carbon dioxide reported in Keeling and Whorf (2004) are based on one of the earliest and well known observation site in Mauna Loa, Hawaii. We here fit the annual data from this source to the results of integrating the differential equation (as explained in detail in Appendix D) from Petschel-Held et al. (1999).

$$\frac{d\tilde{C}}{d\tilde{t}} = \bar{B}\tilde{F} + \bar{\beta}\tilde{E}_{net} - \bar{\sigma}\left(\tilde{C} - \bar{C}_0\right)$$

Where  $\tilde{E}_{net}$  is the portion of the global carbon use rate emitted into the atmosphere and  $\tilde{F}$  is the cumulative global anthropogenic carbon emission. Here the constant  $\bar{\beta}=0.47$  ppm CO<sub>2</sub>/Gtonne-C converts from emissions units of Gtonne of elemental carbon to conventional atmospheric carbon concentration units of parts per million by volume of carbon dioxide in the atmosphere. Very long term clearance of atmospheric carbon into deep oceans is not accounted for in this equation, so the "saturation effect" term  $\bar{B}\tilde{F}$  produces a long term impact on atmospheric carbon concentrations that would not be seen if the clearance term  $-\bar{\sigma}(\tilde{C}-\bar{C}_0)$  drove the atmospheric carbon concentration  $\tilde{C}$  back to its preindustrial value  $\bar{C}_0$  in the limit where  $\bar{\beta}\tilde{E}$  necessarily eventually approaches zero. For this calculation a base value  $\bar{C}_0=281$  ppm was taken from the 1800 value from ice core data reported by Etheridge et al. (1998). This simple approach appears to be an adequate approximation because the atmospheric CO<sub>2</sub> concentration inferred from ice core data for before 1800 varies only slightly from this value, being 282 ppm in 1500 and averaging 279 ppm from 1500–1800.

The cumulative emissions  $\tilde{F}$  appearing on the right-hand side of the above equation for atmospheric carbon loading are estimated using the function

$$\tilde{F} = \sum_{i} \tilde{F}_{i} = \sum_{i} \left( \bar{E}_{i} / \tilde{\nu}_{i} \right) \int_{0}^{a_{i}} da_{i} E_{i} / \left( a_{i} z_{i} \right)$$

where the emissions rate summed over emitting regions is written as  $\tilde{E}_{net} = \sum_i \bar{E}_i E_i$ , where the scale  $\bar{E}_i$  and dimensionless carbon emissions functions  $E_i(a_i)$  are determined for each region *i* as described in Chapter 5.

This linear response model for  $\tilde{C}$  given above is used for calibrating the parameters based on

the Mauna Loa time series data on carbon concentration, with the result shown in Fig.7.2. For this calibration the clearance rate coefficient  $\bar{\sigma}$  and the atmospheric concentration in 1959 were adjusted to give the fit indicated by the dashed curve. (The value of  $\tilde{C}$  at the the time of the first Mauna Loa data is taken to be an adjustable parameter because the reported measurement value is taken not to be a point through which the fitting curve must exactly pass but rather to be a random sample of from a probability distribution for which the fitted value maximizes the likelihood of the data given the theory by adjusting the estimated fitting parameters.) For the solid curve in Fig 7.2 the phases and amplitudes corresponding to the two dominant periods in the discrete Fourier power spectrum for the residuals between the data and the dashed curve are also adjusted via least squares. The residual differences between the data and the solid curves are shown in Fig 7.3. Over the



Figure 7.2: Biennially averaged Mauna Loa measurements of atmospheric carbon dioxide in parts per million by volume (ppmv) vs. fits with periodic corrections included (solid curve) and omitted (dashed curve)

data calibration time frame, the saturation effect  $\bar{B}\tilde{F}$  has so little impact on the result that the parameter  $\bar{B}$  is difficult to calibrate using this data. Instead, an a priori estimate 0.1 for the ratio  $\bar{B}/\bar{\beta}\bar{\sigma}$  based on ocean chemistry is taken from Petschel-Held et al. (1999). The value of this ratio has little effect on the results over the data calibration period, but it completely determines the long-term quasi-equilibrium state (neglecting deep ocean mixing) and is thus important for the part of the projection to the more distance future shown in Fig 7.4. It should be emphasized that the comparatively modest maximum level of atmospheric  $CO_2$  shown in Fig 7.4 depends on the assumption of continued linear decline in the carbon intensities of energy production as shown in Fig 5.9. Should there be another change in the slope of carbon/energy versus integrated used that leads to a greater carbon emissions, the resulting peak atmospheric carbon levels would be higher. Since there is no evidence of this in the available data, analysis of such possibilities requires a model of the factors determining future carbon emissions and lies outside the scope of this thesis.



Figure 7.3: Difference between biennially averaged Mauna Loa measurements of atmospheric carbon dioxide in parts per million by volume (ppmv) and fit with periodic corrections included



Figure 7.4: Projections of atmospheric carbon concentration through the year 2200

#### 7.4 Carbon and Temperature Response

Elevated levels of  $CO_2$  in the atmosphere have significant impact on various factors that influence climate such as temperature, sea level, precipitation rates and others. Assessments of "climate sensitivity" are essentially based on various climate models that test the temperature response for a doubling of  $CO_2$  concentration in the atmosphere. More than a century ago Arrhenius predicted a temperature rise of 5°C to 6°C for a doubling of atmospheric concentration in his classic paper (Uppenbrink, 1996). Modern estimates have always fallen in the range from a low of 1.5°C to a high of 4.5°C. One the known deficiencies of many climate models is the absence of systematic integrated uncertainty analysis for these estimates. IPCC's Third Assessment Report (TAR) presented a range of values for the climate sensitivity with projected temperature rise between 1.4°C to 5.8°C for a doubling of  $CO_2$  without assigning any probability for these estimates. However, Webster et al. (2003) have recently presented maximum likelihood estimates for temperatures rises resulting from a doubling of  $CO_2$  concentration: 2.5 percent likelihood for a temperature rise of 4.9°C by 2100 in the absence of any emission reductions and 3.2°C for an "aggressive" emission reduction policy. It is quite straightforward to conclude that increasing levels of carbon dioxide in the atmosphere should cause corresponding increases earth's surface temperature because of the ability of  $CO_2$  to trap infrared radiation from the sun. Sophisticated climate models have estimated global temperatures over the past millennium, which is punctuated by fluctuations caused by volcanic activities and moderated by changes in solar irradiance. However, rapid deviations from the mean surface temperature coincided with the increase in the atmospheric levels of  $CO_2$  and other greenhouse gases since industrialization. This is more pronounced in the observed deviations during the twentieth century. Although natural factors can explain part of the temperature variability, global mean temperature responds linearly to estimated changes in atmospheric levels of greenhouse gases observed so far to a good approximation (Scott et al., 2000). The change in atmospheric  $CO_2$  concentration due to fossil fuel combustion is estimated to cause most of the observed temperature increases during the twentieth century (Thomson, 1997).

Paleoclimate records also attest the correlation between greenhouse gas concentrations and temperature changes. Petit et al. (1999) provide evidence from the Vostok drilling expedition, which yielded the deepest ice core ever recovered and helped track the temperature history of the past 420,000 years. The  $CO_2$  levels recorded in these ice sheets correlated "strongly" with Antarctic temperatures, thus supporting the idea that greenhouse gases contributed significantly to the transition from the glacial to interglacial period. This correlation is particularly relevant in the context of current concerns about the implications of increasing greenhouse gas levels for earth's future climate. The earth's surface temperature depends on a delicate balance between the amount of solar radiation absorbed by surface and atmosphere and the amount of outgoing radiation. The former is determined by the surface temperature and reflectivity (albedo), while the latter is determined by the greenhouse gases and particulates. The increase in the concentration of greenhouse gases has led to an "enhanced greenhouse effect" and resulted in a "positive forcing" of the climate system. Similarly, the anthropogenic increases in aerosol concentration have resulted in increases surface reflectivity and thus a "negative forcing." Though the effect of aerosols have moderated the warming of earth, future net forcing are expected to be "unequivocally positive and substantial in magnitude" in the absence of any significant effort to reduce emissions (Charlson et al., 2005).

#### 7.5 Temperature Response Model

Here we present the result of calibration of temperature response based on a differential equation from Petschel-Held et al. (1999) that is linear in the temperature response. This equation is

$$\frac{d\tilde{T}}{d\tilde{t}} = \bar{\mu} \operatorname{Ln}[\tilde{C}/\bar{C}_0] - \bar{\alpha} \left(\tilde{T} - \bar{T}_0\right)$$

The dependence of the atmospheric temperature response on atmospheric carbon concentration is nonlinear, although for small increases in the atmospheric concentration over its preindustrial value  $\bar{C}_0$  the response is approximately linear. The global temperature data obtained from Jones et al. (2001) is fit to the solution of the above linear differential equation (details of which are provided in Appendix D). As with the atmospheric carbon loading response described above, one of the parameters in this model (in this case  $\bar{\alpha}$ ) has too little effect over the data calibration period to be fit with a well-defined value using a simple least squares approach. Thus  $\bar{\alpha}$  needs to be fit with experience from physical climatology models, which has a reference value of  $\bar{\alpha}$ =0.17 taken from Petschel-Held et al. (1999). The parameters that are adjusted to obtain the fit shown by the dashed curve in Fig 7.6 are the coefficient  $\bar{\mu}$  (which is a measure of the relative importance of changes in opacity to infrared re-radiation and thermal inertia) and the difference  $\bar{T}_0$  between the preindustrial temperature and the reference value in the database.

In this case the data extends far enough back in time that the initial temperature can be approximated as equal to the preindustrial mean temperature, which is estimated along with the parameter  $\bar{\mu}$  with a least square procedure. In this case the data used go back far enough that it appears to be adequate to start the numerical integration with an initial temperature equal to the base temperature. This procedure gives the result shown in Fig 7.5 for the dashed curve without periodic corrections and by the solid curve in Fig 7.5 with periodicity corrections. The residual differences between the data and periodicity-corrected fit is shown in Fig 7.6.

The periodic corrections illustrated in Fig 7.5 remove obvious violations of the assump-



Figure 7.5: Data for quinquennially averaged increase in global average temperature increase over pre-nineteenth century base versus fit without (dashed curve) and with two periodic components (solid curve)

tion that residuals between the data and fit are independently and identically normally distributed, as can be seen by comparing Fig 7.5 and Fig 7.6. Over the range of years used for the model calibration, these periodic corrections thus provide an empirical description of the difference between the background trend toward higher global average atmospheric temperature and five-year averages of its actual value. However, greater attention to the underlying reasons behind such fluctuations is needed before extrapolating differences from the background trend into the future. In particular, the pause in growth in global average temperature from c. 1940 to c. 1975 may in part be related to changes connected with poor



Figure 7.6: Difference between quinquennially averaged increase in global average temperature and fit with two periodic components.

control of emissions other than  $\text{CO}_2$  from use of fossil fuels. The underlying reason for such a hiatus may be different that for the earlier one c. 1870–1905 and not necessarily repeat in the future. The underlying trend toward higher temperatures, on the other hand, is widely agreed to be connected to increases in atmospheric carbon loading and is thus sensible to extrapolation into the future as shown in Fig 7.7. This extrapolate should be viewed as average around which fluctuations of five-year average temperature on the order of  $\pm 0.1^{\circ}$ C may occur but without a known phase or amplitude that can be usefully extrapolated into the distant future. There are a number of other important qualifications that should be kept in mind concerning the result shown in Fig 7.7. First and foremost, this result rests upon an assumption of indefinite continuation of the trends of linear decline in the carbon intensity of production illustrated in Fig 5.9. For all practical purposes, this effectively assumes that an explicit or implicit global agreement on limiting carbon emissions will before long augment the regional pollution constraints and other factors that so far have been responsible for continuing reductions in the carbon intensity of energy production. Second, this projection relies on very simple models of the evolution of atmospheric carbon loading and the global



Figure 7.7: Increase of global average temperature over preindustrial base value and projections through 2200

average temperature response. Third, what is really useful in the context of the large uncertainties in such extrapolations is not yet another single scenario but rather a probability distribution for possible future outcomes. Ways in which the present work might readily be extended to account for these consideration are discussed in Chapter 8.

## Chapter 8 Conclusion

This chapter summarizes the theme and scope of the dissertation research presented in the previous seven chapters, and highlights potential areas for future work based on the methods developed in this thesis. Chapter 1 outlined four principal contexts—economic, security, resource depletion and environmental impact—in which energy plays a predominant role. This broad overview was presented to place energy issues in contemporary context relevant to public policy, and to motivate the approach and discussion in succeeding chapters on the basis of five qualitative conclusions: (1) there are no obvious prospects for a recurrence of the kind of devastating global international struggles over control of energy resources that punctuated the first half of the twentieth century; (2) it appears reasonable to treat large economic aggregates as gradually evolving in a state not drastically far from an equilibrium; (3) concerns about the environmental effects of burning coal and exporters self-imposed restrictions on oil exports are likely to continue for the readily foreseeable future; (4) any pause in reductions in the carbon intensity of energy production resulting from market internalization of regional pollution costs is likely to be temporary, until the effects of global warming become serious enough to precipitate effective global cooperation on reduction of carbon emissions; and (5) there is reasonably likely to be a significant increase well into the twenty-first century in the global use of nuclear energy.

Chapter 2 surveyed the literature relevant to energy studies and modeling. It was noted that a significant number of energy studies since the 1950s reflected the "spirit of time" and its prevailing attitudes. These attitudes included a transition from technological exuberance reflected in assumptions about continuing exponential growth to the influence of neo-Malthusian ideas and environmentalism tending toward normative analysis. These approaches are in some cases in contrast to the approach taken here of modeling transition to energy sustainability on an analytic rather than normative basis.

Chapter 3 described the sources and methods used for constructing time series data for population, GDP, and other socioeconomic indicators. Chapter 4 outlined the sources and methods used for constructing energy consumption time series for nine different commercial primary energy sources (oil, gas, coal, nuclear, hydro, geothermal, tidal, solar, and wind). These efforts provide a unique country-by-country dataset for energy econometric analysis and climate change studies. In particular, they allow the convenient aggregation of data into any desired set of country groupings and the selection of any desired set of time series during the industrial era. As an example for energy econometric modeling and fuel fractions analysis, the 220 countries in the database were grouped into a specifically defined "tropical" and "temperate" regions for analytical convenience and simplicity. The majority of countries in each of these two regions share a common demographic profile in terms of where they stand in the path of demographic transition from high to low fertility.

Using the tropical/temperate global disaggregation and the databases described in Chapters 4 and 5, region-specific modeling results were presented in Chapters 5 and 6. In Chapter 5 a population model was developed and population growth rates used for calibrating an index of economic development. In the same chapter results were presented for models of per capita GDP growth, total GDP, energy use, and carbon use. All the results are compatible with a gradual transition to energy sustainability. Chapter 6 presented results for the fractions of energy use from various sources by grouping nine commercial energy sources into pairs of competing energy types. In combination with the idea of experiential learning and fuel resource depletion, analysis based on this division provided estimates for future evolution of the fractional shares, annual use rates, and cumulative use of individual energy sources. The unified approach adopted here helps to conceptualize and understand the dynamics of evolution of importance of various energy resources over time.

Chapter 7 presented results for two simple models for the atmospheric response to anthropogenic carbon emissions. The model for atmospheric carbon loading uses a pair of linear differential equations with a linear response to carbon emissions. The model for evolution of global average temperature uses a linear differential equation with a driving function that is nonlinear in the anthropogenic increase in atmospheric carbon loading. After calibrating the adjustable parameters in these models that affect near-term behavior and choosing physically appropriate values for the parameters that determine long-term behavior, projections of future atmospheric carbon loading and global average temperature were made.

#### 8.1 Advances

The work in this thesis attempts to advance the state of the art in a number of different ways. First, as noted above a useful and comprehensive database has been assembled in the readily usable spreadsheet form. PERL coding has also been developed to automate the process of updating the nine energy source time series included in this database. Second, this work has developed a set of formulas based on readily available standard functions for the evolution of population, GDP, carbon intensity of energy production, energy use, carbon use, and the fractions of energy provided by five different groups of resource. All of these models have been calibrated against available data using methods that leave the residuals between data and theory in general without obvious significant deviation from being independently and identically distributed, both by visual observation and by statistical tests for residual periodicity and correlation of nearest neighbors in time series. Fissile material sources for nuclear energy have further been broken down into analytic formulas for fractions from uranium mining, spent fuel reprocessing, and other uranium resources; and the impact of uranium resource depletion on the long term evolution of the market fraction captured by nuclear energy has been modeled using information on costing available in the literature. While the analytic formulas developed for GDP, energy use, and carbon emissions are based on the approximation of small departures from quasistationary equilibrium, they are derived with a utility maximization procedure that can readily be extended to model recovery from drastic perturbations, such as the recovery of economies affected by World War II and late twentieth century economic restructuring of centrally planned economies. The capitalization rate that determines the timescale for such recovery has been calibrated systematically against available data on interest and lending rates, GDP growth rates, and depreciation rates and found to be qualitatively compatible with the time frame observed for such recoveries. Other parameters taken to be global constants have also been systematically calibrated against data from the literature. This includes the labor and capital fractions of production and most notably the inverse of the inter-temporal substitutability of consumption that defines the per capita utility of consumption, which had been assigned various and seemingly somewhat arbitrary values in some previous studies. In addition in Appendix F of this thesis exact and suitably approximate statistical methods are presented for sampling probability distributions for all of the parameters calibrated against observational data. Moreover, the formulas provided for evolution of population, GDP, and energy use and carbon emissions are all developed in terms of continuous logistic functions that map all time onto a unit interval and allow convenient boundary value analysis that guaranties self-consistent solutions extending arbitrarily far forward and backward in time. The modeling of increments over preindustrial base values allows for qualitatively sensible extrapolations back to quasistationary nearly subsistence preindustrial economies.

