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Modeling of CO_2 Emissions and Uranium Resources

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Abstract

One of the features of our rapidly changing society is its increasing hunger for energy. Viewed in this perspective, global climate change is one of the biggest challenges for humankind. The difficulty will be how to meet the needs for energy services, while simultaneously combating the emission of Green House Gases, given that these emissions cannot be cut abruptly. Based on the work of Singer and Rethinaraj, this thesis presents enhancements to the LOGICAL model and uses this to forecast development in population, gross domestic product, energy consumption, and carbon emissions. These forecasts are then used to explain and model a budget approach (in analogy with the work of Messner et. al.) to mitigating global climate change. An important difference from Messner et. al. is the assumption of 3.5°C instead of 2°C as guardrail.

A great number of technologies have been put forward to reduce the carbon intensity of energy production. The present work focuses on nuclear energy from two different viewpoints. First, the question of sustainability of uranium resources is analyzed with the result that more than enough uranium ore is available to satisfy demand for more than a century. Alternate sources such as reprocessing, or extraction from seawater will stay economically unattractive. Second, an analysis of the carbon intensity of nuclear energy production (which is of course not zero), leads to the conclusion that this intensity is small compared to fossil fuels. Just how much smaller, depends on the energy sources for the remainder of the electric grid. Results range from 32gCO_2e/kWh in a de-carbonized environment to 65gCO_2e/kWh if the grid is dominated by coal-fired power plants.

All mathematical formulae used in the thesis are included in the appendix.

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1. Introduction

One of the features of our rapidly changing society is its increasing hunger for energy. Viewed in this perspective, global climate change is one of the biggest challenges for humankind. The difficulty will be how to meet the needs for energy services, while simultaneously combating the emission of Green House Gas(es) (GHG), given that these emissions cannot be cut abruptly.

Much has been argued about how the "remaining" amount of emissions up to a given limit (which corresponds to a certain increment in temperature) could be allotted. It is questionable whether an agreement can be struck before the developed countries really start to feel the impact of climate change, which is widely thought to require over 2°C increase in global average temperatures over its preindustrial value. Due to both physical inertia in the climate system and institutional inertia that has to be overcome in order to limit further increases in atmospheric GHG concentrations, the overall global average temperature increment may be substantially higher. This thesis therefore assumes a limit of +3.5°C to the rise of mean temperatures. This increase translates into an atmospheric CO_2 concentration of about 761ppm for a reasonably likely evolution of other factors affecting the earth's energy balance [SQ+07].

Especially two points are debated between developed and developing countries: (1) whether the cumulative emissions by each country have to be accounted for, and (2) whether quotas on the emission of atmospheric carbon dioxide will be allocated per capita. To get a negative answer to these questions would have several advantages for the already developed nations. However, as public acceptance and moral issues are playing a crucial role in reaching an agreement, those questions can in the long run only be answered in the affirmative if concurrence of both developing countries and those sympathetic to them is sought. The position of developing countries is likely to remain, demanding a per-capita allocation of CO_2-budgets, ideally for the population level that these countries will *eventually* have to support. In the context of this approach, modeling illustrates that by about 2100 the US- and EU-budget will be in the negative, while China will have used up its quota almost completely (which is almost certainly an understatement, as modeling China offers some challenges). However, many unused "emission-rights" will still lie in Africa and India. Therefore, to foreshadow some results of this thesis, in addition to the reduction of emissions, some form of trade has to take place between developed and developing countries.

Regarding the reduction of CO_2 emissions, nuclear energy seems to be a technology (among others) that is able to achieve that at least in the electricity sector. However, the question of whether nuclear technology will be the principle one to be used to fulfill this goal hinges not only on its ecological desirability in respect to climate change, but also on political, psychological, economic, and technological issues. The political and psychological aspects are evolving around what is usually deemed the 3S-Complex: Security, Safety and Safeguards, and will not be subject to examination in this thesis. Rather, the emphasis is laid on the future availability of appropriate fuel resources in order to assess the longevity of use of nuclear technology. That fuel resources are sufficient to make nuclear energy competitive with other low-carbon-emissions energy sources in the long term is a necessary requirement for nuclear energy to have a profound long-term impact, but that has been called into question.

It is myopic to look at the future of nuclear energy without considering energy production and consumption in general. These are in turn inseparable from both economic development and environmental protection. Therefore, in order to investigate the environmental impact of uranium production and consumption, a model was needed which linked economic and environmental factors to energy production and consumption (in addition, Chapter 2 gives an introduction to econometric modeling). Especially since periods of high oil prices starting in 1973 and 1998 twice brought those subjects to governmental and public attention, scores of studies on this issue have been published. An early study is "A time to choose" [F$^+$74], published by the Ford Foundation. In that study, three scenarios were identified, with the aim of projecting energy demand in the United States until the year 2000. Steady growth of the Gross Domestic Product (GDP) was assumed. Of the three cases, the "historical growth"-, as well as the "zero energy growth"-scenario are of particular interest. The historical growth scenario mirrors typical government and industry projections by log-linearly extrapolating historical energy consumption. Today, this growth path is resembled by the A2 family of scenarios, developed by the Intergovernmental Panel on Climate Change (IPCC). The results were (and are) troubling from an environmental perspective, because of the vast increase in energy production and the resulting challenges for environmental protection and global climate change.

By contrast, the "zero energy growth"-scenario tried to solve the dilemma by assuming a decoupling between economic growth and energy usage. If so, economic re-orientation combined with zero energy growth above that of 1985 could still produce a GDP-growth at the same level as before. A demand for an economic re-orientation towards one relying primary on services instead of production is also a main point of a BUND-study on Germany's Energy future [L$^+$83]. This theory is still supported by some [vW$^+$09], suggesting that very substantial decreases in energy use per unit of economic production are feasible. The intended purpose is a drastic reduction in energy growth and averting the consequences mentioned above, albeit by reconverting the entire economy rather than primarily emphasizing higher energy efficiency. When searching for today's corresponding scenario by the IPCC, one would find close resemblance in the B2-family.

In order to model interdependencies of economy and climate change, Singer and Rethinaraj [Ret05] have developed an econometric-based analysis. The model was named LOGICAL, since it uses a *log*istic function of time as an *i*ndependent variable and *cal*ibrates results against historical data. In their approach, annual data from 1820 on describing energy consumption and economic growth has been aggregated for a set of geographical regions.