These advances should be useful in a variety of ways. Perhaps the most significant will be the easy availability of an up to date and comprehensive database that can support detailed time-series analysis. Also, some other studies, particularly those of the "bottom-up" variety that concentrate on more detailed analysis of various energy sources, are based on detailed numerical codes whose details can be difficult to keep fully transparent to outside interested parties. Moreover, when numerical methods are used that rely on direct integral optimization methods, the long discrete time intervals used can make calibration against detailed time series awkward and computationally burdensome. Thus the methods presented here have the potential to free other researchers who would review or use them from some of the restrictions to which other approaches are prone.

The rest of this chapter describes some of the uses to which the present work might usefully be put. It starts with a particular example of a policy-relevant question: if spent fuel reprocessing eventually becomes economically competitive with other sources of fissile material for nuclear energy, how long might spent fuel be stored before reprocessing. The answer to this question has important consequences for design of new nuclear power systems that may support expansion of nuclear energy use along the lines outlined in Chapter 6. If the likely spent fuel storage time is only a few decades, then it may not be desirable to design for reactor-site or off-site dry cask storage capacity for the lifetime reactor fuel load. Otherwise such designs would best be accounted for when a new round of nuclear power systems is built. The preliminary analysis presented in the following section suggests that the latter may be the case. After this comes qualitative discussion of the how the present work might be used as the basis for systematic analysis of outcome probabilities. Of particular interest in this regard is the future of global average atmospheric temperature. Systematic analysis of outcome probabilities could also be usefully applied to the spent fuel storage problem.

#### 8.2 Spent Fuel Management Futures

An important question concerning the future of nuclear power is the management of spent nuclear fuel. There is a view that reprocessing is prohibitively expensive and that suggests that permanent disposal may be a more appropriate economic solution (Bunn et al., 2005). Some proponents of nuclear power cite the current political difficulties of permanent disposal and argue for reprocessing as a better solution (Regalbuto, 2005). The alternative of interim storage is an effective approach to handle the problem of spent nuclear fuel for the next 100 years or even more. We discuss interim storage not as a final stage of the nuclear fuel cycle but essentially as an appropriate arrangement until the economic and political questions surrounding permanent disposal and reprocessing are resolved. Since surface storage in dry casks is a questionable approach for the entire hazardous life time of the spent fuel, "permanent" disposal will be required in long term. Although planned interim storage times typically range from 50-100 years, there is consideration of going up to 300 years. Haapalehto and Wilmer (2003) say that if permanent disposal siting continues to be a problem that cannot be overcome, then interim storage in excess of 100 years may also become a reality. In any case, given discount rates in excess two percent derived here from empirical evidence, there should be major economic advantages to postponing permanent large scale commissioning of expensive "permanent" disposal facilities unless and until it becomes clear that the material placed in them will not subsequently be dug up for reprocessing.

Around 10500 mtHM (metric tons of heavy metal) of spent fuel are generated worldwide every year and only one third of this is usually reprocessed (Haapalehto and Wilmer, 2003). Accumulation of spent fuel has been growing worldwide and is expected to increase when more reactors come online in Asian countries. This makes the issue of interim storage even more urgent. Currently spent fuel is stored temporarily at reactor sites. Since most countries worldwide are adopting a "wait and see" policy, estimating the delay time (the time between the discharge of fuel from reactor and the final decision to reprocess or dispose of it permanently) is of interest.

From the fuel fractions model we have estimated the time delay from burning to reburning nuclear fuel for the two regions. The tropical region has a longer time delay because nuclear power growth is faster in that region on a percentage basis, building up a large stock of spent fuel during early times when uranium ore is cheap compared to the long term use and reprocessing rates.



Figure 8.1: Time between fuel burn and reburn after reprocessing for the tropical and temperate regions

#### 8.3 Scope for Future Work

Systematic probabilistic outcome assessment based on parameters calibrated with historical time series requires sampling of probability distributions for modeling parameters as constrained by data fitting. Appendix F of this thesis describes approximate methods for accomplishing this. These can also provide a useful numerical starting point for more precise sampling methods such as rejection sampling based on the ratio between approximate and exact probability distributions or similar outcome weighting methods. When applied to long term projections, uncertainties about the future of restrictions on carbon emissions should also be accounted for. One way to do this is to continue the linear extrapolations of decreasing carbon intensity or coal use provided in this thesis until a target level of carbon intensity is reached (e.g. about one half of the value for a coal-dominated economy), and then assume a "new age of coal" at about constant carbon intensity of energy production until carbon emissions have accumulated to a point that triggers a return to declining carbon intensity of production (e.g. when the atmospheric carbon load has about doubled from its preindustrial value). The model described in this thesis is set up so that parameters such as these, and indeed all of the other parameters not calibrated against specific databases, can be sampled from probability distributions with various degrees of a priori uncertainty (e.g. log-normal probability distributions with a 67 percent chance of the sampled parameter lying within a factor such as 1.2, 1.3, or 1.4 of the reference parameter value). One advantage of the various analytic and simply integrable methods develop here is that a large number of recalibrations and samplings of calibrated parameters can readily be done for various assumptions about those limited number of parameters whose values cannot be directed calibrated against data streams. Such analysis could be usefully applied both to the global warming question and the problem of spent nuclear fuel storage. With a modest extension of the models developed here, such an approach could also be applied to the question of medium-term oscillation or long-term evolution of fossil fuel prices.

# Appendix A Energy Units and Conversion

Energy data including production, trade, bunkers, and apparent consumption are converted to standard units. In order to make energy comparisons across countries it is essential to convert them to a preferred standard unit. Our choice of unit for all energy consumption data is Exajoule (10<sup>18</sup> joule). The conversion factors used for converting various primary and secondary fuels in terms of thermal energy equivalent are given in the next page. Unless otherwise stated within brackets the fuels on the left hand side are originally provided in thousands of metric tons (Ktonne). The right hand side provides the conversion factor to convert from original units to Exajoule.

Fuel Type	<b>Conversion Factor</b>
Hard coal	$29.3076 \times 10^{-6}$
Lignite	$11.2834 \times 10^{-6}$
Hard coal briquettes	$29.3076 \times 10^{-6}$
Lignite briquettes	$19.6361 \times 10^{-6}$
Gas coke	$26.3768 \times 10^{-6}$
Coke-oven coke	$26.3768 \times 10^{-6}$
Brown coal coke	$19.6361 \times 10^{-6}$
Crude petroleum	$41.8680 \times 10^{-6}$
Natural gas liquids	$45.1923 \times 10^{-6}$
Aviation gasoline	$43.9614 \times 10^{-6}$
Motor gasoline	$43.9614 \times 10^{-6}$
Natural gasoline	$44.8992 \times 10^{-6}$
Jet fuel	$43.1994 \times 10^{-6}$
Kerosene	$43.1994 \times 10^{-6}$
Naphtha	$44.1289 \times 10^{-6}$
Gas diesel oil	$42.4960 \times 10^{-6}$
Residual fuel oil	$41.4996 \times 10^{-6}$
Feedstock	$43.9405 \times 10^{-6}$
Plant condensate	$44.3131 \times 10^{-6}$
Liquefied petroleum gas	$45.5440 \times 10^{-6}$
Electricity $(10^6 \text{ kWhr})$	$9.47 \times 10^{-6}$

Table A.1: Source: Energy Conversion Factors (UNSD, 2005)
# Appendix B Interpolation Methods

This appendix describes statistical methods used for reconstructing continuous time series data on population, GDP, energy production and consumption, real and lending interest rates from various source files. Interpolation methods for handling the problem of missing or unreported data and the underlying assumptions are described. In addition, PERL programs developed for automating linear and geometric interpolation procedures are presented.

## **B.1** Introduction

It is not uncommon to find missing or unreported values in the time series data for various socioeconomic indicators in the source files provided by national or international statistical agencies. These "holes" create a considerable problem for using the source files in any statistical modeling framework. There are various methods for estimating the probable values that lie between two reported data points in time. Computer aided programs like *Mathematica* and MATLAB, which are frequently used for interpolating experimental data in science and engineering, can also be used for interpolating the types of time series data used in this thesis. Since the data files used here are significantly large, using these programs is not always an efficient approach in terms of execution speed and memory management in desktop computers. Microsoft Excel has inbuilt functions for interpolating missing values in time series data using various types of regression functions, but their utility is limited for handling large and complex databases. Moreover, the piecemeal approach of doing various manipulations using the Excel front end are prone to human error and inconsistencies as

well as being time-consuming. We here present methods for automating these procedures using the PERL programming language. Two types of interpolation for addressing the issue of missing or unreported values in the source files are followed here. These are linear interpolation and geometric interpolation.

### **B.2** Linear Interpolation

Linear interpolation is suited for lending and real interest rates, since these do not generally steadily grow exponentially in time. Linear interpolation can be performed using least squares techniques by fitting a linear regression trend line to the available data and interpret the missing values from the linear fit. We use the PERL script given at the end of the appendix, which performs linear interpolation by using the following Excel formula to calculate the interpolation step value given as

In the above formula "end" and "start" represent the reported values bracketing missing data, with "COLUMN(end)" and "COLUMN(start)" representing the cell addresses of the two available data points in a slice of the time series. Using this step value between successive data points, the PERL script linearly interpolates the entire source file. The following example shows how the given Perl code performs successive interpolation. Suppose COL-UMN(A3) and COLUMN(A9) have reported values 9 and 11 in the annual time series, the missing or unreported values for five years can be linearly interpolated as follows A3: =9

A4: =A3+B3; B3: =(A9-A3)/(COLUMN(A9)-COLUMN(A3))A5: =A4+B4; B4: =(A9-A4)/(COLUMN(A9)-COLUMN(A4))A6: =A5+B5; B5: =(A9-A5)/(COLUMN(A9)-COLUMN(A5)) A7: =A6+B6; B6: =(A9-A6)/(COLUMN(A9)-COLUMN(A6)) A8: =A7+B7; B7: =(A9-A7)/(COLUMN(A9)-COLUMN(A7)) A9: 11

### **B.3** Geometric Interpolation

In financial statistics geometric growth rate is used to calculate the compound growth rate over discrete time intervals for payment of interest and other transactions. Since population, GDP, and energy data are provided at equal time intervals, mostly annually, the compound growth model used in financial calculations is more appropriate for treating these estimates. The annual average growth rate over time between which data is available is calculated using the following expression

$$r = (\text{end/start})^{\frac{1}{n+1}} - 1$$

In the formula "end" and "start" represent the actual values in a slice of the time series and n is the number of empty cells. n + 1 is also the number of years over which the compound growth rate needs to be calculated. Using the same example given in the discussion on linear interpolation we can perform the geometric interpolation as follows

In Appendix G (see attached CD-ROM) the PERL code for performing linear and geometric interpolation is given. This code enormously simplifies the interpolation procedures done in programs like MS Excel. For convenience, however, the input files (source files) are always provided in a tab-delimited text format generated by Excel with the country name in the first column, country codes (ISO Codes) in the second and the time series data from the third column onwards. Two header rows with a label and dates and one footer row with data source information are also included in each resulting Excel time series file. The codes provided in Appendix G do linear and geometric interpolation for any type of time series data from 1950 to a specified end year (here 2002). Interpolation for longer time series starting from an earlier period can also be performed using these codes, but the corresponding years should be added in the code.

# Appendix C Utility Maximizing Energy and Carbon Use

Analytic methods are derived here for approximating evolution of capital and labor applied to a two-sector model of production of gross domestic product and energy input to gross domestic production, accounting for the effect of depletion of fossil carbon resources. The rate of population growth is used to calibrate a logistic development index. The GDP production function is log-linear in capital, labor, energy input, and the development index. The energy production function is log-linear in capital, labor, development index, and a production efficiency factor that decreases linearly with fossil carbon depletion. The evolution of capital and labor in each sector is determined by maximizing the total time-integrated discounted utility of per capita consumption. Regularity boundary conditions enforce balanced growth in the early time limit and sustainable use of renewable resources in the large time limit. A capitalization rate is defined as the sum of depreciation social discount rates. Analytic results are obtained by expanding in three small parameters: the fraction of total capital applied to energy production, the ratio of the development rate to the carbon depletion rate, and the ratio of the capitalization rate to the development rate. In this approximation, the production function used is consistent with a piecewise linear evolution of the carbon intensity of energy production with cumulative fossil carbon depletion. The model so derived is suitable for fitting historical data and extrapolations of the use of energy and atmospheric carbon emissions for any groups of countries for which the assumptions made are an adequate approximation.

# C.1 Choice of Model

There is a tension in the choice of energy use models between richness and simplicity. Complex models can provide detailed descriptions of fuel substitution and short-term variability at the expense of requiring computational coding that can be difficult to calibrate systematically against historical data streams. Simple empirical formulas can be easier to calibrate but require additional ad hoc assumptions when applied outside of a limited parameter range. Here we are interested in constructing models for extrapolation up to century or more into the future, to be used in studies of global climate change and long-term storage of spent nuclear fuel, with emphasis on preparing for quantifying uncertainties inherent in such extrapolations. For this purpose it is helpful to have a model which yields analytic results. It also suffices to use approximations commensurate with uncertainties about how well the underlying formulation will hold up over a long time period. On the other hand, it is also useful to have an underlying formulation that can readily be generalized to deal with complex phenomena, such as the economic shocks that occurred during the WWII and with the collapse of the Soviet Union. To this end we start here with a utility optimization model formulated to deal with such transient phenomena but also derive approximate analytic solutions whose adjustable parameters can readily be calibrated against data from less turbulent times.

The approach taken here is to maximize the total time-integrated discounted utility of per capita consumption in a two-sector production model. Everything modeled here is for the increment over specified base values before the onset of exponential growth at a rate comparable to that in the latter part of the nineteenth century and part of the twentieth century. Thus these base values needed to be subtracted from the data when calibrating the model and then added back to the model results to estimate total current and future values.

The GDP production function is log-linear in capital, labor, energy input, and a development index. The energy production function is log-linear in capital, labor, development index, and a production efficiency factor that decreases with fossil carbon depletion. The development index is a measure of "social capital" that often survives major political upheavals, after which economic production returns on a "capitalization timescale" to its previous growth pattern. Denoting the development index as a, and its complement as z = 1-a, the equation  $da/dt = \nu za$  leads to the logistic result

$$a = \frac{1}{1 + e^{-\nu t}}$$

where t is the time from the point where a = 1/2 in units of capitalization time defined below. Noting that low population growth rates tend to correlate with higher levels of economic development, we model population growth as  $dP/dt = \nu zP$ . This leads to a logistic evolution for population and allows the data on the annual fractional population growth to be used to calibrate the parameters defining how the development index evolves with time. Gross domestic product is taken to be proportional to

 $Y = (a^{\eta}((1-\beta k)K)^{\alpha}((1-\beta l)a)^{\omega})^{\varphi}w^{\beta}$ 

for some constant  $\eta$  to be calibrated against time series data. Here  $\beta k$  is the fraction applied to energy production of total capital K, leaving  $(1 - \beta k)K$  left for final goods production. The product  $\beta l$  is the fraction of labor applied to the energy production rate, which is proportional to

$$w = pa^{\zeta} (kK)^{\alpha} (la)^{\omega}$$

Here  $\zeta$  is another constant to be calibrated against time series data. We assume that the total labor supply is proportional to population, which in turn is proportional to a since both evolve logistically. We also choose units for total capital K and energy use rate w so that both approach unity in the long time limit if  $p \to 1$  then. We also assume constant returns to scale for labor and capital, and energy as factors of final goods production and

for labor and capital as factors of energy production. Thus

$$\omega = 1 - \alpha$$

and

$$\varphi = 1 - \beta$$

To account for the impact of depletion of fossil carbon on energy productivity, we set

$$p = (h-1)q + 1$$

Here

$$q = (1 - ((f/g) - 1)^2)(1 - u)$$

and

$$g = f_k(1 - s_k u) \text{ for } u_{k-1} \le u \le u_k$$

Here u is cumulative fossil carbon use in units of the maximum amount of fossil carbon ever used. Also  $u_0 = 0$ , and the other  $u_k$  are break points for a piecewise linear approximation with intercepts  $f_k$  and slopes  $m_k = f_k s_k$  determined empirically for the period for which historical data is available and otherwise according to postulates about the future of energy economics. For the present purposes it will be seen that it suffices to know that the productivity coefficient p maximizes with respect to the carbon intensity of energy production f when f = g. This will turn out to be the only result needed when the solutions are expanded to lowest order in the generally small ratio of the fossil carbon depletion rate to the development rate. The quadratic form that yields this result is included to allow exploration of the "shadow effect of future fossil fuel depletion" within a sensible utility maximization framework should this ever be desired. The model is formulated so that the ratio of energy produced with given capital and labor and development level with f = g = 1 initially to the renewables-only limit f = 0 is a number h that can be estimated from an understanding of available technologies for production of energy from fossil and renewable sources.

In the units used here, gross domestic product (GDP) is  $Y/\alpha$  and the amount of it left over for consumption C after accounting for depreciation rK and the time rate of change of capital  $\dot{K}$  is

$$C = (Y/\alpha) - rK - \dot{K}$$

To obtain the evolution of the control functions K, k, l, f, and u, we maximize the total time-integrated per capita utility  $(C/a)^{1-\theta}/(1-\theta)$  of consumption, discounted at a rate  $e^{-\rho t}$ . Multiplying  $e^{-\rho t}(C/a)^{1-\theta}/(1-\theta)$  by the function of time a which is proportional to population and integrating over time gives

$$\int_t^\infty dt \, a e^{-\rho t} (C/a)^{1-\theta} / (1-\theta)$$

The desired result can be obtained by maximizing any multiple of this plus any function independent of the control variables, subject to the carbon balance constraint

$$\dot{u} = \epsilon_0 f w$$

where  $\epsilon_0$  is generally small when u is measured in units of the total amount of fossil carbon ever to be used.

Defining

$$\mathcal{L} = a^{\theta} e^{-\rho t} C^{1-\theta} / (1-\theta) + \kappa \beta a^{\theta} e^{-\rho t} C^{-\theta} (fw - \dot{u}/\epsilon_0)$$

we perform the maximization using the Lagrangean equations

$$0 = \frac{\delta \mathcal{L}}{\delta k} = \frac{\delta \mathcal{L}}{\delta l} = \frac{\delta \mathcal{L}}{\delta f} = \frac{\delta \mathcal{L}}{\delta u} - \frac{d}{dt} (\frac{\delta \mathcal{L}}{\delta \dot{u}}) = \frac{\delta \mathcal{L}}{\delta K} - \frac{d}{dt} (\frac{\delta \mathcal{L}}{\delta \dot{K}})$$

Here the notation  $\delta \mathcal{L}/\delta X$  is used to denote partial derivatives of  $\mathcal{L}$ —considered as a function  $\mathcal{L}(k, l, f, u, \dot{u}, K, \dot{K})$ —with respect to X for any one of the indicated seven arguments, and the notation d/dt is used for the differentiating such results then considered as a function of the single variable t. The Lagrangean multiplier for the carbon balance constraint is written in the form  $\kappa \beta a^{\theta} e^{-\rho t} C^{-\theta}$  because each Euler-Lagrange equation will be multiplied by  $a^{-\theta} e^{\rho t} C^{\theta}$ , so that  $\beta \kappa$  is the resulting function of interest. One can obtain analytic results for this problem when it is appropriate to expand in the "capital fraction of energy"  $\beta$ , the dimensionless fossil carbon depletion rate  $\epsilon_0$ , and a third parameter

$$\epsilon_1 = \nu \theta \xi$$

where

$$\xi = \eta/\omega$$

For the cases of interest here  $\epsilon_1$  turns out to be reasonably small when time is measured in units of the inverse of the sum of the depreciation rate and the social discount rate, i.e. in units where

$$r + \rho = 1$$

As shown below under the heading "Evolution of GDP," the parameter  $\epsilon_1$  is a measure of the "departure from quasistationary equilibrium," which is the fractional amount by which the full solution differs from what would be obtained at any time if the value of a were frozen and the solution in the one-sector ( $\beta \rightarrow 1$ ) limit allowed to relax to a stationary equilibrium. Here the analysis is carried out only to lowest surviving order in  $\beta$  and  $\epsilon_0$  and through first order in  $\epsilon_1$ . Analytic results can be obtained through arbitrary order in  $\epsilon_1$  and are given below for GDP through second order in this parameter. Since the solutions expanded to higher order in  $\epsilon_1$  are only asymptotically convergence, they only provide substantial additional accuracy when the terms of third and higher order in  $\epsilon_1$  are of too small to be of interest here. Thus terms higher than second order in  $\epsilon_1$  are not reported here, and the terms of second order in  $\epsilon_1$  are described only to allow confirmation that they are small.

# C.2 Simplifying the Euler-Lagrange Equations

The variations  $\delta \mathcal{L}/\delta k = \delta \mathcal{L}/\delta l = \delta \mathcal{L}/\delta f = 0$  yield algebraic results. First we show that k = l exactly and then that k = 1 and f = g through the order of terms retained here. To demonstrate these things, we first note that for m = k, l, or f

$$(\beta^{-1}e^{\rho t}a^{-\theta}C^{\theta}/K)\delta\mathcal{L}/\delta m = (\beta^{-1}/K)\delta C/\delta m - (\kappa/K)\delta(fw)/\delta m = 0$$

#### **C.2.1** l = k

Multiplying the immediately previous result for m = l by  $\alpha/\omega$  and working out the derivatives gives

$$\beta^{-1}(k/K)\delta C/\delta k - \lambda(k/K)f\delta w/\delta k = 0 = (\alpha/\omega)\beta^{-1}(l/K)\delta C/\delta l - (\alpha/\omega)(l/K)f\delta w/\delta l$$

Noting that

$$k\delta C/\delta k = \alpha^{-1}k\delta Y/\delta k = \alpha^{-1}Y\delta \operatorname{Log}[Y]/\delta k = Yk\varphi\delta \operatorname{Log}[1-\beta k]/\delta k + (\beta/\alpha)Yk\delta \operatorname{Log}[w]/\delta k$$

and

$$(\alpha/\omega)l\delta C/\delta l = \omega^{-1}k\delta Y/\delta l = \omega^{-1}Y\delta \text{Log}[Y]/\delta l = Yl\varphi\delta \text{Log}[1-\beta k]/\delta l + (\beta/\omega)Yk\delta \text{Log}[w]/\delta l$$

we have

$$\beta^{-1}(k/K)\delta C/\delta k = -(Y/K)\varphi k/(1-\beta k) + Y/K$$

$$(\alpha/\omega)\beta^{-1}(l/K)\delta C/\delta k = -(Y/K)\varphi l/(1-\beta l) + Y/K$$

 $\mathbf{SO}$ 

$$-(Y/K)\varphi k/(1-\beta k) + 2Y/K - \alpha(\kappa/K)fw = 0 = -(Y/K)\varphi l/(1-\beta l) + 2Y/K - \alpha(\kappa/K)fw = 0$$

Given the identical roles played by k and l on the left and right hand sides of this equation, we have the exact result

$$l = k$$

With this exact result the GDP,  $Y/\alpha$  in the units used here, can be expressed more simply than above by substituting in k = l and the expression for w to obtain

$$Y = a^{\eta \varphi + \zeta \beta + \omega} K^{\alpha} (1 - \beta k)^{\varphi} k^{\beta} p^{\beta}$$

Next we show that  $k \approx 1$  and hence  $l \approx 1$  through the order of terms kept here if  $\kappa/K$  is negligible (which in turn will be shown below). Defining a quantity

$$F = K/Y$$

proportional to the capital intensity of production, the above result for k can be written

$$-\varphi(k/F)/(1-\beta k) + 1/F - \alpha(\kappa/K)fw = 0$$

**C.2.2**  $\kappa/K \sim \epsilon_0$  and  $k \approx 1$ 

Multiplying the Euler-Lagrange equation  $\delta \mathcal{L}/\delta u = d(\delta \mathcal{L}/\delta \dot{u})/dt$  by  $(\epsilon_0/\beta)K^{-1}e^{\rho t}a^{-\theta}C^{\theta}$  gives

$$(\epsilon_0/\beta)K^{-1}e^{\rho t}a^{-\theta}C^{\theta}(\delta \mathcal{L}/\delta C)(\delta C/\delta Y)Y\delta \mathrm{Log}[Y]/\delta u - (\kappa/K)\delta(fw)/\delta u$$

$$= K^{-1}e^{\rho t}a^{-\theta}C^{\theta}d(Ke^{-\rho t}a^{\theta}\kappa/K)/dt$$

This follows from using the differentiation chain rule on  $\delta \mathcal{L}/\delta u$ , eliminating factors that cancel, and writing  $\kappa = K\kappa/K$  inside the time derivative in preparation for its expansion using the product rule. Noting that  $\delta \text{Log}[Y]/\delta u = \beta \delta \text{Log}[p]/\delta u$  and that the remaining terms multiplying  $\epsilon_0/\beta$  on the right hand side are of order 1, it can be seen that

$$\kappa/K \sim \epsilon_0$$

This means that the correction containing the factor  $\kappa/K$  in the above equation

$$-\varphi(k/F)/(1-\beta k) + (1/F) - \alpha(\kappa/K)fw = 0$$

is of order  $\epsilon_0$ , so to lowest order in both  $\epsilon_0$  and  $\beta$  we have

 $k \approx 1$ 

The ordering  $\kappa/K \sim \epsilon_0$  is somewhat more transparent if the products given here in the Euler-Lagrange equation that results from variation with respect to carbon depletion u are computed to give

$$(\epsilon_0/\alpha)F^{-1}\delta \mathrm{Log}[p]/\delta u = (\kappa/K)\delta(fw)/\delta u + (\dot{K}/K - \theta\dot{a}/a + \rho)\kappa/K + d(\kappa/K)/dt$$

With the coefficients of  $\epsilon_0$  and the terms multiplying  $\kappa/K$  on the right of order unity in the units used here, this confirms that  $\kappa/K \sim \epsilon_0$ . If terms of next order in  $\epsilon_0$  were to be retained, then it would be necessary to integrate this equation to determine the evolution of  $\kappa/K$ .

#### C.2.3 $f \approx g$

Taking the m = f version of the above result for  $\delta \mathcal{L}/\delta m = 0$  gives

$$(\beta^{-1}/K)\delta C/\delta f - (\kappa/K)\delta(fw)/\delta f = 0$$

Dropping the  $\kappa/K \sim \epsilon_0$  term and noting that  $\delta C/\delta f$  is proportional to  $\delta p/\delta f$ , we have, after neglecting terms of order  $\epsilon_0$ , that  $\delta p/\delta f \approx 0$ . Since p = (h-1)q - 1 and  $q = 1 - ((f/g) - 1)^2$ , this gives  $\delta q/\delta f \approx 0$  and hence

 $f \approx g$ 

#### C.2.4 Evolution of GDP

Multiplying the Euler-Lagrange equation  $\delta \mathcal{L}/\delta K = d(\delta \mathcal{L}/\delta \dot{K})/dt$  by  $e^{\rho t}a^{-\theta}C^{\theta}$  gives, to lowest order in  $\beta$ , since  $\delta \mathcal{L}/\delta K = a^{\theta}e^{-\rho t}\delta(C^{1-\theta}/(1-\theta))/\delta K = a^{\theta}e^{-\rho t}C^{-\theta}\delta C/\delta K$  and  $\delta C/\delta K = \alpha^{-1}Y\delta \text{Log}[Y]/\delta K - r = F^{-1} - r$  and  $\delta \mathcal{L}/\delta \dot{K} = -\delta \mathcal{L}/\delta C = e^{-\rho t}a^{\theta}C^{-\theta}$ 

$$F^{-1} - r = \rho + \theta(\dot{C}/C - \dot{a}/a)$$

Since  $r + \rho = 1$  with the units of time used here, this can be rewritten as

$$1 - F^{-1} = \theta(\dot{a}/a - \dot{C}/C)$$

It is convenient to define a function

$$G = (\alpha^{-1} - r)K/C$$

which is proportional to the capital intensity of consumption, in addition to the function F = K/Y proportional to the capital intensity of production. With these definitions we

have

$$\dot{C}/C = \dot{K}/K - \dot{G}/G$$

and

$$\dot{K}/K - \dot{F}/F = \dot{Y}/Y = (\eta + \omega)\dot{a}/a + \alpha\dot{K}/K$$

where the final equality is computed from  $Y = a^{\eta \varphi + \zeta \beta + \omega} K^{\alpha} (1 - \beta k)^{\varphi} k^{\beta} p^{\beta}$  to lowest order in  $\beta$ . Solving this for  $\dot{K}/K$  by collecting terms and dividing by  $\omega = 1 - \alpha$  gives

$$\dot{K}/K = (1+\eta/\omega)\dot{a}/a + \omega^{-1}\dot{F}/F$$

so that

$$\dot{C}/C = (1+\eta/\omega)\dot{a}/a + \omega^{-1}\dot{F}/F - \dot{G}/G$$

and

$$1-F^{-1}=-(\eta\theta/\omega)\dot{a}/a-(\theta/\omega)\dot{F}/F+\theta\dot{G}/G$$

Noting that  $a = 1/(1 + e^{-\nu t})$  gives  $e^{-\nu t} = a^{-1} - 1$  and  $\dot{a} = \nu e^{-\nu t} a^2 = (1 - a)a = za$  gives  $\dot{a}/a = \nu z$ . With the above definition  $\epsilon_1 = \nu \theta \xi = \nu \theta \eta / \omega$  we thus have

$$1 - F^{-1} = -\epsilon_1 z - (\theta/\omega)\dot{F}/F + \theta\dot{G}/G$$

Dividing the above capital balance equation  $C = Y/\alpha - rK - \dot{K}$  by K and using the expression just derived for  $\dot{K}/K$  gives the companion equation

$$(\alpha^{-1} - r)G^{-1} = \alpha^{-1}F^{-1} - r + \omega^{-1}\dot{F}/F - (1 + \eta/\omega)\nu z$$

From these equations it is clear that in the long-term, steady-state limit where  $z \to 0$ and  $t \to 0$  we have  $F \to 1$  and  $G \to 1$ . This is what motivated the choice of constants in the definitions of these functions proportional respectively to the capital intensities of production and consumption. Using the differentiation chain rule and the result  $\dot{a}/a = \nu z$ , we can rewrite  $\dot{F}/F = \nu z F'/F$  and  $\dot{G}/G = \nu z G'/G$  with

$$F' = dF/da$$
 and  $G' = dG/da$ 

and multiply by the product  $(1 + \epsilon_1)FG$  to obtain

$$(1+\epsilon_1)FG - (1+\epsilon_1)G = -\epsilon_1 z(1+\epsilon_1)FG - \epsilon_1 \eta^{-1} z(1+\epsilon_1)F'G + \epsilon_1 \eta^{-1} \omega^{-1} zG'(1+\epsilon_1)F$$

$$(\alpha^{-1} - r)(1 + \epsilon_1)F = \alpha^{-1}G - r(1 + \epsilon_1)FG - (1 + \eta/\omega)\nu z(1 + \epsilon_1)FG + \epsilon_1\omega^{-1}\eta^{-1}\theta^{-1}(1 + \epsilon_1)F'G$$

Asymptotically convergent expansions for  $(1 + \epsilon_1)F$  and G in  $\epsilon_1$  can be obtained iteratively starting from the approximate solutions

$$(1 + \epsilon_1)F \approx (1 + \epsilon_1 a)$$
 and  $G \approx 1/(1 + \gamma_1 \epsilon_1)$ 

where

$$\gamma_1 = (1 - (1 + \xi^{-1})/\theta)/(\alpha^{-1} - r)$$

These forms are chosen to exactly match regularity boundary conditions so that only solutions away from the  $a \to 0$  and  $z \to 0$  boundaries but not the solutions at these boundaries need to be expanded in powers of  $\epsilon_1$ . For F = K/Y this condition is that  $\dot{F}/F = \dot{K}/K - \dot{Y}/Y \to 0$  as  $a \to 0$ , so that the initial exponential growth timescales for capital and consumption are the same and thus in balance. Taking the  $a \to 0$  limit of  $(\alpha^{-1} - r)G^{-1} = \alpha^{-1}F^{-1} - r + \omega^{-1}\dot{F}/F - (1 + \eta/\omega)\nu z$  and solving for G in this limit gives  $G \to 1/(1 + \gamma_1\epsilon_1)$  with the above definition of  $\gamma_1$ .

The result for F through first order in  $\epsilon_1$  can be seen by inspection of the above equation

for  $(1 + \epsilon_1)F \approx G - (1 + \epsilon_1)G$  to be

$$F \approx F_1 = (1 + \epsilon_1 a) / (1 + \epsilon_1)$$

This result can be substituted in the second of the pair of equations used for the expansion to give

$$G \approx (1 + \epsilon_1 \gamma_1 a) / (1 + \gamma_1 \epsilon_1)$$

This result can then be inserted back into the first of the pair of the equations used for the expansion to give

$$F \approx F_2 = (1 + \epsilon_1 a + \epsilon_1^2 \gamma_2 az)/(1 + \epsilon_1)$$

where

$$\gamma_2 = 1 + \xi/\omega + \gamma_1(1 - \xi^{-1})$$

Note that the second order correction to F is proportional to az, so that  $\epsilon_1^2 az$  has a maximum value of  $(\epsilon_1/2)^2$  at a = z = 1/2 and vanishes at both boundaries.

An expression for Y can be found, also keeping only the terms lowest order in  $\beta$ , using  $FK = Y \approx a^{\eta+\omega}K^{\alpha}$  to obtain

$$K^{\alpha} = a^{\alpha(1+\eta/\omega)} F^{\alpha/\omega}$$

Inserting this into  $Y \approx a^{\eta+\omega} K^{\alpha}$  gives, in the approximation of keeping terms up to order  $\epsilon_1^n$ in this expansion,

$$Y = a^{1+\eta/\omega} F_n^{\alpha/\omega}$$

Since the expansion in  $\epsilon_1$  is only asymptotically convergent and the other approximations used in the calculation make it of little utility to have a result accurate to order  $\epsilon_1^2$  when  $\epsilon_1$ is small, it generally should suffice to use  $F_1$  in equations upon which model calibration and extrapolations are based. The difference between this result and that obtained using  $F_2$  can then be used as check on the consequences of discarding terms of order  $\epsilon_1^2$ .

# C.3 Integrating the Carbon Balance Constraint

Changing the independent variable in the above carbon balance equation to a and approximating  $k = l \approx 1$  in the above energy production function gives

$$\nu zadu/da = \epsilon_0 f p a^{\zeta + \omega} K^{\alpha}$$

Inserting the above-mentioned result  $K^{\alpha} \approx a^{\alpha(1+\eta/\omega)} F^{\alpha/\omega}$  in the carbon balance gives, to lowest surviving order in  $\beta$  and  $\epsilon_0$ ,

$$\nu zadu/da = \epsilon_0 f p a^{\psi} F_2^{\alpha/\omega}$$

where

$$\psi = 1 + \zeta + \alpha \xi$$

Since in the approximation used here f and p are known functions of cumulative fossil carbon depletion u, this differential equation can be reduced to the quadratures

$$\int_0^u du/fp = (\epsilon_0/\nu) \int_0^a da \ a^{\psi} F^{\alpha/\omega}/(za)$$

#### C.3.1 Integration over Development Index, a

Expanding in  $\epsilon_1$ , the integrand on the right hand side can be approximated as

$$a^{\psi} F_2^{\alpha/\omega} / (za) = (a^{\psi-1}/z)(1 - \epsilon_1 z \alpha/\omega + \epsilon_1^2 z (1 + \gamma_2 a - (1 - \alpha/\omega)z/2))\alpha/\omega$$

Using z = 1 - a, and  $\alpha = 1 - \omega$ , we have

$$1 + \gamma_2 a - (1 - \alpha/\omega)z/2 = (1 + (2\omega\gamma_2 + \omega - \alpha)a)/2\omega$$

Thus

$$a^{\psi} F_{2}^{\alpha/\omega} / za = a^{\psi-1} / z - \epsilon_{1} a^{\psi-1} \alpha / \omega + \epsilon_{1}^{2} (a^{\psi-1} + a^{\psi} (2\omega\gamma_{2} + \omega - \alpha)) \alpha / 2\omega^{2}$$

and the integral on the right hand side can be approximated to order  $\epsilon_1^2$  as

$$(\epsilon_0/\nu)\int_0^a daa^{\psi}F_2^{\alpha/\omega}/za \approx (\epsilon_0/\nu)a^{\psi}(\Phi - (\epsilon_1/\psi)\alpha/\omega + \epsilon_1^2(1/\psi + (2\omega\gamma_2 + \omega - \alpha)a/(\psi + 1))\alpha/(2\omega^2))$$

and to order  $\epsilon_1^1$  as

$$R = (\epsilon_0/\nu)a^{\psi}(\Phi - (\epsilon_1/\psi)\alpha/\omega)$$

Here the function  $\Phi = a^{-\psi} \texttt{Beta}[a, \psi, 0]$  is a special case of Lerch's transcendent

$$\Phi = \sum_{k=0}^{\infty} \frac{a^k}{\psi + k}$$

and  $\text{Beta}[a, \psi, 0] = \int_0^a da a^{\psi-1}/(1-a)$  is an incomplete Beta function. This can be computed by power series expansion or by using inbuilt *Mathematica* functions (Wolfram, 2003).