The terminal boundary condition is a sustainable fossil-energy-free economy in the infinite time limit. Results are plotted usually to the year 2150. Because of the extensive use made of the LOGICAL algorithms here, their key features will be explained in detail, as well as any changes made to them for the purpose of this thesis (Chapter 2) and additional suggestions for their improvement on the basis of the results presented.

The results of energy consumption predictions are then used to forecast carbon emissions. A short interpretation of the CO_2 extrapolations will be given in Chapter 3, while emphasis will be laid on the effects of these results in respect to budgeting cumulative emissions. From that, possible options for trade or transfer of these budgets will be deducted.

Looking again at nuclear energy, it is unclear if it has a clear advantage over other forms of "clean" energy production. There is uncertainty about the clout of nuclear energy when it comes to reducing emissions, as no technology for producing energy is completely free of these. To assess this, a calculation of the carbon footprint of nuclear power is presented in Appendix A.

To evaluate the resource situation, we not only have to take into account how much ore we will be able to mine in the future. So-called secondary sources like reprocessing and uranium from unorthodox sources like extraction from seawater have to be looked at. Several works have been published on that account, with very different results. The International Atomic Energy Agency (IAEA) [PB02] states that there will be enough uranium for future generations, even with an increased usage of nuclear power. On the contrary, works like that of Dittmar [Dit09] are claiming that the uranium supply will run dry in the very near future.

Deffeyes and MacGregor [DM80] wrote a classic article on world uranium resources, in which they link uranium concentration and content of the earth's upper crust. Currently, uranium is mined from a variety of sources, including veins, fossil placers, and sandstones. Eventually, also lower grade deposits may become economically viable for extraction, continuing as progressively lower grade ores are tapped. The relationship for grade vs. amount in the earth's crust was approximated by a log-normal distribution by Deffeyes and MacGregor. Schneider and Sailor [SS08a] took this up and developed several models to extract prices from the extrapolation Deffeyes and MacGregor gave. They used a simple crustal model where the relationship between cumulative global uranium extraction Q and uranium price P is expressed by

$$Q/Q' = (P'/P)^{\alpha\beta} \tag{1.1}$$

where $\{Q', P'\}$ are reference points. Here α and β describe the relationship between quantities and ore grade, and between ore grade and price, respectively.

The probable development of prices in various IPCC scenarios will also be examined using an algorithm that has been developed to express how uranium prices depend on cumulative uranium use (Chapter 4). Together, the results will give a clearer picture on how high the price can be expected to rise during a 150-year time window. For heuristic purposes, results for very long time frames have been examined in order to get some idea of when uranium supply limitations might become significant.

The interested reader will find the detailed mathematics of the models used, and algorithms developed, in Appendix B.

2. Energy Modeling

2.1. The History of Growth Theory

2.1.1. Early Theories

The significance of economic growth theory is highlighted by the fact that its first and most famous work, Adam Smith's *Wealth of Nations* [Smi04], is nearly 350 years old. At the turning point between pre-industrial and industrial development in Europe, Smith tried to explain divergent standards of living. His reasoning and major parts of his analysis retain their relevance to the present day.

Smith proposed a supply-driven growth model[1], where Y is output or GDP, K is capital, L represents labor (population) and T is land:

$$Y = Y(L, K, T) \tag{2.1}$$

Therefore, the growth in output is a function of the growth in all those factors, *plus* productivity (φ):

$$dY/dt = f(d\varphi/dt;\ dL/dt;\ dK/dt;\ dT/dt) \tag{2.2}$$

He believed that because division of labor is advancing productivity, it is consequently the driving force behind economical growth, although it "is limited by the extent of the market" [Smi04]. Therefore, free access to markets, both domestic and foreign, is crucial. However, as increasing productivity requires increasing investments by entrepreneurs, the capital stock has to rise accordingly. The easiest way to achieve that, is to reduce the living standards of workers, thereby reducing the cost of labor.

Smith did not see growth as eternally rising. He proposed a "stationary state" [Smi04], where population growth and capital accumulation ceased.

Karl Marx (1867-1894) provided some very influential modifications of the classical picture. Firstly, Marx did not believe that the connection between cost and supply of labor was endogenous. In his scheme, wages simply depended on the amount of unemployed laborers. Secondly, he provided through his famous "enhanced reproduction schema"[2] [Mar71] a two-sectoral growth model, which introduces such critical ingredients as the concept of a "steady-state" growth equilibrium.

[1] Smith formulated his thesis without mathematical formulae, but those can be extracted

[2] reproduction here means re-producing commodities, etc., so they can be consumed again and again (re-consumed). Marx differentiates the four phases: production, distribution, circulation and consumption. Because this process generally accelerates with time, (i.e. higher production leads to falling prices and higher consumption), Marx calls his scheme "enhanced".

Like Smith, Marx believed the rate of profit to have a declining tendency in the long term. However, this is not brought about by competition increasing wages (Smith), but rather the "rising organic composition of capital" [Mar71], which he defined as the ratio of "constant capital" (c) (commodities, raw materials, machinery) and "variable capital" (v) (wages, or rather the product of wages and employed labor). With "surplus" (s), the rate of profit (r) can be written as

$$r = s/(v + c) \tag{2.3}$$

and dividing by v

$$r = (s/v)(v/(v + c)) \tag{2.4}$$

yields the inverse of the "organic composition of capital" (c/v) and the "exploitation rate" (s/v). Bearing in mind that (r) has a long term tendency to fall, Marx proposes two ways in which the capitalist can boost his rate of profit again: Firstly, he can substitute labor by machinery, thereby releasing labor into unemployment, which in turn reduces v by simultaneously reducing employed labor and wages and increases (r) (see eq. (2.3)). Secondly, he can exploit his labor force more extensively. The rise in (s/v) in turn increases (r) (see eq. (2.4)).