#### C.3.2 Integration over Carbon Depletion, u

With the approximations used here, the effect of carbon depletion on energy production efficiency is p = (h - 1)(1 - u) + 1. Defining

$$b = (h-1)/h$$

this can be written

$$p = h(1 - bu)$$

Also with the approximations used here, the carbon intensity of energy production is proportional to  $f = f_k(1 - s_k u)$  for  $u_{k-1} \le u \le u_k$ . Denoting the left-hand-side integrals up to the break points  $u_k$  as  $R_k$ , we have, for u such that  $u_{k-1} \leq u \leq u_k$ ,

$$R - R_{k-1} = (1/(hf_k)) \int_{u_{k-1}}^{u} \frac{du}{(1 - s_k u)(1 - bu)} = \frac{1}{hf_k(s_k - b)} \text{Log}[\frac{1 - bu}{1 - s_k u} \frac{1 - s_k u_{k-1}}{1 - bu_{k-1}}]$$

The solution for u is conveniently written in terms of a function x using

$$x_{k-1}e^{hf_k(s_k-b)(R-R_k)} = x = (1-bu)/(1-s_ku)$$

where

$$x_k = (1 - bu_k)/(1 - s_k u_k)$$

Solving for u in each range  $u_{k-1} \leq u \leq u_k$  gives

$$u = (x - 1)/(s_k x - 1)$$

Note that x = 1 when  $a = 0 = u_0$ . Since u is measured in units of the total amount of carbon ever used (i.e., in the limit  $f \to 0$ , for the largest value M of k we have  $s_M = 1$ . Thus  $x \to \infty$  as  $a \to 1$  and  $u \to 1$ , where also R diverges as Log[z]=Log[1-a] due to the 1/z factor in its  $\epsilon_0^0$  term.

## C.4 Dimensional Formulas

All of the formulas above are given in terms of dimensionless variables, but for comparison with time series data it is convenient to convert them to dimensional form. Using an overbar for dimensional constants and an overtilde for dimensional variables, we can write the development index as

$$a = 1/(1 + e^{-\tilde{\nu}(\tilde{t} - \tilde{t}_0)})$$

where  $\tilde{\nu}$  is the initial population growth rate and  $\tilde{t}_0$  its inflection time (where *a* has zero second derivative with respect to time and a = 1/2). The total population can be written as  $\tilde{P} = \bar{P}a$ . Data on the fractional population growth rate can be used to calibrate the parameters  $\tilde{\nu}$  (e.g. in 1/yr) and  $\tilde{t}_0$  (e.g. in yr) independently of the limit population, using the formula

$$d \operatorname{Log}[P]/d\tilde{t} = \tilde{\nu}z = \tilde{\nu}(1-a)$$

In the approximation used here, the gross domestic product expression is proportional to  $a^{1+\eta/\omega}F_1^{\alpha/\omega}$ . Denoting  $\bar{G}_{DP}$  as the long-term limit of gross annual production rate (e.g. in trillions of 1990US\$/yr purchasing power parity), the gross domestic product can be written in the form

$$\tilde{G}_{DP} = \bar{G}_{DP} a^{1+\xi} F_1^{\alpha/\omega}$$

The parameter  $\xi$  is related to the fractional GDP growth rate

$$d \text{Log}[\tilde{G}_{DP}/a]/d\tilde{t} = \tilde{\nu} z a d \text{Log}[\tilde{G}_{DP}/a]/da$$

where

$$d \operatorname{Log}[\tilde{G}_{DP}/a]/da = \xi + (\alpha/\omega) d \operatorname{Log}[F_1]/da$$

Here  $d\text{Log}[F_1]/da \sim \epsilon_1$ , so to the lowest order in  $\epsilon_1$  we have that  $d\text{Log}[\tilde{G}_{DP}/a]/da$  is the constant  $\xi$ . This will provide a useful starting point for calibrating the constant  $\xi$  against time-series data. (Note that a is proportional to the increment of population over its base value, so  $d\text{Log}[\tilde{G}_{DP}/a]/da$  is the fractional increase with development of the incremental GDP over the base value per capita of population increment over the base value.)

In dimensional units the carbon intensity of energy production can be written as  $\tilde{f} = \bar{f}f$ , where  $\bar{f}$  is its initial value (e.g. in Gtonne/EJ), and the integrated carbon use as  $\tilde{u} = \bar{u}u$ , where  $\bar{u}$  is the maximum amount of carbon ever used (e.g. in Gtonne=billions of metric tons). In these units a piecewise linear approximation to the relationship between these quantities can be written

$$\tilde{f} = \bar{f}_k (1 - \bar{s}_k \tilde{u})$$

where the quantities used in the equations above are then  $f_k = \bar{f}_k/\bar{f}$  and  $s_k = \bar{u}\bar{s}_k$ . With the approximations used here annual energy production can be written in dimensional units as

$$\tilde{w} = \bar{w} p a^{\psi} F_1^{\alpha/\omega}$$

where  $\bar{w}$  is the long-term limit energy use rate (e.g. in EJ/yr). Given a slow rate of fossil fuel depletion (i.e. small  $\epsilon_0$ ) over the time for which data is available and a small capitalization delay (i.e. small  $\epsilon_1$ ) we have approximately constant values  $p \approx h$  and  $F_1 \approx 1$  so that

$$\operatorname{Log}[\tilde{w}] = \operatorname{Log}[\bar{w}] + \psi \operatorname{Log}[a] + \operatorname{Log}[p] + (\alpha/\omega) \operatorname{Log}[F_1]$$

is approximately linear in  $\text{Log}[\bar{w}]$  and Log[a]. This can provide a useful starting point for calibrating  $\text{Log}[\bar{w}]$  and  $\psi$  against time-series data. Alternatively, carbon use rate can be written

$$\operatorname{Log}[\tilde{E}] = \operatorname{Log}[\bar{E}] + \operatorname{Log}[f] + \psi \operatorname{Log}[a] + \operatorname{Log}[p] + (\alpha/\omega) \operatorname{Log}[F_1]$$

Here  $\bar{E} = \bar{f}\bar{w}$  is a constant on the order of the maximum carbon use rate (e.g. in Gtonne/yr). This formulation provides a useful alternative starting point for calibrating  $\text{Log}[\bar{E}]$  and  $\psi$  against time-series data on carbon use, for studies where the primary output of interest is the carbon emissions rate

$$f_E \bar{E}$$

with  $f_E$  the amount of carbon emitted in the atmosphere per unit carbon burned.

# Appendix D Atmospheric Response to Carbon Emissions

Estimates of the increase of atmospheric carbon loading and global average temperature driven by analytic expressions for carbon emissions are computed using a set of three successively integrable differential equations. The first of these simply integrates anthropogenic carbon release as a "hysteresis" term input to a linear equation for response of atmospheric carbon loading. Since the atmospheric response to carbon emissions is linear in this model, the atmospheric carbon loading from each source can be tracked individually. In the limit of small increase in over preindustrial atmospheric carbon loading, the equation for the increase in global average temperature over the preindustrial value is also linear. As the infrared absorption effect of atmospheric carbon saturates, there is a global reduction from the response computed in the linear approximation that cannot be ascribed to separately to different sources. This correction is computed separately.

## D.1 Reduction to Quadratures

Here we develop solutions to three successively integrable differential equations describing atmospheric response to anthropogenic increases in carbon emissions. Here the global anthropogenic carbon emission rate (e.g. in Gtonne/yr)

$$\tilde{E} = \sum_{i} f_E \bar{E}_i E_i$$

is summed over the dimensional scale parameter  $f_E \bar{E}_i$  for each contributing emitter group times a dimensionless function  $E_i$  of a logistic function of time

$$a_i = \frac{1}{1 + e^{-\tilde{v}_i(\tilde{t} - \tilde{t}_{0i})}}$$

The equations to be solved are

$$d\tilde{F}/d\tilde{t} = \tilde{E}$$
$$d\tilde{C}/d\tilde{t} = \bar{B}\tilde{F} + \bar{\beta}\tilde{E} - \bar{\sigma}(\tilde{C} - \bar{C}_0)$$
$$d\tilde{T}/d\tilde{t} = \bar{\mu}Log[\tilde{C}/\tilde{C}_0] - \bar{\alpha}(\tilde{T} - \bar{T}_0)$$

where physical chemistry relates  $\bar{B}$  to  $\bar{\beta}\bar{\sigma}$  through a dimensionless constant  $\gamma_0 \approx 0.10$ :

$$\bar{B} = \gamma_0 \bar{\beta} \bar{\sigma}$$

#### D.1.1 Atmospheric Carbon Loading

Here  $\tilde{F}$  is cumulative anthropogenic carbon emissions. Define

$$c = (\tilde{C} - \bar{C}_0) / \bar{C}_0$$

as the increase in atmospheric carbon concentration divided by the preindustrial value  $\bar{C}_0$ and

$$T = (\tilde{T} - \bar{T}_0)/\bar{T}$$

as the increase in global average temperature over a base equilibrium value  $\bar{T}_0$  in convenient units  $\bar{T}$  (e.g. <sup>o</sup>C), and note that

$$dc/d\tilde{t} + \bar{\sigma}c = \tilde{F}\bar{B}/\bar{C}_0 + \tilde{E}\bar{\beta}/\bar{C}_0$$

and

$$dT/d\tilde{t} + \bar{\alpha}T = (\bar{\mu}/\bar{T})Log[1+c]$$

Noting that  $d/d\tilde{t} = d/da_i(\tilde{\nu}_i z_i a_i)$  for each *i* where  $z_i = 1 - a_i$ , cumulative anthropogenic emissions can be integrated as

$$\tilde{F} = \sum_{i} \tilde{F}_i = f_E \sum_{i} (\bar{E}_i / \tilde{\nu}_i) \int_0^{a_i} da_i E_i / (a_i z_i)$$

To decompose the increased atmospheric carbon load into source contributions as  $c = \sum_i c_i$ , define the dimensionless sources

$$S_i = (\bar{B}/\tilde{\nu}_i)(\tilde{F}_i/\bar{C}_0) + (\bar{\beta}/\tilde{\nu}_i)(\tilde{E}_i/\bar{C}_0)$$

and

$$\sigma_i = \bar{\sigma} / \tilde{\nu}_i$$

Then

$$dc_i/da_i + \sigma_i c_i/(z_i a_i) = S_i/(z_i a_i) = (a_i/z_i)^{\sigma_i} d((z_i/a_i)^{\sigma_i} c_i)/da_i$$

Thus the atmospheric loading responses to the sources  $S_i$  are

$$c_{i} = (z_{i}/a_{i})^{\sigma_{i}} \int_{0}^{a_{i}} da_{i} (a_{i}/z_{i})^{\sigma_{i}} S_{i}/(z_{i}a_{i})$$

#### D.1.2 Temperature Response

For  $c \ll 1$  we have  $Log[1+c] = c - c^2/2 + c^3/3 - c^4/4 + \dots$  Thus the initial global average atmospheric temperature response can also be attributed here individually to the various anthropogenic emission sources that contribute to increased atmospheric carbon loading by solving

$$dT_i/da_i + \alpha_i T/(z_i a_i) = \mu_i Log[1 + c_i]/(z_i a_i)$$

where

$$\alpha_i = \bar{\alpha}/\tilde{\nu}_i; \ \mu_i = \bar{\mu}/\tilde{\nu}_i$$

(Here we have used  $Log[1 + c_i]$  instead of  $c_i$  in this equation so that the exact global result is recovered if all of the emissions are lumped together.) The solutions for  $T_i$  are

$$T_{i} = \mu_{i}(z_{i}/a_{i})^{\alpha_{i}} \int_{0}^{a_{i}} da_{i}(a_{i}/z_{i})^{\alpha_{i}} Log[1+c_{i}]/(z_{i}a_{i})$$

Denoting the difference between the approximation from summing these response as

$$\Psi = \left(\sum_{i} T_{i}\right) - T$$

the equation for  $\Psi$  is

$$d\Psi/d\tilde{t} + \bar{\alpha}\Psi = \bar{\mu}(\sum_{i} Log[1+c_i] - Log[1+\sum_{i} c_i])$$

## D.2 Power Series Expansions

We are interested here in contributions of the form  $f_E \bar{E}_i E_i(a_i)$  to carbon emissions where we have power series expansions for the  $E_i$  of the form

$$E_{i} = \sum_{j=1}^{J_{i}} a_{i}^{j\psi_{i}} \sum_{k=0}^{K_{J_{i}}} e_{ijk} a_{i}^{k}$$

This allows a term by term integration of the above expression for c if the  $z_i$  dependence in its integrand is expanded as  $1/z_i = 1 + a_i + a_i^2 + \dots$  For two contributions of the form  $\overline{E}_i E_i a_i$  to carbon emissions this equation can be written is the form

$$z_2 a_2 d\Psi/da_2 + \alpha_2 \Psi$$

$$= \mu_2 c_1 c_2 (1 - (c_1 + c_2) + (c_1^2 + 6c_1 c_2/4 + c_2^2) - (c_1^3 + 10c_1^2 c_2/5 + 10c_1 c_2^2/5 + c_2^3) + \dots) = \Omega$$

The solutions for  $T_i$  can similarly be expanded as double power series. The solution for the degree of relief from global temperature rise due to opacity saturation,  $\Psi$ , can also be expanded as a power series using the relations between the other  $a_i$  and a chosen independent variable  $a_n$ :

$$a_{i} = \frac{1}{1 + e^{-\tilde{\nu}_{i}(\tilde{t}_{0n} - \tilde{t}_{0i})} (z_{n}/a_{n})^{\tilde{\nu}_{i}/\tilde{\nu}_{2}}}$$

For example, for two regions only, we have

$$\Psi = \mu_2 (z_2/a_2)^{\alpha_2} \int_0^{a_2} da_2 (a_2/z_2)^{\alpha_2} \Omega/(z_2a_2)$$

where  $\Omega$  is taken from the above expansion of logarithms. However, in practice it is generally more convenient to integrate the equation for T numerically and if desired also for  $\Psi$ . Thus division between regional contributions and globally mixed contribution to warming from effects of regional carbon emissions is more of conceptual than computationally practical use.

# Appendix E Fuel Fractions Formulas

This appendix defines a model of the fraction of the total energy use rate from from various energy sources operationally defined here as "primary": oil, natural gas, coal, water driven (hydroelectric, geothermal electric, and tidal electric), new renewables (solar thermal electric and wind electric), and nuclear energy sources. The energy source for the nuclear component is then broken down into contributions from conventional uranium mining with current enrichment tails assay, spent fuel reprocessing, and other sources.

Formulas are given here for the evolution of fractions of a given amount of energy use from five different sources: fluid fossil fuels, coal, nonfossil energy nuclear, and "new renewables" (wind electric and solar thermal electric). All but coal phase in through an exponentially saturating process of "learning by doing." Coal is the original dominant industrial energy source and thus starts with a market fraction of one. As fossil fuels are used, the fractions from fluid fossil fuel and coal eventually decline and asymptotically approach zero. The model for the nuclear power fraction accounts for declining availability of nuclear fuel from conventional mining, continuing sources of supply from spent fuel reprocessing, and lumped recovery of the isotope uranium-235 from other sources (e.g. phosphates, mining of other metals, further enrichment of old enrichment tails, brine, and eventually enormous potential supplies from open ocean seawater, collective referred to here as "byproduct" uranium even if in the case of seawater uranium recovery may be the main operational objective). The model for water-driven electricity production (hydroelectric, tidal electric, and geothermal) accounts for limitations on the availability of suitable sites. The competition between energy sources is examined and calibrated in pairs including fluid fossil vs. other, coal vs. non-fossil, water-driven vs. other nonfossil, and nuclear vs. "new renewables."

Define  $x_k$  as the cumulative use for a given region of a fuel which has been competing for a fraction for a fraction of total cumulative use

$$u_k = x_k + u_{k+1}$$

The relevant fuel fractions are then

$$f_1(x_1) = \frac{dx_1}{du_1} = \frac{\text{fluid fossil}}{\text{primary energy}}$$

$$f_2(x_2) = \frac{dx_2}{du_2} = \frac{\text{coal}}{\text{other than fluid fossil}}$$

$$f_3(x_3) = \frac{dx_3}{du_3} = \frac{\text{water driven}}{\text{total nonfossil}}$$

$$f_4(x_4) = \frac{dx_4}{du_4} = \frac{\text{new renewables}}{\text{nonwater nonfossil}}$$

$$f_5(x_5) = \frac{dx_5}{du_5} = \frac{\text{conventional uranium ore}}{\text{toal nuclear energy}}$$

$$f_6(x_6) = \frac{dx_6}{du_6} = \frac{\text{"byproduct" uranium}}{\text{"unconventional" fissiles}}$$

The cumulative amount of nuclear energy  $u_7$  obtained from fissile materials in spent nuclear fuel is calculated at the end of this chain using the formula  $u_7 = u_6 - x_6$ . For estimating the minimum allowed delay between discharge from fresh fuel burning and the burning of fissile material from its reprocessing, fuel from "fresh" (conventional and byproduct uranium) sources is assumed to be stored until it is mixed with reprocessed material in a proportion that allows approach to an eventual steady state use of such loading mixtures. Fresh fuel loadings are assumed to be recoverable separately from previously reprocessed materials so that the discharge from fresh fuel can eventually be reprocessed. Previously fresh fuel loadings are assumed to be reprocessed only once, since current estimates of the cost of subsequent reprocessing suggest that additional rounds of reprocessing may be more expensive than recovery of fresh uranium from other sources even if open ocean water sources eventually need to be tapped. Current estimates suggest that one reprocessing cycle may just be economically competitive with recovery of other "unconventional" uranium sources even in the long term. Here for simplicity it is assumed that this is the case, so that an eventual essentially steady state is reached with the mixture of reprocessed and fresh nuclear fuel sources being determined by the conversion ratio (the ratio of energy content recovered from reprocessed fuel to that produced when the fresh fuel that gave rise to it was burned). This approach can be readily generalized to the case where reprocessed fuel eventually becomes cheaper than other unconventional sources, and thus the stock of spent fuel eventually declines to a minimum determined by the minimum economic delay until reprocessing. However, unless reprocessing becomes substantially cheaper than other fissile fuel sources, the time for this to occur is so long that the approximation made here still gives useful insight into possible interim spent fuel storage times.

### E.1 Non-nuclear Energy Fuel Fractions

For each depletable resource there is assumed to be a cost balance, at the point where it captures a fraction f of the total of its share of the market, of the general form

$$C_0 = D\chi + Ae^{-Rx} + Bf$$

The term  $D\chi$  describes the effect of resource depletion. For times where the technology for using the resource is not fully mature, the exponential term describes how "learning by doing" reduces the cost of using the resource. The use of a constant on the left hand side of the equation implies that for the competing energy source depletion and experiential learning are either negligible or subsumed in the right hand side when the model is calibrated against observations. Coal is assumed to be a mature technology during times of interest, so one can set A = 0 when fitting the function  $f_2$  to data. Only the ratios  $C_0/B$ , D/B, A/B and the exponent R are needed to define f(x) in each case. For uranium depletion, a model for  $\chi$  is given below. With these simplifications we can express various fuel fractions as follows

$$f_1 = c_{13}(1 - e^{-c_{12}x_1}) - c_{11}x_1$$

$$f_2 = c_{21} - c_{22}x_2$$
$$f_3 = 1 - c_{33}(1 - e^{-c_{32}x_3})$$
$$f_4 = c_{43}(1 - e^{-c_{42}x_4}) + \epsilon_3\chi$$

The solution of  $f_1 = c_{13}(1 - e^{-c_{12}x_1}) - c_{11}x_1$  for integrated fossil fuel use  $x_1$  is done using the approximation  $f_1 = c_{13}(1 - e^{-c_{12}x_1})$  to integrate the equation up to a matching point where  $x_1 = \text{ProductLog}[c_{12}c_{13}/c_{11}]$ . Here ProductLog[Z] is the solution for W in the equation  $Z = We^W$ . Beyond this point the solution for  $f_1$  is integrated in the approximation  $f_1 = c_{13}(1 - c_{11}x_1)$ . Choosing the switchover point  $\text{ProductLog}[c_{12}c_{13}/c_{11}]$  for this procedure guarantees continuity of the fossil fuel fraction  $f_1$  and approximately minimizes the small approximation error involved in this convenient method for providing a fully analytic solution. The values of  $c_{11}$ ,  $c_{12}$ ,  $c_{13}$  are adjusted to minimize the summed square deviations between the data and the model. The underlying assumption, which is well satisfied given the results of the data fits described in this thesis, is that the experiential learning effect is essentially saturated before resource use has proceeded to the point where resource depletion has a significant effect on market fractions.

The solution of  $f_2 = c_{21} - c_{22}x_2$  for integrated coal use  $x_2$  is by piecewise integration of two such linear approximations. The one of these for the lower values of integrated coal use  $x_2$  is forced to make  $f_2 = 1$  at the outset of use of other energy sources and to fit the second piecewise linear fit at an adjustable matching value for  $x_2$ . This matching value and the slope and intercept of the linear fit for larger  $x_2$  are adjusted to minimize the summed square deviations between the data and the model.

The market fraction for the water-driven part of total nonfossil energy is the complement of the fraction  $c_{33}(1 - e^{-c_{32}x_3})$  for the sum of nuclear energy and new renewables, which shows a simple learning by doing pattern starting with zero market fraction in the limit  $x_3 \rightarrow 0$ of zero cumulative experience. Resource depletion for uranium has so far had negligible impact on nuclear energy costs, so its effect is neglected when calibrating the models against available data on market fractions.

The solution of  $f_3 = 1 - c_{33}(1 - e^{-c_{32}x_3})$  for the cumulative water-driven electricity production  $x_3$  is done by simple analytic integration of this formula. The values of  $c_{23}$  and  $c_{33}$  are adjusted to minimize the summed square deviations between the data and the model.

There is so little experience so far with market penetration of new renewables (wind electric and solar thermal electric) that the available data on market fractions is only sufficient to give a rough estimate of the initial market penetration rate. This is  $f_4 \approx c_{43}c_{42}x_4$  with  $1 - e^{-c_{42}x_4} \approx c_{42}x_4$  expanded for small values of  $c_{42}x_4$  and the effect  $\epsilon_3\chi$  of uranium resource depletion taken to so far be negligible. To avoid the fit being dominated by the very early startup phase, the available data is averaged over equal intervals in  $x_4$  with the interval size being the maximum spacing in  $x_4$  between data entries in the time series. The effect of this is to lump several early data points into an average value to be equally weighted in the fit with the latest data point, in the usual case where the rate of use of new renewables has recently been increasing with time.

When integrating each of the four equations for  $f_1$ ,  $f_2$ ,  $f_3$ , and  $f_4$ , an integration constant is needed to match the available data on each type of integrated energy use. This is because each of the equations used is not a particularly good approximation for very early times. In particular, the learning-by-doing models integrated forward from zero cumulative experience give only a trivial null solution. This is an indication of the fact that at some early time a "jump-start" investment is needed within the context of this approximation to start the process of market penetration. In the case of nuclear and "new renewable" energy sources this start-up investment was provided through central government subsidy, while in the case of early hydroelectricity and the first oil well and natural gas pipeline this was done by entrepreneurs hoping for a later recovery of start-up costs. In the private sector this can be a "dicey business," and indeed in the case of the first oil well a letter canceling the investors' financial commitment was in the mail when E.L. Drake first struck oil (Yergin, 1991). Given the vagaries of early market penetration, an appropriate procedure is to integrate fuel fractions equations backwards in integrated energy source use, fitting only data back to a point where the model remains a reasonable approximation, and avoiding the very early startup phase. In each case an adequate approximation for the present purpose can be obtained by forcing the integration results to exactly match the available data point. An improvement on this is to adjust the integration constant to minimize the mean square deviation between the cumulative energy use data and the integrated result. This is what is done here for all but the coal fraction, where for this particularly simple case the integration constant is found simultaneously with least squares fitting of the slope and intercept parameters in the fuel fractions model to the integrated result.

## E.2 Nuclear Energy Sources

Only for long term extrapolations is the effect of uranium resource depletion taken to be significant. Even then, reactor costing estimates suggest that the effect will be small; hence the use of the symbol  $\epsilon_3$  as a coefficient of the factor  $\chi$  in the above equation, with  $\chi$ itself taken to asymptote to one. To model the effect of uranium resource depletion, let  $x_5$  be energy obtained from cumulative mined uranium. Let  $x_m$  be the limit amount ever conventionally mined and  $y_5 = x_5/x_m$ . Let  $\chi = c_u/c_m = y_5^{1/m_5} = y_5^{\gamma-1}$  be the ratio of the inflation adjusted cost of mined uranium to its limit value when  $y_5 = 1$ . The literature on uranium ore resources suggest that  $2 \leq m_5 \leq 3$ , and here we assume a reference value of  $m_5 = 2.3$  from Deffeyes and MacGregor (1980). Let  $y_5 = x_5/x_m$  be the cumulative nuclear energy from conventional ore normalized to the energy content of the limit amount from use of uranium ore and  $v_5 = u_5/x_m$  be the similarly normalized cumulative total use of nuclear energy, and assume the following

$$f_5 = dy_5/dv_5 = 1 - (c_u/c_m)^{m_4} = 1 - y_5^{m_4/m_5} = 1 - y_5^{1+\epsilon_2}$$

The solution of the equation  $f_4 = c_{43}(1 - e^{-c_{42}x_4}) + \epsilon_3\chi$  for the division of nonwater-nonfossil energy into new renewables and nuclear energy is done in two intervals, one for  $y_5 << y_0$ where the approximation  $f_4 = c_{43}(1 - e^{-c_{42}x_4})$  is used, and another for  $y_5 >> y_0$  where the equation  $f_4 = c_{43} + \epsilon_3\chi = c_{43} + \epsilon_3y_5^{\gamma-1}$  is solved by expansion through first order in the small parameter  $\epsilon_3$ . Making the substitution  $x_4 = x_m y_4$ , the equation  $f_4 = dy_4/dv_4 =$  $c_{43}(1 - e^{-c_{42}x_m y_4})$  can be integrated directly and inverted to yield  $x_4$  as a function of  $u_4 = v_4 x_m$ and hence the cumulative nuclear energy use  $u_5 = u_4 - x_4$ . Here  $u_5$  is the difference between cumulative nonwater-nonfossil energy use  $u_4$  and cumulative use  $x_4$  of new renewables. For the approximation  $f_4 = dy_4/dv_4 = c_{43} + \epsilon_3 y_5^{\gamma-1}$ , for larger values of cumulative use of conventional uranium, it is convenient to replace the variable  $y_4$  by its equivalent  $y_4 =$ 

of conventional uranium, it is convenient to replace the variable  $y_4$  by its equivalent  $y_4 = v_4 - v_5$  so that  $1 - dv_5/dv_4 = c_{43} + \epsilon_3 y_5^{\gamma-1}$  or  $dv_5/dv_4 = 1 - c_{43} - \epsilon_3 y_5^{\gamma-1} = (1 - c_{43})(1 - \epsilon_5 y_5^{\gamma-1})$  where  $\epsilon_5 = \epsilon_3/(1 - c_{43})$ . Using the chain rule then an equation to be integrated to find a relationship between the previously calculated cumulative normalized non-water non-fossil use  $v_4 = u_4/x_m$  and the cumulative normalized energy  $y_5 = x_5/x_m$  from conventional uranium ore is

$$dy_5/dv_4 = (dy_5/dv_5)(dv_5/dv_4) = (1 - c_{43})(1 - y_5^{1 + \epsilon_2})(1 - \epsilon_5 y_5^{\gamma - 1})$$

It is reasonable to assume that the fraction or nuclear energy derived from conventional ore will stay large until near the time when the limit extraction cost is reached, so that like  $m_5$  the exponent  $m_4$  also exceeds 2. The present work is limited to cases where  $(m_4/m_5) - 1 \ll 1$ (i.e.  $\epsilon_2$  is small) and the results shown in the main text are for  $\epsilon_2 = 0$ . The only case considered in detail here is that for which  $\epsilon_2 = 0$  (i.e. the same exponents for the increase in conventional uranium ore resource endowment with extraction cost and for the increase in market fraction of unconventional fissile fuel sources with increase uranium costs). For this case the solution of  $y_5 = 1 - e^{-v_5} = \epsilon_3 \chi$  is

$$v_5 = -\mathrm{Log}[1 - y_5]$$

Expanding the solution to this equation through first order in  $\epsilon_3$  and  $\epsilon_4$  gives the following

$$y_5 = y_{50} + \epsilon_3(1 - y_{50})(\text{Beta}[y_0, \gamma, 2] - \text{Beta}[y_{50}, \gamma, 2])$$

where

$$y_{50} = 1 - (1 - y_0)e^{-(1 - c_{43})(v_4 - v_0)}$$

The matching point  $v_0$  for the value of  $v_4$  that separates the two approximations obtained is conveniently chosen where  $e^{-c_{42}x_4} = \epsilon_3(x_5/x_m)^{\gamma-1}$ , so as to minimize the error made when each of these terms is dropped respectively for the regions  $v_0 \leq v_4$  and  $v_0 \geq v_4$ . This is because the term  $e^{-c_{42}x_4}$  decreases rapidly for  $v_0 < v_4$  while the term  $\epsilon_3(x_5/x_m)^{\gamma-1}$ increases for increasing  $v_4$ . There is thus little to be gained in accuracy of the lower region approximation by using larger values of  $v_0$ , but much to be lost therein for much lower values. Given a choice for the matching value  $v_0$ , the matching value  $y_0$  is determined from the lower region approximation and the integration constant for the upper region chosen to guarantee continuity.

$$1 - (dy_6/dy_4) = b_6(1 - dy_5/dy_4)$$

The model used for the split of nuclear energy  $u_6 = u_5 - x_5$  from unconventional fuel sources into "byproduct" and "reprocessing" is that a fixed fraction of unconventional fuel comes from each of these two sources. For simplicity here we set the fraction from reprocessing to be  $f_R = 1/(1 + 1/c_R)$  and from byproduct to be  $f_6 = 1 - f_R$ . Here a reference value of the conversion ratio of  $c_R = 1/3$  gives  $f_R = 1/4$  and  $f_6 = 3/4$ . This is specific example of the choice noted above to allow use of "byproduct" uranium (including from seawater) and reprocessed fissile materials to remain in balance over the long term.

From a narrowly conceived market competition basis, it might seem odd that reprocessing is assumed to grow with conventional uranium resource depletion in proportion to the use of other unconventional uranium sources. In particular, recovery of uranium as a byproduct of phosphate production is likely to be substantially cheaper than spent fuel reprocessing unless a substantial growth in uranium use requires more recovery from phosphates than is possible even given a projection near doubling of global population and concomitant agricultural production. However, both France and Japan appear committed to continued spent fuel reprocessing, Russia has considerable experience with this albeit a shortage of capital for upgrading to post-Soviet safety standards, and India and China are also at least nominally interested in reprocessing despite the much lower price of fresh uranium on the global market. In the case of France, Japan, and possibly India, this reflects a historical political commitment to some measure of energy independence and perhaps also previous or ongoing connection with the nuclear weapons potential of maintaining reprocessing capabilities.

If reprocessing capacity alone were the sole determinant of the rate of production of electricity from reprocessed nuclear materials, than something more like an absolute constant value of such energy production might be a more appropriate assumption. However, particularly in the case of Japan, licensed and operational reactor capacity for using reprocessed fuel has not yet kept pace with the rate of shipment of spent fuel for reprocessing. As the stock of unburnt reprocessed material increases, however, the political incentive for actually burning it is likely to increase. Thus a growing rather than fixed absolute rate of actual burning of reprocessed fissile material in the conglomerate set of countries committed to reprocessing seems more appropriate. The most important feature of the approach adopted here is that
it does model a smooth transition between low levels of burning of reprocessed fuel at early times when conventional uranium ores are cheaper to longer term future where reprocessing comes more economically competitive as more cheaply exploitable conventional uranium ores are depleted.

## E.3 Limited Sites for Water-driven Electricity Generation

For the renewable energy models as described so far, there is no limit to how far their use may be scaled up in competition with other energy sources, including with nuclear energy in the long run. In practice, there are a limited number of favorable locations for some renewable energy sources. This is true to a small extent for windmills mounted on passes in hilly areas, but after this there remains a vast potential on planar areas at mid-latitudes, for example in China and west of the Mississippi River in the United States. Likewise for solar thermal electric energy there is a vast potential area in dry sunny regions near 30 degrees latitude, if economies of scale can be exploited cheaply enough to allow solar thermal electricity generation to compete on a large scale. For large hydroelectric dams, however, this is not the case. In the case of the United States, development of large scale hydroelectric dams has saturated at about a fraction 0.53 of the nominal potentially economic amount reported by the United Nations. In developing countries large scale dams are proving increasingly difficult to site due to concerns about impact on the environment and people, particularly in areas where population density is comparatively high compared to what it was in areas when and where large scale hydropower penetrated the U.S. market.

For the calculations done here is is assumed that the results of the above model are applicable only up to a given fraction of the economic potential reported by the United Nations with this fraction different from region to region. For a two-region division into largely "tropical"/developing countries and largely "temperate"/developed countries, for reference values this limit is set to 0.53 for developed countries and  $0.5 \times 0.53$  for developing countries. The  $0.5 \times 0.53$  and 0.53 factors correspond to annual thermal equivalent energy production of 17.6 EJ and 37.8 EJ respectively for the "tropical"/developing and "temperate"/developed countries region global disaggregation used in this thesis. Beyond this limit the incremental increase in hydropower calculated from the above formulas is multiplied by a specified constant (with a reference value of 0.5 for all regions). This is done because it would be inappropriate to assume an abrupt shut-off of growth in water-driven electricity production once the "dam limits" specified here are reached. There are three reasons for this. First, adequate economic incentive and allowing enough time for gradual adaption to spin-off effects from large hydropower projects may allow some further construction of large dams. Second, additional geothermal, tidal, or even wave power potential may be developed after limits on large dam construction are reached. Third, there is considerable potential for small scale hydropower projects that do not involve the construction of dams on such a large scale that major population displacements or regional environmental damage occurs for individual projects. The reference value for the factor which decreases the rate of expansion of water-driven electricity generation is chosen with these considerations in mind.

### E.4 Specification of Constants

Finally here it remains to specify the reference values for the constants  $c_{43}$ ,  $x_m$ , and  $\epsilon_3$  used in above formulas describing the competition between nuclear and new renewable energy sources. The sum  $c_{43} + \epsilon_3$  is the limit fraction of nuclear plus new renewable energy supplied by new renewables. In the historical database only wind electric and solar thermal electric energy are included in new renewables. If the early twenty-first century new renewables would capture most of a market share  $c_{43}$  if they continued their recent very rapid rate of market penetration. This assumes that costs of mined uranium continue to have a very small impact on the competitiveness of nuclear power during the rapid "learning by doing" period for new renewables. In the context of the present model, after initial rapid market penetration the market share of new renewables very gradually increases to a limit value of  $c_{43} + \epsilon_3$  uranium resource depletion slowly increases the cost of nuclear energy.