The twentieth century added three new theories: evolutionary economy, (post-)keynesianism and neo-classicism. Joseph Schumpeter (1883-1950) pioneered the evolutionary concept by forming the term "creative destruction" [Sch76], describing the impact enterpreneuers have on existing structures by furthering invention. Following the works of Charles Darwin[3], Schumpeter proclaimed that only the entrepreneur who continually uses invention to adapt his business, can prosper [Sch06]. A simple (negative) example may be the demise of Polaroid in instant photography soon after digital cameras provided instant viewing of the pictures in a much cheaper way [TLH04]. But Schumpeter carried his argument further: Because the influence of invention disturbs the equilibrium of a circular flow,[4] the results are to be observed in a periodic form. In accordance with Kondratiev[5], he suggested a model in which the four main cycles (Kondratiev, Kuznets, Juglar and Kitchin (c.f. section (2.2.4))), can be superimposed to form a composite waveform [Sch82]. The phenomenon of business cycles is still being discussed, not without controversy. Milton Friedman[6] suggested that economies are not subject to some sort of intrinsic periodic cycle, but rather suffer disturbances to their equilibrium which ought to be treated with perturbation theory. However, Korotayev and Tsirel recently showed by employing spectral analysis, that Kondratiev, Juglar and Kitchin cycles are present in the world GDP at an acceptable level of statistical significance (c.f. (2.2.4)) [KT10].

Arguably the most important figure of the twentieth century in economics was John M. Keynes[7]. His book, *General Theory of Employment, Interest and Money* [Key36] changed

[3] Charles Robert Darwin (1809-1882), English naturalist
[4] which he defines not quite as Marx did, but close enough for our reasons
[5] Nikolai Kondratiev (1892-1938), Russian economist
[6] Milton Friedman (1912-2006), American economist and Nobel laureate
[7] John Maynard Keynes CB, (1883-1946), British economist

the way the world looked at the economy and the role of government in society permanently, and virtually over night. The work was groundbreaking in that it pioneered formal macroeconomic growth theory. Until then, economists expected that perturbations were healed by the forces of the market itself, without the need of political interference to return to equilibrium. When the Great Depression[8] occurred, there was need for an explanation why these forces failed and the recession went deeper and deeper. Keynes explained that if wages and prices are too inflexible to accommodate downward trends, demand can drop sharply and the economy can be caught in a continuous state of underemployment. To remedy the situation, he assigned the state an interventionist role, flanked by a devaluating monetary policy. Keynes explained that by pursuing an anti-cyclic economic policy, depressions could be avoided. Consequently, Keynesianism is based on demand (C) instead of production (Y). It was the prominent growth theory after the second World War until the late 1970s, when it failed to explain the prevalent "stagflation"[9].

The third prominent economic theory is based on the classical system of Smith and Ricardo, hence called neo-classical. Its creators, Jevons, Walras, Edgeworth and Pareto, are credited with transforming the study of economics into a rigorously mathematical scientific discipline, hereby often borrowing their concepts from nineteenth century physics. At the beginning, this process was often overtly simple and only the names of the variables changed. What had been energy in physics became Utility[10]. Therefore, the physics concept of potential energy became the "sum of utility," and kinetic energy changed to flow of money. Unfortunately, the new formalism did not satisfy several quintessential physical principles even closely; i.e. neither in neoclassical economics, nor in economic reality is the sum of payments and utility conserved. Nevertheless, this new formality allowed the employment of vector fields and multidimensional tensors to formulate mathematically complex new theories.

2.1.2. The Solow Model

In 1937 Hicks tried to absorb the macroeconomic thoughts of Keynes into neoclassical economics to form the "neoclassical synthesis," which did not take hold. The functional birth of neoclassical growth theory as we know it is closely linked to the year 1956, when Robert Solow[11] in his article [Sol56] assumed a substitutional macroeconomic production function, which undermined the methodological foundation of post-Keynesian theory. Solow showed how economic policy can raise an economy's growth rate by raising its savings rate. But the model also predicts that such an increase in growth cannot last indefinitely. In the long run, the growth rate will return to the rate of the economy's technological progress and its growth in labor force, which neoclassical theory conceives as independent

[8] The Great depression started with the stock market crash on October 29, 1929 (known as "Black Tuesday") and lasted until the early 1940s.
[9] Stagnation + Inflation = Stagflation. In Keynesianism, Inflation is one of the primary triggers for growth.
[10] defined as economic satisfaction and wellbeing
[11] Robert Solow (born 1924), American economist and Nobel laureate

(hence "exogenous growth theory"). A common prediction is that *any economy will always converge towards a steady state rate of growth.* This is especially prominent in cases like Germany or Japan after WWII, whose growth in GDP can readily be modeled using this mechanism. The model can be described using five equations:

Macro-production function

$$Y = AK^{\alpha}L^{\omega} \tag{2.5}$$

Y presents the total production in an economy (GDP). A represents the impact of "technology,"[12] K is capital and L is labor. Most neoclassical models use a Cobb-Douglas function like the one above. α and ω are the elasticities of capital and labor respectively. Usually, $\omega = 1 - \alpha$. In this case, the production function has *constant returns to scale*[13].

GDP equation

$$Y = C + G + I \tag{2.6}$$

In Solow's model, net exports and government spending are not accounted for. Hence the GDP is the addition of private consumption C, public consumption G and savings/ investments I.

Savings function

$$I = sY \tag{2.7}$$

Capital accumulation works by continually saving a certain share of production, expressed by the savings rate s.[14]

Change in capital

$$\Delta K = sY - rK \tag{2.8}$$

with r being the depreciation rate.

Change in workforce

$$L_{t+1} = L_t(1 + \vartheta) \tag{2.9}$$

where ϑ is the population growth function, and the time variable is treated as a series of discrete steps.

One of the primary motivators of this model is the assumption of *diminishing returns*. I.e. capital is always produced based on known technology. However, technology improves over time, which makes new capital more productive than old capital [HS06]. In general, if one factor is increased while all other factors and technology stay constant, the increments of the output become smaller over time. Without technology, the model predicts a steady state. This steady state reverts to a path of balanced growth, if multifactor productivity advances are balancing the diminishing returns.

[12] Actually, A represents multifactor productivity. However, it is often generalized as technology

[13] if L and K are each increased by 20%, Y increases by 20%

[14] A constant savings rate is a key oversimplification that will be addressed later.