As water-driven electrical energy is limited in overall development potential and to particular geographic locations, here the discussion of the technical basis for estimating the competition between nuclear and renewable energy sources is limited to consideration of new renewables, in particular to support an estimate of the parameter  $c_{43}$ . Both wind and solar energy are time-varying sources of electrical energy. In populated temperate regions winds are often strongest in the spring and autumn when energy demand is low, and wind energy also varies substantially and irregularly in many locations from day to day. Insolation of course varies both diurnally and seasonally, and in some locations can also have irregular variations due to changes in cloud cover. All of this puts limits on the fraction of energy from these resources which can readily be absorbed by regional electricity distribution systems. These energy sources are most useful when conserving on the use of expensive fluid fossil fuels being burnt in low-capital-cost but energy-inefficient power plants. Except in the case of reliable solar thermal electric energy meeting peak summer air conditioning load demand, they are less useful for displacing low-fuel-cost but also capital intensive nuclear power. Nevertheless, conventional new renewable energy sources would appear to have the potential to compete for about fifteen percent of the market that would otherwise be provided by nuclear power. The discussion in the previous paragraph is limited to wind-electric and solar thermal electric energy sources without energy storage capacity. In some cases it may be possible to combine these systems with energy storage for electricity production to mitigate the shortcomings they suffer due to variable availability. Examples include locating them near pumped hydroelectric power generators to lift water back over dams and storage of compressed air to increase the efficiency of gas-fired power plants. In the longer run as fluid fossil fuels become more expensive, the generation of transportation fuels during periods of excess renewable energy supply may become economically competitive. With sophisticated metering and time-sensitive demand control, it may also become economic to use excess renewable energy supply to replace fluid fossil fuels in other ways. Examples could include recharging batteries of plug-in hybrid vehicles on windy nights or in daytime parking garages when excess solar electric power supply is available. There is also the possibility that metered electricity may eventually be provided by other new renewable technologies, such as bioelectric energy generation using genetically engineered plants. Considering all of these possibilities, it seems reasonably likely that the limit market fraction of new renewables could be about twice the fifteen percent limit imposed by the current energy generation and transmission system, rising to a slightly higher value characterized by the value of  $\epsilon_3$  estimated below as conventional uranium responces gradually become comparatively more expensive. Thus the reference value chosen here is  $c_{43} = 0.3$ .

A reference value for the equivalent thermal energy content in ZJ of the asymptotic limit of global conventional uranium ores is estimated for a reference case as

$$x_{tot} = (360/130)^{m_5} x_r (7.5 \times 3600 \times 24/0.38)/10^6.$$

Here US\$360/kg at 1995 prices is an estimate of the ultimate cost of recovering uranium metal from seawater, US\$130/kg at 1995 prices is a reference cost of uranium ore at which Singer (1997) estimated the availability of  $x_r \approx 20$  Mtonne (rounded from a nominal result of 18.8 Mtonne) of uranium metal in conventional ores, 7.5 GWdays-electric is an estimate of the burn-up for natural uranium-fueled reactors, or for enriched uranium reactors in terms of input uranium to the enrichment process. Also,  $3600 \times 24$  the number of seconds per day, 0.38 a reference thermal to electric conversion efficiency, and  $10^6$  a factor for converting the result to EJ. For reference values of the exponent  $m_5 = 2.3$  and  $x_r = 20$  this corresponds to a very sizable global conventional uranium ore resource endowment of 355 ZJ thermal energy equivalent. Since mined uranium is readily transportable, this global endowment is apportioned to each region in proportion to its limit rate of nuclear energy use. The parameter  $\epsilon_3$  that quantifies the impact of uranium ore depletion is estimated using an extension of a simplified version of the model of Bunn et al. (2003), referred to here as the "Fetter" model after one of the authors who worked on its mathematical formulation. For this purpose the dependence of the nuclear market fraction f of nuclear plus renewable electricity generation is approximated in the form

$$c_0 + mc_u - c_h = (1 - f)E$$

Here  $c_0 + mc_u$  is a linear approximation to the dependence of the cost of nuclear electricity on the cost  $c_U$  of mined uranium,  $c_h$  is the cost of renewable electrical energy generation that would drive the nuclear market share f to zero, and E is a proportionality constant to be evaluated by substituting in reference values  $c_r$  and  $f_r$  respectively for  $c_u$  and f. Substituting in the above expression  $c_u = c_m y_5^{\gamma-1}$  and solving for f gives

$$f = 1 + c_h/E - c_0/E - (mc_m/E)y_5^{\gamma-1}$$

Thus we identify the coefficient  $mc_m/E$  with  $\epsilon_3$ . For this purpose an estimate of  $c_h = 0.045$ US\$2003 per kilowatt hour is taken from the cost of electricity in the U.S. Pacific Northwest, which competed with a nuclear power project to its demise locally and ships power at an expense increasing with distance to capture a sizable share of the west coast market that might otherwise be supplied with nuclear power for baseload electricity generation. For the reference values,  $f_r = 0.473$  is used from the developed region database, and the following calculation is used for  $c_r$ , also denoted by the symbol **Cr**:

cost = 10;kgperlb = 2.2; U3O8perU = (3 \* 238.0289 + 8 \* 15.9994)/(3 \* 238.0289);Cr = costkgperlbU3O8perU For the computation of the slope m describing the increase in US\$2003 per kilowatt hour per unit \$/kg increase in mined uranium costs, the slope of the more complicated Fetter model cost of nuclear electricity dependence on  $c_u$  is fit at a reference uranium cost of  $c_m =$ \$300/kg. (Since the Fetter model with its reference values has a cost of electricity nearly linear in  $c_u$  the choice made for  $c_m$  makes little difference. In outline, what the Fetter model does is to break down the cost of electricity production into the cost **cleu** of supplying fabricated low enriched uranium fuel, interest and principal costs **ccap** on capital outlay, and a cost of operations and maintenance **com**. The most complicated input to this is **cleu**, which is broken down into the cost of mined uranium, conversion to uranium hexaflouride, enrichment, and fuel fabrication. Much of the complication in these formulas comes from adjusting the enrichment tails assay to an optimum determined by the cost of uranium feed.

The *Mathematica* formulas for fitting this model with an approximation that is linear in the cost of mined uranium are as follows

Cm = 300; xf = 0.00711; xp = 0.037; B = 43; M = 24000;  $\epsilon = 0.33;$  fc = .005; fs = .005; Cc = 6; Cs = 100; tu = 2; tc = -0.5; ts = 0;i = .05;

$$\begin{split} j &= 1 + i; \\ \ln[\text{Cu.}] := \text{Log}[10, (1 - \text{fs})\text{Cs}/(j^{\text{ts}-\text{tc}}\text{Cc} + j^{\text{ts}-\text{tu}}\text{Cu}/(1 - \text{fc}))] \\ \text{xt}[\text{Cu.}] := 10^{\wedge}(-0.1631(\ln[\text{Cu}])^2 + 0.47055\ln[\text{Cu}] - 2.6453) \\ R[\text{Cu.}] := (\text{xp} - \text{xt}[\text{Cu}])/(\text{xf} - \text{xt}[\text{Cu}]) \\ \text{ff} &= 0.01; \\ \text{Clf} &= 250; \\ \text{tf} &= 0.01; \\ \text{Clf} &= 250; \\ \text{tf} &= 0.5; \\ \text{ccap} &= 0.0475; \\ \text{com} &= 0.0112; \\ \text{Fc} &= 1.128; \\ \text{Cdd} &= 565; \\ \text{tleu} &= 0; \\ V[\text{x.}] := (2x - 1)\text{Log}[x/(1 - x)]; \\ S &= V[\text{xp}] - V[\text{xt}[\text{Cu}]] - R[\text{Cu}](V[\text{xf}] - V[\text{xt}[\text{Cu}]]); \\ \text{Cleu} &= \frac{1}{(1-\text{ff})}(\frac{R[\text{Cu}]}{1-\text{fs}}(\frac{\text{Cu}}{(1-\text{fc})j^{\wedge}\text{tu}} + \frac{\text{Cc}}{j^{\wedge}\text{tc}}) + \frac{S \text{Cs}}{j^{\wedge}\text{ts}}) + \frac{\text{Clf}}{j^{\wedge}\text{tf}}; \\ \text{cleu} &= (\text{Cleu}j^{\text{tleu}} + \text{Cdd})\text{Fc}/(MB\epsilon); \\ \text{celec} &= \text{cleu} + \text{ccap} + \text{com}; \\ m &= D[\text{celec}, \text{Cu}]/.\text{Cu} \to \text{Cm} \\ \text{celecatm} &= \text{celec}/.\text{Cu} \to \text{Cm}; \\ \text{colecatm} &= \text{celec}/.\text{colecatm} &= \text{celec}/.\text{colecatm} = \text{celec}/.\text{colecatm} = \text{celec}/.\text{colecatm} = \text{c$$

Here (2x-1)Log[x/(1-x)] is a function of V[x] called the value function and is evaluated as V[xp] for the product uranium-235 product assay xp from enrichment, V[xf] for the natural uranium feed assay xf and the optimum tails assay

xt[Cu]=10<sup>(-0.1631</sup>(lu[Cu])<sup>2</sup>+0.47055lu[Cu]-2.6453) for uranium feed cost C, where lu[Cu] depends on the cost Cc of conversion to uranium hexaflouride, the cost (S Cs) of uranium enrichment separative work, the small process loss fractions fs and fc, time delays ts-tc and ts-tu between paying for each process and using the product, and the annual cost inflator

j=1+i for real interest rate i. The unit cost **Cleu** of farbricated fuel also includes a fuel fabrication cost **Clf** with associated interest charge determined by the delay time **tf**. The cost of electricity **celec** includes costs **cleu** for low enrichment uranium, **ccap** for paying for capital investment, and com for operations and maintenance. In these formulas **B** is the assumed burn-up based on the mass of enriched uranium metal in fuel loadings, **M** is a units conversion factor for the burn-up, **Fc** is a fuel carrying factor to account for delay between fuel costs and electricity production revenues and  $\epsilon$  is and reference thermal to electric conversion efficiency (The effect of whose slight difference from  $\epsilon = 0.33$  the value of 0.38 used generically for electrical energy sources elsewhere in this thesis is subsumed in uncertainties in estimates of the product Fc/(MB) which it divides. Also, D[celec, Cu] = dcelec/dCu, and the notation  $/.Cu \rightarrow Cm$  indicates evaluating the result with Cu replaced by Cm. This costing model assumes waste management through direct disposal with a cost of **Cdd** per kg, which adds a contribution to overall costs which is small compared the capital payoff costs. In principle the model should be reworked to allow for a longer time delay before incurring direct disposal costs due to the use of interim spent fuel storage, but in practice this would have very little impact on the slope of the dependence of electricity costs on mined uranium costs and is in any case small compared to the uncertainty in estimating capital costs. The reader is referred to Bunn et al. (2003) for the logic behind the choice of reference values in this calculation. The value of the impact of uranium cost increases on nuclear power fraction resulting from this procedure is  $\epsilon_3=0.153$ .

## Appendix F

## Calibrating and Sampling Energy Model Parameters

Statistical methods for calibrating and sampling parameters used in modeling energy and carbon use are described in this Appendix. These include four global parameters and seven additional parameters calibrated against time-series data for groups of data reporting regions. For the four global parameters methods are described for obtaining a beta distribution for the capital fraction of production, a normal distribution derived from suitably weighted data pertinent to the social discount rate, and Student's t-distributions for the depreciation rate and the inverse of the intertemporal substitutability of consumption.

For regionally specific parameters, approximately linear regressions are computed with up to two dominant periodic corrections, and neighboring data is averaged as needed to limit correlation in residuals between data and theory below a specified level of statistical significance. Methods for deriving and sampling marginal Student's t-distributions integrated over the amplitudes of contributions to periodicity corrections are described for these parameters. Additional global parameters in equations describing atmospheric carbon loading and global average temperature response also have probability distributions that can be approximated by Student's t-distributions in a manner similar to that for regionally specific parameters. Approaches to adapting to these methods to atmospheric response parameters are also described.

## F.1 Introduction

Statistical methods for calibrating and sampling parameters in energy use models are described here. These are presented in four parts: global "capitalization delay" parameters without periodicity corrections, regional parameters calibrated against time series data with periodicity corrections, global atmospheric temperature response parameters calibrated with periodicity corrections, and maximum likelihood estimates for fuel fractions model parameters.

There is only one relevant parameter used in this approach which has an effect on the calibration of energy and carbon use models but whose effect on the calibration is expected to be so small that it can not itself usefully be calibrated against the data. This is the ratio h of energy production efficiency with negligible fossil fuel depletion to that with no fossil fuel use, given otherwise the same levels of economic development and of capital and labor applied to energy production. An examination of the range of electricity production costs and of technologies that can substantially reduce energy required for transportation and heating suggests the h - 1 may be of order unity, i.e.  $h \sim h_0$  where  $h_0 \approx 2$ . To account for uncertainties in such estimates, one can sample normal distributions for Log[h] with mean  $Log[h_0]$  and standard deviation  $\sigma_h$ . For  $\sigma_h$  a simple and appropriate probability distribution would have it taking the values  $\{0.2, 0.3, 0.4\}$  is 1:2:1 binomially-weighted distribution.

## F.2 "Capitalization Delay" Parameters

Four parameters taken to be global constants are needed for estimation of the maximum "departure from quasistationary equilibrium" in a utility optimization model for the evolution of gross domestic product. These are the capital share of production  $\alpha$ , the inverse  $\theta$  of the intertemporal substitutability of consumption, the depreciation rate  $\bar{r}$ , and the social discount rate  $\bar{\rho}$ .

#### **F.2.1** Capital Share of Production, $\alpha$

Following Bickel and Doksum (2001, p.13) the Beta distribution for  $\alpha$  has the form

$$\alpha^{\hat{r}-1}(1-\alpha)^{\hat{s}-1}/\text{Beta}[\hat{r},\hat{s}]$$

Denoting the average over the data set by  $\langle \rangle$ , this function is fit to reported set of estimates for various countries by solving the equations (Bickel and Doksum, 2001)

$$< \mathrm{Log}[\alpha] >= \texttt{PolyGamma}[\hat{s}] - \texttt{PolyGamma}[\hat{r} + \hat{s}]$$

$$< \text{Log}[1 - \alpha] >= \text{PolyGamma}[\hat{s}] - \text{PolyGamma}[\hat{r} + \hat{s}]$$

Here the (complete) Euler Beta function defined as  $\text{Beta}[\hat{r}, \hat{s}] = \Gamma[\hat{r}]\Gamma[\hat{s}]/\Gamma[\hat{r}+\hat{s}]$ , with

$$\Gamma[X] = \int_0^Z Z^{X-1} e^{-Z} dZ$$

the usual Gamma function. Also,  $\operatorname{PolyGamma}[X] = d\operatorname{Log}[\Gamma[X]]/dX$  is the simplest form of a polygamma function known as the digamma.

The function

$$\texttt{InverseBetaRegularized}[\texttt{Random}[], \hat{r}, \hat{s}]$$

where Random[] is a uniform random number between 0 and 1 provides a random sample of this distribution. Here InverseBetaRegularized[ $X, \hat{r}, \hat{s}$ ] is the solution for X in the equation

$$Beta[X, \hat{r}, \hat{s}]/Beta[\hat{r}, \hat{s}] = Random[]$$

Estimates for the labor share of capital  $\omega = 1 - \alpha$  used to estimate  $\alpha$  for this purpose for 31 countries are taken from Gollin (2002).

## F.2.2 Inverse, $\theta$ , of the Intertemporal Substitutability of Consumption

An estimate

$$\hat{\theta} = 1 - \hat{m}_{\theta}$$

for the inverse of the intertemporal substitutability of consumption is obtained from a least squares fit of the form

$$Log[(100 - wellbeing)/100] = m_{\theta}Log[wealth/1000] + b_{\theta}$$

to a set of  $n_{\theta} = 41$  estimates of self-reported "wellbeing" as a function of average per capita income "wealth" from various countries. A random sample of the marginal distribution for  $\theta$  integrated over  $b_{\theta}$  can be obtained by sampling the Student's t-distribution with  $n_{\theta} - 2$ degrees of freedom

$$1 - \hat{m}_{\theta} + \texttt{Random}[\texttt{StudentTDistribution}[n_{\theta} - 2]]s_{\theta}/\sqrt{n_{\theta}}$$

where

$$s_{\theta} = \sum_{j=1}^{n_{\theta}} (\text{Log}[(100 - \text{wellbeing})/100]_j - (m_{\theta}\text{Log}[\text{wealth}/1000]_j + b_{\theta}))^2 / (n_{\theta} - 1)$$

Here Random[StudentTDistribution[ $n_{\theta} - 2$ ]] is the value of X obtained by setting the cumulative standard t-distribution equal to a random number between 0 and 1:

$$\int_0^X dZ (1+Z^2/N)^{-(N-1)/2}/(\text{Beta}[N/2,1/2]\sqrt{N}) = \text{Random}[1]$$

The data set used consists of estimates of well being, and purchasing power estimates of per capita income from Maddison (2003) adjusted for inflation with the U.S. consumer price

index for the slight difference in years in which the surveys on self-reported wellbeing were done (USDOL, 2005).

#### F.2.3 Depreciation Rate

An estimate

$$< \operatorname{Log}[\tilde{r}] >$$

of the logarithm of the depreciation rate is and its variance are used to provide samples

$$s_r = \sum_{j=1}^{n_r} (\text{Log}[\tilde{r}]_j - < \text{Log}[\tilde{r}] >)^2 / (n_r - 1)$$

can be used to provide samples

$$< \mathrm{Log}[\tilde{r}] > + \mathtt{Random}[\mathtt{StudentTDistribution}[n_r - 2]]s_r / \sqrt{n_r}$$

of the logarithm of the depreciation rate. The estimated depreciation rates used for this are for estimates for the United States for 1948-78 inclusive are taken from Bischoff and Kokkelenberg (1987).