Convergence Towards a Balanced Growth Path

The Solow-Swan Model suggests that from any starting point, consumption and capital are converging towards a balanced growth path. This means that countries with a high per-capita capital stock grow slower than those with a lower one. In other words, there is a negative correlation between initial capital stock (or income) and its rate of growth. However, this convergence is not *absolute* (i.e. all countries converge on a single balanced growth path) [SW05, BS04]. Absolute convergence can only be observed between homogeneous countries, e.g. those of the Organisation for Economic Co-operation and Development (OECD). In practice, individual factors like the savings- or birth rate, or the investment in human capital are important for determining a country's balanced growth path. Consequently, one can only observe *relative convergence*, i.e. that nations which are farther away from *their* balanced growth path are growing faster. Literature confirms that when compensating for structural differences, (relative-)convergence can easily be substantiated with statistical significance [SW05]. This can even be conveniently seen on a logarithmic plot of the GDP of some of the G8 nations (Figure (2.1)). It should however be noted, that severe disruptions in a country's structure (like the transition from the USSR to the Commonwealth of Independent States (CIS) or major economic reforms like those of the 1980s in China) can alter the slope of its growth path (which can also also be observed in Japan during the Bakamatsu[15] (1853-67)).

2.1.3. Endogenous Growth Theory

During the 1980s, the inability of neo-classical growth theory to explain its exogenous variables (savings rate and technological progress) was found increasingly unsatisfying. As a result, endogenous growth theory tries to overcome this shortcoming by building *macroeconomic* models out of *microeconomic* foundations [Rom90]. Assumptions are that companies and inventors are trying to "produce" technological progress in order to maximize their own productivity and gain a lead over competitors. This newly produced technological knowledge can be sold, or bought by other actors which increases their capability of innovation. In contrast to Keynesian or neo-classical theory, technological progress does not "simply happen", it has to be financed. Moreover, governments can stimulate the economy by easing technological change (by providing subsidies). There are, however, some drawbacks to this theory. Firstly, endogenous models tend to be quite complex, and secondly, they cannot rely on the concepts of diminishing returns anymore, otherwise growth would become unsustainable with time. Thirdly, and most importantly, the returns to scale are not constant, which means that the balanced growth path can be altered. This has been especially challenged by Jones [Jon95], who shows by analyzing time-series historical data that no evidence for this behavior exists. Radical system changes, however, can indeed alter the growth path of a country, which means that the details of this behavior remain

[15] "Late Tokugawa Shogunate", final years of the Edo period when the Tokugawa shogunate came to an end. It is characterized by the demise of isolationism and the Opening of Japan.

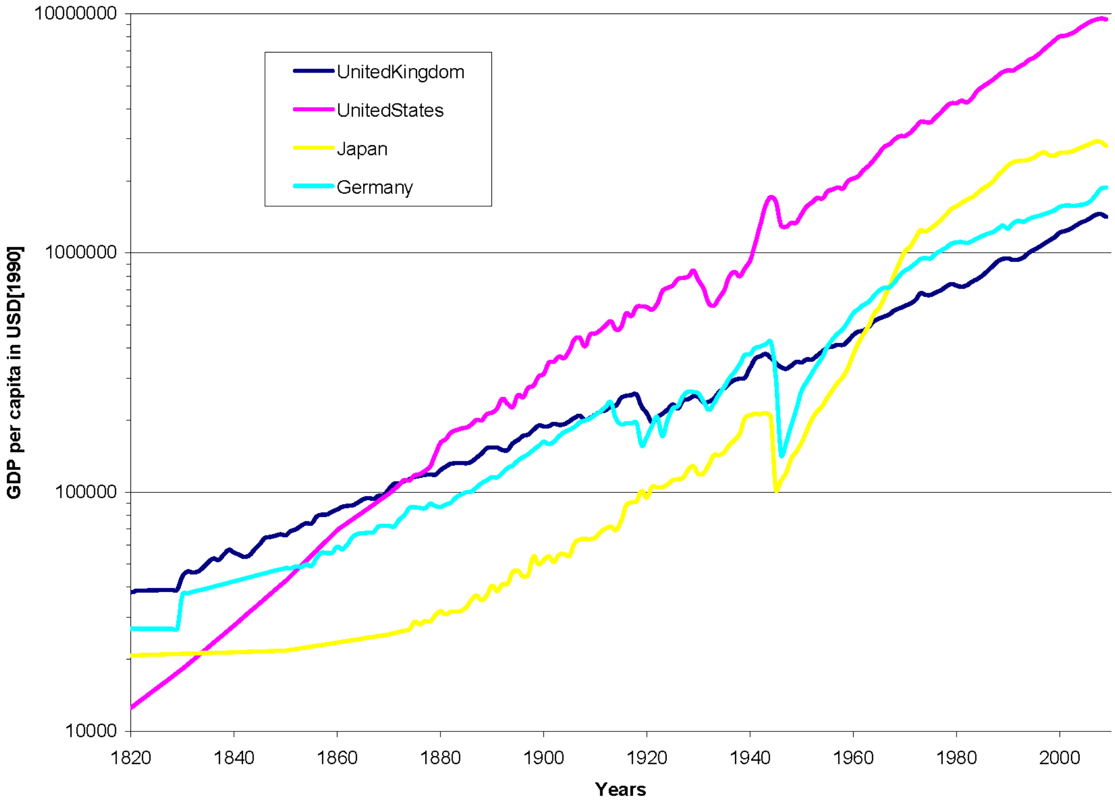


Figure 2.1. – GDP per-capita in the United States, United Kingdom, Germany and Japan 1820-2007

debated.

2.1.4. Growth-Theoretical Aspects of Climate Modeling

While the main focus of much research in climate modeling is put into the natural rather than social sciences, every model dealing with the industrial era is dependent on predictions for the amount of anthropogenic GHG emissions. The total of human-made emissions can be linked to the total production of an economy, since all the emissions ultimately occur when producing something.[16] Therefore a coupling can be established between an entity's (the world's, a country's, etc.) emissions and its GDP. Although this coupling is not linear, it is nowadays hardly disputed. Holtz-Eakin and Selden tried to quantify this relationship, which they found to be diminishing over time [HS95]. Nevertheless, the coupling between the GDP and GHG-emissions establishes economic growth theory as a source term for

[16] to a certain extent this is valid for services as well; e.g. in case of transportation, energy is used to fuel propulsion

climate modeling.

However, most often uncertainties in economic development have not been as systematically examined as error margins in atmospheric modeling. Therefore, economic prediction models used to forecast emissions are often quite simple. Regularly, extrapolation at the historic growth rate is deemed sufficient. However, this rate has varied, and simple extrapolation precludes incorporating any response to economic or atmospheric conditions. At the other extreme, one could formulate a general equilibrium model which embodies these responses by calculating their period-by-period equilibria. Nevertheless, the parameterization of such a model, while assuring that the model is still being driven by data instead of more arbitrarily chosen parameters, places an enormous informational burden on analysts.