#### F.2.4 Social Discount Rate

Following a model of maximization of the utility of per capita household income described by Barro and Sala-i-Martin (1995) and motivated by earlier work of Ramsey (1928), the social discount rate is estimated from real interest rates less growth rates of per capita gross domestic product. The real interest rate is the nominal lending rate less the rate of inflation. For cases with high inflation rates this involves taking the difference of two large numbers. On the assumption that the values of these numbers are distributed with common variance, it is thus approximately appropriate to weight the contribution from each pair thereof by the inverse of the square of the lending rate. The data on lending and inflation rates comes from the World Bank (2005), and economic growth rates are computed from data from Maddison (2003). Under these assumptions, the probability distribution for one observation can be rewritten as  $(2\pi\tilde{\sigma}^2\tilde{w}_j^2)^{-1/2}e^{-(\tilde{y}_j-\tilde{\rho})^2/(2\tilde{\sigma}\tilde{w}_i)^2}$ . Here

$$\tilde{y}_j = \tilde{\Re}_j - \hat{\theta}\tilde{g}_j$$

where  $\tilde{\Re}_j$  are real interest rates and  $\tilde{g}_j$  refers to the rates fractional annual growth in GDP. Multiplying these distributions gives

$$L(\tilde{\rho}, \tilde{\sigma} | \boldsymbol{\tilde{y}}) = \frac{\sum_{j=1}^{n} e^{-((\tilde{y}_j - \bar{\rho})/\tilde{w}_j)^2/2\tilde{\sigma}^2}}{\prod_{j=1}^{n} 2\pi \tilde{\sigma}^2 \tilde{w}_j^2}$$

Following Box and Tiao (1972), this will be multiplied by a prior probability distribution for  $\sigma$  proportional to  $1/\sigma$  to obtain the posterior probability distribution  $P(\tilde{\rho}, \tilde{\sigma} | \mathbf{\tilde{y}})$  for the data given the vector  $\mathbf{\tilde{y}}$  of data from which an estimate  $\hat{\rho}$  is to be found. Maximizing with respect to  $\bar{\rho}$  gives the estimate

$$\hat{\rho} = \frac{\sum_{j=1}^{n} (\tilde{y}_j / \tilde{w}_j)}{\sum_{j=1}^{n} 1 / \tilde{w}_j}$$

What we are interested in here is the marginal distribution for  $\tilde{\rho}$  integrated over the "nuisance parameter"  $\sigma$ . Generalizing the derivation from Box and Tiao (1972) to the weighted case case of interest here, note that

$$\sum_{j=1}^{n} ((\tilde{y}_j - \bar{\rho})/\tilde{w}_j)^2 = \sum_{j=1}^{n} ((\tilde{y}_j - \hat{\rho})/\tilde{w}_j)^2 + (\tilde{y}_j - \hat{\rho})^2 \sum_{j=1}^{n} (1/\tilde{w}_j)^2$$

Defining

$$\bar{s}^2 = 1/\nu_{\rho} \sum_{j=1}^n ((\tilde{y}_j - \hat{\rho})/\tilde{w}_j)^2 \frac{n}{\sum_{j=1}^n (1/\tilde{w}_j)^2}$$

where

$$\nu_{\rho} = n - 1$$

we have that

$$P(\tilde{\rho}, \tilde{\sigma} | \boldsymbol{y}) = \tilde{k}_{\rho} \tilde{\sigma}_{w}^{-(n+1)} e^{(\nu_{\rho} \bar{s}^{2} + n(\bar{\rho} - \hat{\rho})^{2})/2\tilde{\sigma}_{w}^{2}}$$

where

$$\tilde{\sigma}_w = \tilde{\sigma}n / \sum_{j=1}^n (1/\tilde{w}_j)^2$$

Here the constant  $k_{\rho}$  is to be chosen to get the total probability to integrate to 1. Integrating this result over  $\sigma$  exactly follows the similar integration for the unweighted case as in Box and Tiao (1972), thus giving a t-distribution for the marginal probability density for  $\tilde{\rho}$  integrated over  $\sigma$ :

$$P(\bar{\rho}|\boldsymbol{y}) = (\bar{s}/\sqrt{n})/(\text{Beta}[\nu_{\rho}/2, 1/2]\sqrt{\nu_{\rho}})(1 + \frac{n(\bar{\rho} - \hat{\rho})^2}{\nu_{\rho}\tilde{s}^2})^{-(\nu_{\rho} + 1)/2}$$

When n is very large, as in the cases of interest here, this can be approximated by a normal distribution with mean  $\bar{\rho}$  and variance  $\bar{s}^2$ .

# F.3 Marginal T-Distributions for Region-Specific Parameters

#### F.3.1 General Considerations for Region-Specific Fits

There are four time-series data streams of interest here that are taken to be sums from groups of reporting entities. Here these groups will for brevity be called "regions," whether or not they are geographically compact. We also follow a fairly common practice of referring to sequential data as "time" series whether or not a linear function of time is used to index the data.

It is often the case that the residual differences between data and an estimated simple fit to the data are not independently and identically distributed. Consider, for example, the question of how well the mean of the following data set is known:

 $\{30, 31, 28, 29, 30, 71, 73, 69, 68, 69, 40, 38, 39, 40, 43, 60, 61, 59, 59, 62, 50, 52, 46, 51, 51\}$ 

The (here obviously erroneous) assumption that the differences between this data and the mean of are identically and independently normally distribution leads to an estimate of  $50\pm3$  for the mean and standard error from this data. Actually, however, to a good approximation there are only the five repeated data {30, 70, 40, 60, 50} with mean and standard error  $50\pm8$ . More generally, at least to the extent that we wish to use time series data for estimates of the uncertainty in fitting parameters, possible correlations between residuals and simple fits need to be allowed for.

Both near-neighbor correlation and longer term periodicity are to be expected in the data used here. For example, while the difference between fertility and mortality rates may vary from year to year, since only a small fraction of the population generally dies or gives birth in any year there a year-to-year correlation in deviations of population from mean trends is expected. There may also be long-term cyclic trends whose amplitude may or may not damp over time (e.g. due to the growth of high fertility subgroups in the population, which may or may not retain the cultural cohesiveness needed to maintain high fertility over a long time period). Economic production is also known to be cyclic, including both shorter term minor business cycles and generational cycles of learning and to some extent forgetting appropriate mechanisms for preserving macroeconomic stability.

Here we account for the possibilities of both longer term periodicity and nearest neighbor data correlations in "time" series. Periodicity is accounted for by computing a "periodogram" of the amplitudes of a Fourier decomposition of the data (Wei, 1990). Nearest neighbor correlation is dealt with be differencing the data were appropriate and m-ennial averaging, where m is chosen to be the smallest number that adequately reduces the probability that remaining correlations have not occurred by chance.

Moreover, only population and GDP growth rates are needed for calibrating the model used, so the population and GDP data can be differenced to reduce nearest neighbor correlations. The calculation sequence used for each data set is to compute a complete periodogram and make maximum likelihood estimates of the fitting parameters with 0, 1, and 2 of the dominant periodicities allowed for. The periodic corrections are expressed as a linear sum of Sin and Cosin contributions, whose amplitudes can be eliminated analytically when minimizing the sum of the squares between the residuals and the data. The number of periodicities included for each fit is chosen so that the statistical significance of the next most dominant one occurring by chance does not exceed a specified threshold. The maximum number of periodicity corrections for any fit is limited to two, which turns out to be sufficient. This procedure is then repeated as needed with *m*-ennial averaging until the probability of remaining nearest neighbor correlation occurring by chance also does not exceed a specified threshold. For N independent regional fits to four data sets with both nearest neighbor and longer-term periodicity, each with a chance  $\delta$  of a remaining correlation occurring by chance, there is a chance  $(1 - \delta)^{8N}$  that none of them will have residual correlation by chance. A simple cutoff criterion for  $\delta$  is to set  $(1 - \delta)^{8N}$ ="siglimit," where "siglimit"=1/2. For N = 2this gives "siglimit"=0.04. This is a generalization of the procedure described by Wei (1990, p.261) for checking the significant of one of all possible discrete Fourier component amplitudes exceeding a given threshold by chance.

The procedure used here constrains the periods of periodicity to be an integer multiple of the minimum data spacing. According to Wei (1990), based on experience with similar types of data set, the restriction to periods as integer multiple of the minimum data spacing is not an important limitation. In future work this should be checked for the lowest frequency, where this approximation is least accurate, by allowing the lowest frequency to vary continuously. A final introductory comment is that the forms of the underlying fits here are all chosen to be linear to a good approximation. These means that to a good approximation the probability distributions for the fitted parameters are multivariate t-distributions after averaging over a variance that is assumed to be uniform and normal within each fit. In each case the periodic corrections themselves are not of particular interest for long-term extrapolations, for a variety of reasons. For the population growth rate, the periodic corrections turn out to be small. The value of the carbon intensity of production around which its variation occurs

asymptotes to zero. For GDP, the result from the calibration enters only to first order in  $\epsilon_1$  in the rest of the calculation, which is in any case taken to depend on the mean value of the calibration and not on the periodic corrections. For energy and carbon emissions, the amplitudes of the periodic components are also modest, and their effects should generally average out over time. It should also be noted that negative values of the fitting parameters are unphysical, so in case of a usually extremely unlikely event that the sampling process produces a negative value that value is dropped an another one is computed in its place.

#### F.3.2 Periodicity Corrections and Significance Tests

In cases for which periodicity corrections are included, data with even spacing along the abscissa is first prepared if needed. This is accomplished by *m*-ennial averaging, with *m* the minimum value chosen to make the nearest neighbor correlations not significant at the "siglimit" level described above. For the fit of carbon intensity of production vs. cumulative carbon use. Data on cumulative carbon use are linearly interpolated within intervals of each increments in cumulative carbon use, with at least two data points per interval. This procedure not only greatly simplifies the periodogram analysis for this case, it also averages over and down-weights what would otherwise be a large amount of more variable earlier data in cases where the data set extends far back in time.

The use of equally spaced data for each series allows use of a common procedure for estimating the amplitude and statistical significance of periodic corrections. From Wei (1990, pp.261-262), we halve the number n of data points and round down to the nearest integer to obtain

$$n_2 = \lfloor n/2 \rfloor$$

Choosing frequencies

$$f_k = 2\pi k/n$$

let  $e_m$  be the residuals between the uncorrected fit and the data and the Fourier amplitudes be

$$B_k = 2/n \sum_{m=1}^n e_m \text{Sin}[mf_k]$$
$$A_k = 2/n \sum_{m=1}^n e_m \text{Cos}[mf_k]$$

for  $k = 1 \dots n_2$ , except that  $A_k = 1/n_2 \sum_{m=1}^n e_m \cos[mf_k]$  if n is even. The periodicity amplitudes are

$$p_k = (n/2)(A_k^2 + B_k^2)$$

except that  $p_{n_2} = nA_{n_2}^2$  if n is even. Ordering these in a sequence  $P_j$  of decreasing size with increasing index j, the significance criterion for each amplitude is to an excellent approximation (Wei, 1990)

$$(\nu_M + 1 - j)(1 - \tau_j)^{\nu_K - j}$$

where

$$\tau_j = P_j / ((\sum_{m=1}^{n_2} P_m) - (\sum_{m=1}^{j} P_m))$$

Here

$$\nu_M = n - M$$

where M is the number of parameters in the fit (one for the GDP growth rate model and two for the others).

The test used for significance of nearest neighbor correlation is

$$\sum_{m=1}^{n-1} (e_m - \mu)(e_{m+1} - \mu) / \sum_{m=1}^{n} (e_m - \mu)^2$$

where the mean value of the residuals is

$$\mu = (1/n) \sum_{m=1}^{n} e_m$$

#### F.3.3 Population Growth Rate and Development Index

The logistic development index  $a = 1/(1 + e^{-\tilde{\nu}(\tilde{t}-\tilde{t}_0)})$  increases approximately linearly with time near the inflection point where a = 1/2. The next term in the power series expansion around a = 1/2 that gives linear approximation

$$a \approx 1/2 + (\tilde{t} - \tilde{t}_0)\tilde{\nu}/4$$

is  $-\tilde{\nu}^3(\tilde{t}-\tilde{t}_0)^3/48 \approx -(4/3)(a-1/2)^3$ . The population growth rate  $\tilde{\nu}z = \tilde{\nu}(1-a)$  used here to calibrate the logistic model parameters is thus usefully written in the form

$$\tilde{\nu}z \approx \tilde{\nu}/2 - (\tilde{t} - \tilde{t}_0)(\tilde{\nu}/2)^2 = (\tilde{\nu}/2)(1 + \tilde{t}_0\tilde{\nu}/2) - (\tilde{\nu}/2)^2 = \theta_1 + \theta_2\tilde{t}$$

where,  $\theta_2 = -(\tilde{\nu}/2)^2$  and  $\theta_1 = (\tilde{\nu}/2)(1 + \tilde{t}_0\tilde{\nu}/2)$  and thus  $\tilde{t}_0 = (2/\tilde{\nu})(2\theta_1/\tilde{\nu} - 1)$ , and

$$\tilde{\nu} = 2\sqrt{-\theta_2}$$
 and  $\tilde{t}_0 = (\theta_1/\sqrt{-\theta_2} - 1)/\sqrt{-\theta_2}$ 

(For uniform notation between  $\theta_1$  and  $\theta_2$ , the overbar on  $\theta_2$  is omitted even though it has dimensions, in this case of 1/time.)

Inserting these expressions into  $\tilde{\nu}z = \tilde{\nu}(1-1/(1+e^{-\tilde{\nu}(\tilde{t}-\tilde{t}_0)}))$  gives a formulation for  $d\text{Log}[\tilde{P}]/d\tilde{t}$  that can be fit with a function that is nearly linear in  $\{\theta_1, \theta_2\}$  for a reasonably wide range around a = 1/2. This has two advantages. First, an initial estimate of the starting parameters for the full non-linear least square fit can be found from the linear approximation  $\tilde{\nu}z \approx \theta_1 + \theta_2 \tilde{t}$ . Second, the residuals from the nonlinear fit approximately follow the multivariate t-distribution that would result from a linear fit if the residuals are independently and identically normally distributed, after including the requisite number of linear periodic corrections and suitably averaging the input data as described above.

#### F.3.4 GDP Growth Rate

The expression for the logarithmic derivative of GDP vs. development

$$d \text{Log}[\tilde{G}_{\text{DP}}]/da = 1 + \xi + (\alpha/\omega) d \text{Log}[F_1]/da$$

can be rearranged to give

$$\omega(d\text{Log}[G_{\text{DP}}]/da - 1) = \eta + \alpha d\text{Log}[F_1]/da$$

Here  $F_1 = (1 + \epsilon_1 a) / (1 + \epsilon_1)$ , where

$$\epsilon_1 = \nu \theta \xi$$

Since  $\alpha d \text{Log}[F_1]/da$  is of the order the ordering parameter  $\epsilon_1$ , and  $\omega(d \text{Log}[\tilde{G}_{\text{DP}}]/da - 1)$  is known once the calibrations and sampling described above are done, we have here a singleparameter regression that is nearly linear in  $\xi$ . This again allows for a well defined starting point, the average of  $\omega(d \text{Log}[\tilde{G}_{\text{DP}}]/da - 1)$ , for its the complete nonlinear calibration. The marginal distribution for  $\xi$  is also thus approximately a simple t-distribution.

#### F.3.5 Carbon Intensity of Energy Use

The carbon intensity of energy use  $\tilde{f}$  is fit with a piecewise linear function of each region's cumulative carbon use, as follows. First, it is approximated that all primary energy comes from coal with a specified value  $\bar{f}$  of  $\tilde{f}$  before the first oil wells start producing. (This neglects a very small amount of earlier oil mining activity in some regions). Second, the size of the minimum interval in cumulative energy use that contains at least  $n_{\rm av}$  data points is computed, where  $n_{\rm av} \geq 2$ . The data is divided into equal intervals of this length, and a linear fit to the data in each such interval produces a set of estimates  $\tilde{f}_j$  at the midpoints  $\tilde{x}_j$  of these intervals. These pairs are fit with a linear function for the larger values of  $\tilde{x}_j$  and line so obtained is joined at a point  $\tilde{x}_J$  to the  $\bar{f}$  at the value of cumulative carbon production

where oil was first drilled. The number of points along the line so interpolated is adjusted to give minimize the sum of square differences between the set  $\{\tilde{x}_j, \tilde{f}_j\}$  and the fit.

#### F.3.6 Energy or Carbon Use Rates

Either energy or carbon use rates can be used to calibrate two additional parameters, depending on the desired application. When the output is used as a basis for further work on energy supply options, it is appropriate to use the expression

$$\operatorname{Log}[\tilde{w}] = \operatorname{Log}[\bar{w}] + \psi \operatorname{Log}[a] + \operatorname{Log}[p] + (\alpha/\omega) \operatorname{Log}[F_1]$$

where  $F_1$  is defined above, p = h(1 - bu) with b = (h - 1)/h and  $u = (x - 1)/(s_k x - 1)$  with  $x = x_k e^{hf_k(s_k - b)(R - R_k)}$  in each range  $a_{k-1} \le a \le a_k$ , where R is approximated as

$$(\epsilon_0/\nu)a^{\psi}(\Phi - (\epsilon_1/\psi)\alpha/\omega)$$

through first order in  $\epsilon_1$  and  $R_k$  are the values of this expression evaluated at  $a \to a_k$ . A least squares fit linear approximation  $\text{Log}[\tilde{w}] \approx \text{Log}[\bar{w}] + \psi \text{Log}[a]$  provides a unique starting point for values of  $\text{Log}[\bar{w}]$  and  $\psi$  by minimizing the sum of the squares of the residuals between the set of  $\text{Log}[\tilde{w}_j]$ . Over the data range of interest we have  $p \approx h$  and  $(\alpha/\omega)\text{Log}[F_1]$  again small, so a t-distribution approximation is appropriate for  $\text{Log}[\bar{w}]$  and  $\psi$ .