In terms of methodology, energy economics models in literature are usually characterized as progressing "bottom-up" or "top-down". Bottom-up models, like the Market Allocation Model (MARKAL) (Fig. (2.2)) try to meet a given demand of energy by optimizing the mixture of (also given) specific types of energy sources and emission control technologies in a disaggregated approach. Consequently they are not providing any time related predictions, but only paths to efficiently reach a desired goal [RG92]. Besides MARKAL, another often used system is the Model for Energy Supply Strategy Alternatives and their General Environmental impact (MESSAGE), which is a engineering optimization model used for medium- to long-term energy system planning, energy policy analysis, and scenario development [MS95]. The model's current version, MESSAGE-IV, is a UNIX[17] based system that provides information on the utilization of domestic resources, energy imports and exports and trade-related monetary flows, investment requirements, technology substitution, pollutant emissions, as well as trajectories for primary, secondary, and final energy and its substitution processes [RR00a, RR00b]. It is aggregated into eleven macro regions. Being a bottom-up model, a typical application is running the model on a Reference Energy System (RES) (see Figure (2.3)) with specified performance characteristics for each technology. MESSAGE then determines how much of the available technologies and resources have to be used to satisfy a certain demand, while minimizing total discounted energy system costs. Classical exogenous growth theory and general equilibrium models on the contrary are characterized as top-down. Energy is treated a separate sector in the general economy. LOGICAL, the model used in this thesis, has this feature. Another example is the Macroeconomic Model (MACRO), which corresponds to the macroeconomic part of the Model for Estimating the Regional and Global Effects of GHG reductions (MERGE) [MR92, MR04].

Often, both approaches are being combined, to make use of the advantages of both. Prominent examples are MARKAL-MACRO or MESSAGE-MACRO.These include the impact of policies on energy costs, GDP and demand of energy. In case of MESSAGE-MACRO, the link is established by using MESSAGE results on total and marginal costs of energy supply to derive the quadratic demand functions for MACRO. The linked model is iterated until MACRO's resulting energy demands do not deviate from MESSAGE's by

[17] Operating system, trademarked

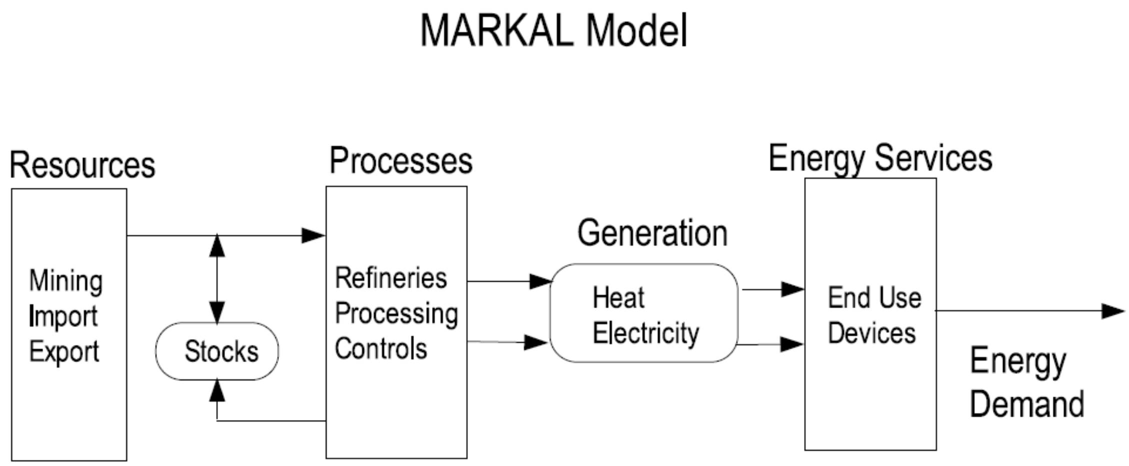


Figure 2.2. – Market Allocation Model (MARKAL) [Ret05]

more than a given fraction [MS00]. However, aggregating these models in a way that their extrapolation is systematically connected to time-series historical data is a considerable task [Nor04].

2.1.5. IPCC-Scenarios

One of the most prominent group of scenarios are those featured by the Intergovernmental Panel on Climate Change (IPCC). The SRES[18]-Scenarios were developed from the IS92 scenarios of the IPCC's Second Assessment Report of 1995. They are a good example for "bracketing" the problem, not in providing "best" and "worst" estimates, but by exploring different paths that are "equally valid" (i.e. self-consistent, albeit not equally likely) to develop a qualitative understanding of the driving forces and to assess associated uncertainties. A total of 40 scenarios are reviewed, separated into four families with certain initial conditions and growth parameters. Each set of parameters can be described by a certain storyline on how the world would be developing over time. However, the possibility of any single emissions path occurring as described is highly uncertain [NS00]. The families are characterized as follows:

A1 The A1 storyline and scenario family describes a future world of very rapid economic growth, low population growth, and the rapid introduction of new and more efficient technologies. Major underlying themes are convergence among regions, capacity building, and increased cultural and social interactions, with a substantial reduction in regional differences in per-capita income. The A1 scenario family is divided into four groups that describe alternative directions of technological change in the energy system.