When the output is used as a basis for work on atmospheric response to carbon emissions then the expression

$$\operatorname{Log}[\tilde{w}] = \operatorname{Log}[\bar{w}] + \psi \operatorname{Log}[a] + \operatorname{Log}[p] + (\alpha/\omega) \operatorname{Log}[F_1]$$

can instead be used for a nonlinear square residuals minimization starting from the result from the approximation  $\text{Log}[\tilde{E}] = \text{Log}[\bar{E}] + \psi \text{Log}[a]$ . Here the additional term of Log[f]where  $f_k(1 - s_k u)$  for  $u_{k-1} \leq u \leq u_k$  also varies comparatively slowly over the data range, so the t-distribution should be a reasonable approximation for  $\{Log[\bar{E}], \psi\}$ .

Whichever of  $\text{Log}[\bar{w}]$  and  $\text{Log}[\bar{E}]$  is estimated in these ways, the other can be found for diagnostic purposes from the relation

$$\bar{E} = \bar{f}\bar{w}$$

#### F.3.7 Marginal T-Distributions

For all of the regional parameters just discussed, the amplitudes of the periodicity corrections can generally be treated as "nuisance parameters," so that we are interested in the marginal distributions of the remaining parameters after integrating over the distributions for these parameters. All of the complete distributions are approximated as multivariate t-distributions, and Box and Tiao (1972, pp. 116-118) describe how to find marginal t-distributions for one or two from multivariate ones of more variables. For most of the cases for such marginal distributions here we have two variables of interest and two or four periodicity amplitudes that need to be integrated over. We discuss these cases first, and then return to the GDP data case with only one variable of interest.

Assume we have found constants  $\{\theta_{11}, \theta_{21}\}$  that minimize the sum of the squares of a linear approximation to a desired two-parameter fit to data and used them as starting points for a numerical search for constants  $\{\theta_{12}, \theta_{22}\}$  that minimize the sum of the squares to find  $\boldsymbol{e}_2.\boldsymbol{e}_2$ , where  $\boldsymbol{e}_2(\theta_1, \theta_2)$  is a vector of the differences between the data and a fitting formula that depends on  $\{\theta_1, \theta_2\}$ . Here  $\boldsymbol{X}.\boldsymbol{X}$  denotes the dot product of two vectors and the notation used here closely follows that of a *Mathematica* calculation that can be used to execute the instructions given here.

For a periodicity with the one dominant frequency  $j_1/n$  derived from the periodogram analysis described above, let

$$s_{3j} = z_j^m \text{Cos}[2\pi j j_1/n] \text{ and } s_{4j} = z_j^m \text{Sin}[2\pi j j_1/n]$$

be the components of vectors  $s_3$  and  $s_4$  with j ranging from 1 to the number of data points, n. Here  $z_j = 1 - a_j$  are components of a vector  $\boldsymbol{z}$  with elements evaluated at times of the ndata points. Then find  $\{\theta_{13}, \theta_{23}\}$  that minimizes  $\boldsymbol{e}_3.\boldsymbol{e}_3$ , where

$$e_3 = e_{22}/z - (2/n)(s_3 \cdot e_2 s_3 + s_4 \cdot e_2 s_4)$$

The exponent m will be taken to be zero for fitting development index parameters using population growth data, where damping of periodicity components is not necessarily expected and the values of  $a_j$  and  $z_j$  are not previously known.

Sampling a probability distribution for  $\{\theta_1, \theta_2\}$  starts with computing the matrix  $\boldsymbol{w}$  whose components are

$$\boldsymbol{w}_{\lambda\kappa} = d^2 \boldsymbol{e}_3 \cdot \boldsymbol{e}_3 / d\theta_\lambda d\theta_\kappa$$

and its inverse

 $\boldsymbol{W} = \boldsymbol{w}^{-1}$ 

In the case of a linear fit  $\boldsymbol{w}$  reduces to the matrix  $\boldsymbol{X'}.\boldsymbol{X}$  used by Box and Tiao (1972, pp. 116-117) to compute a marginal multivariate t-distribution. In the case of a nearly linear fit, the results given here will provide an approximation to the desired probability distribution over a wide range of values in the neighborhood of the maximum likelihood estimators. According to Box and Tiao (1972), the marginal probability distribution in the linear case for two parameters  $\boldsymbol{\theta} = \{\theta_1, \theta_2\}$  with maximum likelihood estimators  $\hat{\boldsymbol{\theta}} = \{\hat{\theta}_1, \hat{\theta}_2\}$  given a vector of data  $\boldsymbol{y}$  is

$$p(\boldsymbol{\theta}|\boldsymbol{y}) = \frac{\Gamma[\nu_k + 2]|\boldsymbol{c}|^{1/2}s^{-2}}{\Gamma[1/2]^2\Gamma[\nu_k/2]\nu_k} (1 + \frac{(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})'.\boldsymbol{c}.(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})}{\nu_k s^2})^{-(\nu_k + 2)/2}$$

Here, with n the number of data points and k = 4 the total number of fitting parameters

$$\nu_k = n - k$$

and in the nomenclature used here

$$s^2 = (1/\nu_k) \hat{e}_3 \cdot \hat{e}_3$$

where  $\hat{\boldsymbol{e}}_3$  is the value of  $\boldsymbol{e}_3$  evaluated at for the maximum likelihood values  $\hat{\theta}$ . The matrix  $\boldsymbol{c}$  is the inverse of the 2×2 matrix  $\boldsymbol{C}$  of the upper left entries of the inverse matrix  $\boldsymbol{W}$ 

$$\boldsymbol{C} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \text{ and } \boldsymbol{C}^{-1} = \boldsymbol{c} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

For a periodicity with the two dominant frequencies  $j_1/n$  and  $j_2/n$  derived from the periodogram analysis described above, also let

$$s_{5j} = z_j^m \text{Cos}[2\pi j j_2/n] \text{ and } s_{6j} = z_j^m \text{Sin}[2\pi j j_2/n]$$

Then find  $\{\theta_{14}, \theta_{24}\}$  that minimizes  $\boldsymbol{e}_4.\boldsymbol{e}_4$  where

$$e_4 = e_{22}/z - (2/n)(s_3.e_2s_3 + s_4.e_2s_4 + s_5.e_2s_3 + s_6.e_2s_6)$$

Then redefining  $\boldsymbol{w}_{\lambda\kappa} = d^2 \boldsymbol{e}_3 \cdot \boldsymbol{e}_3 / d\theta_\lambda d\theta_\kappa$  and  $\boldsymbol{W} = \boldsymbol{w}^{-1}$ , we obtain the same expressions for  $\boldsymbol{C}$ and  $p(\boldsymbol{\theta}|\boldsymbol{y})$  with  $\nu_k = n - k = n - 6$ , and  $s^2 = (1/\nu_k) \hat{\boldsymbol{e}}_4 \cdot \hat{\boldsymbol{e}}_4$ , where  $\hat{\boldsymbol{e}}_4$  is the value of  $\boldsymbol{e}_4$  for the maximum likelihood parameters.

For the case of a probability distribution for the a single parameter  $\xi$  given residuals  $e_2(\xi)$ the same procedure applies with k = 3 for one periodicity correction and k = 5 for two periodicities. In these cases the desired marginal distribution for  $\xi$  reduces to the simple t-distribution which can be sampled using

$$\int_{0}^{t_{\eta}} dZ (1+Z^{2}/\nu_{k})^{-(\nu_{k}-1)/2}/(\text{Beta}[\nu_{k}/2,1/2]\sqrt{\nu_{k}}) = \text{Random}[]$$

to give a random sample for  $\xi$  of

$$\xi = \hat{\xi} + s\sqrt{w_{11}}$$

#### F.3.8 Sampling Bivariate T-Distributions

Setting the bivariate t-distributions described to various constants produces confidence contour functions of  $\{\theta_1, \theta_2\}$  that are ellipses. To sample such a distribution, it is convenient to find a coordinate transformation that makes such contours circles centered on the origin. The radius of these circles than has an F-distribution with  $(2, \nu_k)$  degrees of freedom and the angle  $\Theta$  around the circles is uniformly distributed on the interval  $[-\pi, \pi]$ .

Following Bickel and Doksum (2001, pp. 23-26) we start with the determinant of the symmetric matrix  $\boldsymbol{c}$ 

$$D = (c_{11}c_{22} - c_{12}^2)$$

We then set

$$\sigma_1 = \sqrt{c_{22}/D}; \ \sigma_2 = \sqrt{c_{11}/D}; \ \rho_0 = -c_{11}/\sqrt{c_{11}c_{22}}$$

The sampling procedes first by sampling an F-distribution with  $(2, \nu_k)$  degrees of freedom

$$q_R = 2$$
Random[FRatioDistribution[2,  $nu_k$ ]]

and a unit interval uniform distribution Random[] to get the random angle

$$\Theta = 2\pi (\texttt{Random}[\ ] - 1/2)$$

Then find the ordinate and abscissa for the corresponding point in the form

$$Z_1 = \operatorname{Sin}[\Theta] \sqrt{2q_R}$$
 and  $Z_1 = \operatorname{Cos}[\Theta] \sqrt{2q_R}$ 

The desired sample for  $\{\theta_1, \theta_2\}$  is

$$\{\theta_1, \theta_2\} = (\hat{\theta}_1 + \sigma_1 Z_1 \hat{\theta}_2 + \sigma_2 (Z_1 \rho_0 + Z_1 \sqrt{1 - \rho_0^2}))s$$

While the F-distribution can be conveniently sampled in *Mathematica* using the syntax given here, to visualize the process it can be useful to note, e.g. from Box and Tiao (1972) that for  $(2,\nu_k)$  degrees of freedom its probability distribution can be written as

$$p[Z] = (1 + 2Z/\nu_k)^{-(2+\nu_k t)/2} (2/\nu_k)/\text{Beta}[1/2, \nu_k/2]$$

whence

$$\int_0^X p[Z] dZ = 1 - (1 + 2X/\nu_k)^{-\nu_k/2} (2/\nu_k) / \texttt{Beta}[1/2, \nu_k/2] = \texttt{Random}[1/2, \nu_k/2] = \texttt{$$

This result could, if desired, be explicitly solved algebraically and inserted in the above expression for  $q_R$  to give

$$q_R = 2(2/\nu_k)(1 - \texttt{Random}[]\texttt{Beta}[1/2, \nu_k/2]\nu_k/2)^{-2/\nu_k} - 1$$

### F.4 Atmospheric Response Parameters

As for the energy and carbon use model, there is one parameter in the atmospheric response model that has such a small effect on the calibration results that we do not expect it to be able to be usefully calibrated against available data. This is the parameter  $\gamma_0 = \bar{B}/\bar{\sigma}\bar{\beta}$ . On physical grounds the value for  $\gamma_0/(1 - \gamma_0)$  is expected to lie in the range 0.08 to 0.15 (Petschel-Held et al., 1999). For sampling this parameter one can use a normal distribution for  $\text{Log}[\gamma_0]$  centered around  $\text{Log}[\gamma_0]=\text{Log}[0.10]$  with a standard deviation  $\sigma_B$  chosen as above for  $\sigma_h$ . For a first analysis, the calibration and sampling for the remaining two parameters each in the atmospheric carbon loading and global average temperature response models proceed just as for the cases described above. In these cases, however, the bivariate t-distribution results may only adequately approximate the probability distribution for the model parameters only in the near neighborhood of the maximum likelihood estimators unless the more complicated fitting functions, depending on the degree of departure from linearity of the fitting model and the impact this has on results. For the carbon loading model it may be advantageous to do the fitting for the atmospheric clearance time scale  $\bar{\tau} = 1/\bar{\sigma}$  rather than the clearance rate  $\bar{\sigma}$ . In the limit of small  $\bar{\tau}$  this may make the result closer to linear in this fitting parameter.

Moreover, for the global average temperature periodicities in the differences between the model and the data may have more significant amplitude and persist well into the future, if inferences from the preindustrial period without significant anthropogenic driving terms are a reasonable guide. In this case it may be necessary to sample the full multivariate distribution, without integrating over the periodicity amplitudes viewed as nuisance parameters. Examination of these interesting possibilities is left for future work.

### F.5 Fuel Fractions Parameters

Fuel fractions model parameters are determined by least squares estimators. Methods described above for producing and sampling distribution approximations to probability distributions can also be applied to these fits.

## Appendix G

## Energy Statistics Manual for Users and Programmers

### G.1 Introduction

This section provides basic instructions for generating time series data on various forms of energy production and consumption from the United Nations Energy Statistics Database 2002 (UNSD, 2005). The United Nations Statistics Division provides comprehensive international energy statistics based on the annual questionnaires sent to national statistical agencies of different countries every year. The electronic database used in this thesis for the years starting from 1950 to 2002 contains officially reported information by individual countries about production, imports, exports, bunkers, and stock changes for various primary and secondary conventional, non-conventional, and renewable sources of energy available with the United Nations Statistics Division as of December 31, 2004. The energy statistics yearbook published annually contains time series energy data for five recent years, and is usually released six months after the release of the electronic database. The complete electronic database containing annual energy data from 1950 should be purchased from the United Nations Statistics Division (energy\_stat@un.org). Here we provide complete instructions and PERL source codes for extracting various times series energy data from the electronic database. This instruction manual is sufficient to help set up the electronic files on a personal computer and run various packages for extracting and manipulating the time series data contained in the electronic database. A more detailed description of the source codes and the extracted time series data are provided in the attached CD-ROM. The Program in Arms Control, Disarmament, and International Security (ACDIS) office at the University

of Illinois at Urbana-Champaign retains a stand alone version of the complete electronic repository used in this thesis for internal use, which should be updated periodically.

### G.2 Organization of Electronic Data

The original UN electronic database consists of seven compressed files containing energy data, unit conversion factors, population estimates, and a documentation file: Edata\_1950-1969; Edata\_1970-1979; Edata\_1980-1989; Edata\_1990-1996; Edata\_1997-2002; ConvFct2002; Pop2002; and edataDescrip2002.doc.

Country codes and descriptors, commodity codes and transaction codes for various primary and secondary energy forms, and standard conversion factors are found in the documentation file. The annual energy data for various countries are provided in the first five of the listed compressed files, and provided in a six column tab delimited format as explained in Chapter 4: column 1 containing 3-digit country codes found in Annexes I and II of the documentation; column 2 containing a combined energy code consisting of a 2-digit commodity code and 2-5 digit transaction code described in Annex III of the documentation; column 3 containing the year; column 4 containing the quantity or unit code; column 5 is either blank or \* (which indicates UN estimate in the absence of official record); and column 6 containing the raw data. The various PERL packages written for the purpose interprets these alphanumeric codes and extracts various user-specified time series energy data from the source files.

## G.3 Requirements and Installation

The PERL packages written for the present purpose are platform independent and can be implemented on Windows, Mac OS, and Linux systems. Computers running Mac OS X come preloaded with PERL distribution. For Windows and Linux based computers, there are various free distributions of PERL implementation files. A popular free PERL distribution is ActivePerl 5.8 available from: http://www.ActiveState.com.

### G.4 Code Implementation

First, decompress the energy data files and place the extracted files in a conveniently named directory. Then create a two column tab delimited text file containing the country names and their corresponding ISO codes (a 3-digit numeric code by which each country/region is denoted) and place it in the same folder as the energy files. Finally place all the required PERL packages, which are provided in the attached CD-ROM, in the same folder. Now the files and packages are set up for implementation. For Windows based systems, the PERL packages can be run from DOS command prompt. For Mac OS X based systems, the packages can be run from the "Terminal" window, a UNIX interface utility found in the "Applications" folder. The packages primarysolidliquid.pl, naturalgas.pl, primaryelectricity.pl, and secondarypetro.pl can be used to extract time series data for consumption of various forms of primary and secondary energy sources. The options are specified in each packages for user's convenience. The extracted files are place in a default folder and named appropriately. The output files are given in original as well as common units (EJ). Further manipulation of these output files can be done using the provided PERL codes merge\_1950to2002.pl, interpolate\_startyear1950.pl, disaggregate.pl, and cnameclean.pl. The code merge\_1950to2002.pl is used for adding two files. Disaggregate.pl aggregates and disaggregates the output files to match the list of countries and regions with Maddison (2003). A two column tab delimited text file containing the instructions for aggregation and disaggregation is included for the implementation of Disaggregate.pl. The output files occasionally contain holes, which denote the absence of reported information or UN estimate. Hence the output files are interpolated for missing years using compound annual average growth rate. This is done using the package interpolate\_startyear1950.pl. In order to address consistency of naming convention of countries, the package cnameclean.pl is used. Implementation of this package require a two column tab delimited text file containing the ISO codes and the name of all countries in a specified format. This is done to match the naming convention adopted for population and GDP time series provided by Maddison (2003).

## G.5 CD-ROM Contents

The PERL codes and data files are included in the attached CD-ROM.

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T.S. Gopi Rethinaraj was born on 28 October 1972 in Thanjavur, India, and received his bachelor and master degrees in physics from A.V.V.M. Sri Pushpam College, Bharathidasan University, in 1993 and 1995 respectively. He began his professional career as a science reporter at The Indian Express based in Mumbai from 1996 to 1999, and has also written on science and security issues for Jane's Intelligence Review and Reuters. He received the Bulletin of the Atomic Scientists fellowship in January 1999 for investigative reporting on nuclear security issues. After completing this fellowship, he joined the University of Illinois at Urbana-Champaign (UIUC) in August 1999 as a graduate student in the Department of Nuclear, Plasma, and Radiological Engineering to pursue doctoral research under the supervision of Clifford Singer. His teaching and research interests include energy policy and security, science and technology policy, arms control, Asian security affairs, food and water security in Asia, climate policy, and science journalism. He defended his doctoral thesis in June 2005, and is currently an assistant professor of public policy at the Lee Kuan Yew School of Public Policy, National University of Singapore (NUS).