[18]Special Report on Emissions Scenarios

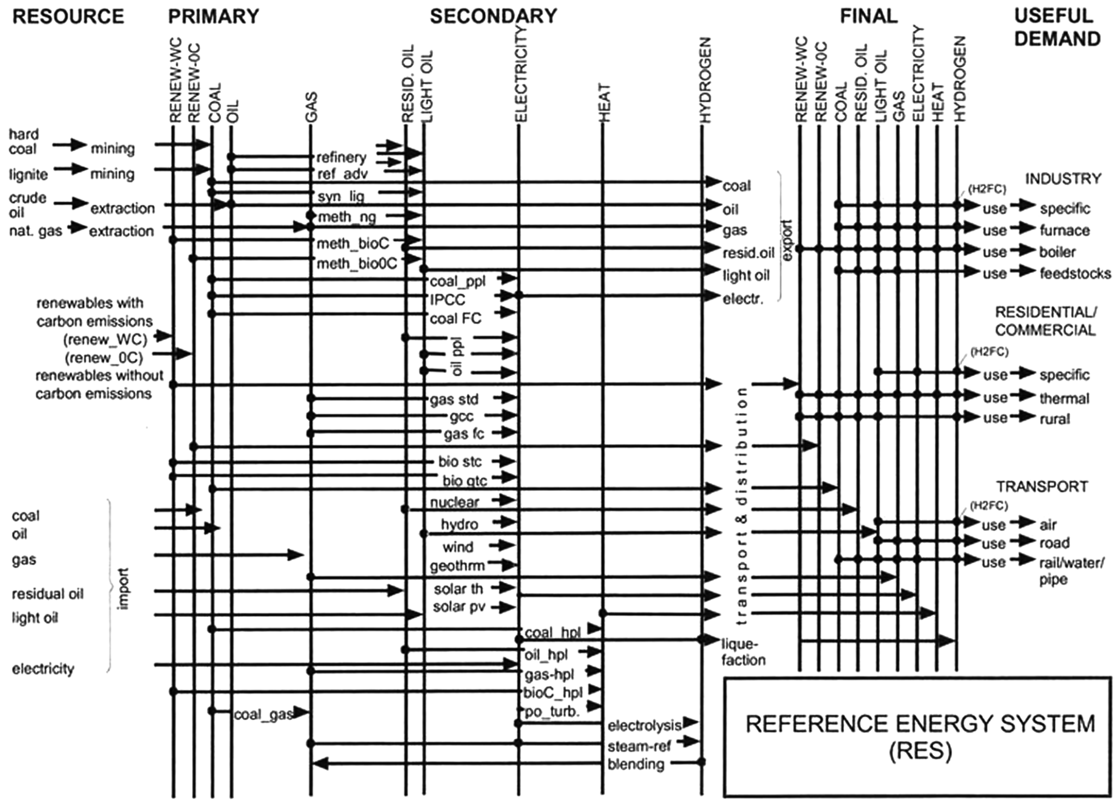


Figure 2.3. – Reference Energy System [Nak00], used as input for the MESSAGE model.

A2 The A2 storyline and scenario family describes a very heterogeneous world. The underlying theme is self-reliance and preservation of local identities. Fertility patterns across regions converge very slowly, which results in high population growth. Economic development is primarily regionally oriented and per-capita economic growth and technological change are more fragmented and slower than in other story lines.

B1 The B1 storyline and scenario family describes a convergent world with the same low population growth as in the A1 storyline, but with rapid changes in economic structures toward a service and information economy, with reductions in material intensity, and the introduction of clean and resource-efficient technologies. The emphasis lies on global solutions to economic, social, and environmental sustainability, including improved equity, but without additional climate initiatives.

B2 The B2 storyline and scenario family describes a world in which the emphasis is on local solutions to economic, social, and environmental sustainability. It is a world with moderate population growth, intermediate levels of economic development, and less rapid and more diverse technological change than in the B1 and A1 story lines. While the scenario is also oriented toward environmental protection and social equity, it focuses on local and regional levels.

These story lines lead to certain traits for every scenario, which are depicted below in Table (2.1).

Family	**A1**				**A2**	**B1**	**B2**
Scenario Group	A1C	A1G	A1B	A1T	A2	B1	B2
Population growth	low	low	low	low	high	low	medium
GDP growth	very high	very high	very high	very high	medium	high	medium
Energy use	very high	very high	very high	high	high	low	medium
Land- use changes	low-medium	low-medium	low	low	medium-high	high	medium
Resource availability	high	high	medium	medium	low	low	medium
Pace and direction of techno-logical change favoring	rapid	rapid	rapid	rapid	slow	medium	medium
	coal	oil & gas	balanced	non-fossils	regional	efficiency & demateri-alization	“dynam-ics as usual”

Table 2.1. – IPCC Scenario Characteristics [NS00]. Each scenario is run through a variety of models (one to six), so that a total number of forty accumulates.

Critique of the SRES-Scenarios

The IPCC has been criticized harshly by Castles and Henderson for its use of Market Exchange Rates (MRE’s) instead of Purchasing Power Parity (PPP) (c.f. Section (2.2.3)) for converting national GDP figures to a common measure (US Dollars) [CH03a], [CH03b]. They argue that scenarios yield improbably high projections of GDP for developing regions because of this. Consequently, the emissions of those regions would grow unnaturally and hence those of the world in toto. When using PPP, the gap in future energy use rates between rich and poor countries would be greater and consequently climate change much less of a problem. The IPCC defends itself by saying that using MRE's instead of PPP indeed elevates carbon dioxide emissions, but that the differences are small in comparison to other uncertainties, and that the discrepancies are diminishing if one moves from emission rates, to cumulative emissions and concentrations, and further on to temperatures [MRE05]. Tol partly agrees, but states that when looking at the impacts of climate change and the cost of its mitigation, differences may become larger again [Tol04]. He argues, that even while differences are not large, the models are showing too much sensitivity on the choice of exchange rate and should therefore be improved.

2.2. LOGICAL

2.2.1. Background and Structure [19]

The idea behind the logistic independent variables calibrated vs. historical data model LOGICAL was to create something more readily data-calibratable than MESSAGE-MACRO, while incorporating the advantages of exogenous growth theory and mitigating some of its drawbacks. The program is written in *Mathematica*. A Cobb-Douglas function of labor, capital and energy is used to model GDP. Productivity is not simply specified as an arbitrary exogenous functional form. Rather, a development index A is defined, which is exogenous to a micro structure that handles technology but is calibrated against data on population growth based on the general observation that countries with the largest population growth rates tend to be least far along in their technological development and organizational efficiency. Another drawback of many models is the way various parameters are being calibrated. LOGICAL is one of the few where those parameters are systematically calibrated against long-term historical data. This approach has the advantage of taking into account the evolution of various factors that influence the future distribution and consumption of energy resources. However, the key is how development is driven in the model, which is by maximizing the total time-integrated discounted utility of per-capita consumption [Ret05]. Thus the governing relation is the so called welfare (W) function, which is to be maximized:

$$W = \int_{t_1}^{t_2} \frac{L\left((C/L)^{1-\vartheta} - 1\right)}{(1-\vartheta)} e^{-\rho t} dt \tag{2.10}$$

with L denoting labor, C consumption and ρ the "social discount rate". Here t is understood as years measured from a reference time τ, hence $t = t_{real} - \tau$.[20] The parameter ϑ is a parameter called the "inverse of the intertemporal substitutability of consumption", in other words how easy it is to delay consumption in order to have a more prosperous future.

[19] Equations and concept of LOGICAL in this and the following sections (if not stated otherwise) courtesy of one or more of the authors of [SRA+07, Ret05, SRA+08, ST07].

[20] e.g. if we chose $\tau = 1980$, the year $t_{real} = 2010$ would be referenced as $t = 30$.

Consumption is defined as in the Solow model, except that units used for capital and time are chosen so that the long term capital limit for K is 1 and production is expressed as $Y\alpha^{-1}$. Thus

$$\begin{aligned} \frac{Y}{\alpha} &= C_s + G + I \qquad \text{(Solow)} \\ \Rightarrow \quad C_s &= \frac{Y}{\alpha} - G - I \end{aligned} \tag{2.11}$$

we combine private and public consumption in $C_s + G = C$ and use the Capital Change Relation $I = rK + \dot{K}$ (with r being the depreciation rate) to get

$$C = \frac{Y}{\alpha} - rK - \dot{K} \tag{2.12}$$

Another important feature of LOGICAL is its discerning between that part of the whole production which is employed in producing energy, and the rest ("general production"). As there will be estimates deducted in later chapters for future CO_2 emissions and uranium production from the amount of energy produced, this distinction is crucial to that purpose.

While still using a Cobb-Douglas relation (like the one Solow used in eq. (2.5)), LOGICAL features βk, βl and βw as the fraction applied to energy production of total capital K and total labor L and total energy per unit time w, respectively. Consequently, e.g. $(1-\beta k)K$ is the amount of total capital left for the production of goods. Consistent with constant returns to scale, the elasticities of capital and labor are taken to be $\alpha = 1 - \omega$ and likewise the relation of the elasticities of general- and energy production $\varphi = 1 - \beta$ respectively. All four parameters are being calibrated separately against historical data. With these choices we get a formula for Y in the production function (eq. (2.5)) of

$$Y = A\left[\left(\left(1-\beta k\right)K\right)^{\alpha}\left(\left(1-\beta l\right)L\right)^{\omega}\right]^{\varphi} w^{\beta} \tag{2.13}$$

While Solow would describe A as technology, the context here is a bit broader and the parameter is therefore called the "development index"[21] The whole concept of filtering out energy production from total production becomes even clearer when looking at the next figure (2.4), where one can easily see the connection between the various parts of the production function.

The energy production and use rate w is also taken to have constant returns to scale with respect to capital and labor:

$$w = pB\left(kK\right)^{\alpha}\left(lL\right)^{\omega} \tag{2.14}$$

Hereby p is taken to be the energy production efficiency, which has a substructure, defined by $p = 1 + (h-1)f$. h is the ratio between the cost in capital and labor of all non-fossil

[21] It is important not to confuse the development index with living standard. While the latter is just another expression for the per-capita GDP, the former represents rather a "social capital". It is a measure of how high developed as society is as a whole; of culture and "maturity". The development index is mostly undisturbed by disruptions to the economy, i.e. wars.

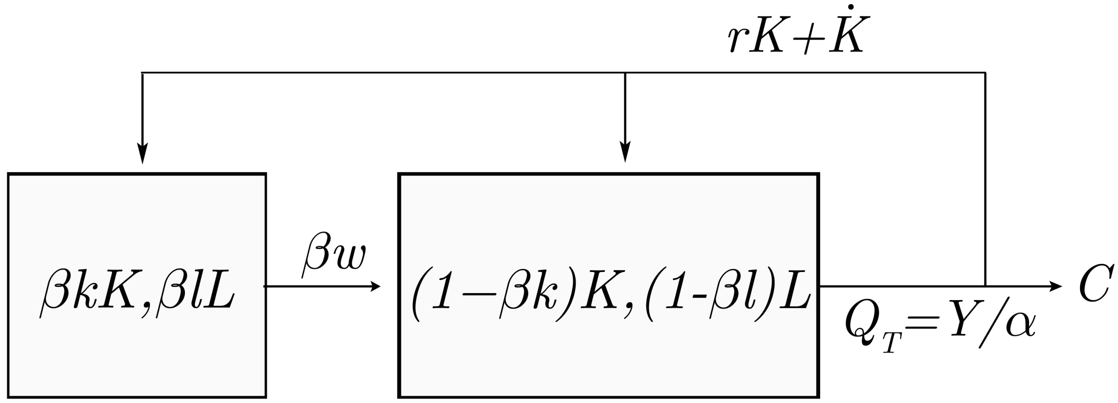


Figure 2.4. – Diagram of the LOGICAL-1 Model

vs. all-fossil energy production. For the examples shown herein, a value of $h = 2$ is used based on currently observed spectrum of energy prices in the United States. f in turn is the (dimensionless and time dependent) carbon intensity of energy production, defined as a ratio of actual carbon intensity of energy use to that if only coal were used. Here only commercial energy sources excluding traditional biofuels are accounted for so f thus ranges from 1 before the use of oil to 0 in the far future. Historically the value of f has been a monotonic non-increasing function of cumulative carbon use, u. A piecewise linear approximation to its dependence on u is used to calibrate f. Thus p ranges time-dependently from h to 1 as u goes from 0 to its long term asymptotic limit value. In the equations as formulated here, units are chosen so that the long-term asymptotic limit value of u is 1.

There is a carbon balance constraint, which is imposed on the overall structure, expressing that the overall increase in cumulative carbon use is proportional to the product of carbon intensity (f) and energy use rate (w).

$$\dot{u} = \epsilon f w \tag{2.15}$$

Here ϵ is a proportionality constant, assumed to be a small parameter with the equations expressed in the units used here, with time expressed in units of the inverse of the sum of the depreciation rate and the social discount rate as discussed further below.

As noted already, historical data suggests in general an anti-proportional connection between the development index A and the population growth rate η_A. Highly developed countries tend to have a low population growth rate and vice versa. While this is not universally valid (e.g. with the notable exception of China as a result of its "one child policy"), it still serves in most cases as a convenient way to calibrate A [SRA$^+$08, SRA$^+$07]. Complimentary to the development index is a parameter called the "underdevelopment index", named Z (hence $Z = 1 - A$). For the cases presented here, a good first approximation to population growth is given by the solution to the equation

$$\dot{A} \Big/ A = \eta_A \nu Z \tag{2.16}$$

Here the definition of population used is the increment over the value of an early industrial era base year, here taken to be the year 1820, for reasons explained later. Likewise, GPD, energy, and carbon use are computed as the difference between current values and those in that base year. In effect a number of people equal to that in 1820 is assumed to continue to subsist on the minimal GPD, energy, and carbon use rates found in 1820, which unfortunately for them has continued largely to be the case (in terms of economic measures of living standards). The development index is not only dependent on the population growth rate; also important is the *initial* population growth rate ν. η is the GDP productivity coefficient and a constant, calibrated against data. With that, we can write Z as

$$Z = \frac{e^{-\nu t}}{1 + e^{-\nu t}} \tag{2.17}$$

For the energy-part of the model, these equations are the same, but to easily distinguish them in context, B is used instead of A.

$$\dot{B} \Big/ B = \eta_B \nu Z \tag{2.18}$$

The above equations should also make it clear that resource use only becomes more closely optimal as the level of development approaches the maximum achievable limit. In other words, it is not assumed that energy or capital are always optimally used, but that nevertheless they can be allocated effectively to recover from major disruptions on about the capitalization time independently inferred from other data in the present analysis [SRA+08, SRA+07].

As the utility maximization (eq. (2.10)) usually poses a non-analytic problem, above equations are expanded in three small parameters to reduce it to an analytical one. Two of them have been already described: β, the "capital fraction of energy" and ν, the "dimensionless initial population and development growth rate for each region". The third is the constant ϵ, used in the carbon balance equation (2.15). It is sufficient to keep only the terms of lowest order in β and ϵ. Details of the expansion in these parameters are given in Appendix B and in [Ret05]. Numerical values for all parameters, both global and region dependent, are listed in Section (B.2) of the Appendix.

LOGICAL-0

While the LOGICAL-1 version of the equations described above are designed to overcome some of the problems of the earlier LOGICAL-0 (presented in [SRA+08, SRA+07]), the solutions for the Euler-Lagrange-equations have not yet been found because of problems with the integral maximization algorithms. In order to present results here, these have been calculated with the older version of the model. In principle, the mechanisms of both are similar, however LOGICAL-0 does not distinguish between the energy part and overall GDP part of production. Therefore, the development indices A and B are the same in that version ($A = B$). Furthermore there is one conceptual difference: While LOGICAL-1 allows for a lag between population growth rate and development index, LOGICAL-0 assumes the growth productivities do not lag behind the development index. Moreover, LOGICAL-0 uses an analytic expansion assuming that the common value of ν is small, carrying out an asymptotically convergence expansion only through first order in a parameter of order ν. The consequence is that in some cases the solutions given here are only rough approximations around times where the product $A \cdot Z$ is a maximum, which occurs where $A = 1/2$, particularly for the case of China for which LOGICAL-0 has the greatest problem with respect to the lag between population growth rate reductions and productivity growth. The effect is that for the regional disaggregation presented below the results are in a few cases primarily empirically reasonably calibrated rather than fully theory-based; in the case of a region dominated by China apparently underestimates of extrapolated GDP, energy, and carbon use rates. It is this problem that the LOGICAL-0 model has in dealing with a China-dominated region that primarily motivated the development of the LOGICAL-1 model described in the previous sections and the accompanying appendix. In LOGICAL-0, the rate at which carbon is burned E defined as

$$\tilde{E} = \bar{E}\alpha^{\psi} f p_1^{\alpha/\omega} \tag{2.19}$$

with a result from solution for welfare maximization of

$$F_1 = \frac{1 + A\delta}{1 + \delta} \tag{2.20}$$

The result for F is gained through first order expansion in δ which is defined as $\delta = \nu\vartheta\xi$ with $\xi = \eta/\omega$ (and $\omega = 1 - \alpha$). δ is understood as the capitalization lag, or a measure for of the fractional difference between the capital stock at any given time and what the capital stock would relax to if all parameters were frozen. η is calibrated against the rate of growth of per-capita GDP for each region J. $\bar{E}$ (here in Gtonne/yr with tonne meaning metric tons) is a constant used for scaling the final carbon burn rate to fit historical time-series data, and $\tilde{E}$ is the carbon use rate in the same units as $\bar{E}$.

2.2.2. Database

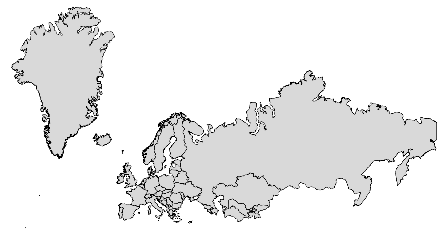
As stated above, the calibration against an extensive time-series database is one of the cornerstones of the LOGICAL approach. The data consists of fourteen data sets, listing population and GDP annually from 1820 on, as well as providing usage figures of twelve different forms of primary energy use rates since 1700 for every country and other reporting unit listed by the United Nations. The data comes from the UN from 1950 on and is mapped onto the current boundaries of UN reporting units. The data from before 1950 is used here only to estimate cumulative use up to the initial date used for time-series calibration; it comes from various literature sources on production, imports, and exports, as discussed by Rethinaraj [Ret05].

The data belonging to each entity and country can be aggregated freely to form reporting regions. Those regions are chosen not necessarily for their political homogeneity, but rather to create regional groupings expected to be roughly energy self-sufficient, as there is no trade model integrated yet into LOGICAL. Currently nine regions are chosen:

ChinaPlus consists of mainland China as well as the provinces of Hong Kong and Macao, as well as North Korea, Mongolia and Taiwan.

SAFTAPlus includes the South Asian Free Trade Association (SAFTA) and consists of Afghanistan, Bangladesh, Bhutan, India, the Maldives, Nepal, Pakistan, Sri Lanka and Iran

EUPlus consists of the EU member states, as well as their overseas dependences La Reunion, St. Helena, the Falkland Islands, and Bermuda. Furthermore, several surrounding countries are also added, those are Armenia, Azerbaijan, Belarus, Georgia, Kazhakstan, Kyrgyzstan, Moldova, the Russian Federation, Tajikistan, Turkmenistan, the Ukraine, Uzbekistan, the successor states of former Yugoslavia, and Switzerland. The affiliation especially of the countries formerly belonging to the Warsaw Pact has to be kept in mind when comparing the findings of this region to forecasts for western Europe made elsewhere.



USAPlus depicts North Amercia and its dependencies, i.e. USA, Canada, Puerto Rico, Guam and the U.S. Virgin Islands.

Pacifica is formed by the myriad of small islands of Polynesia, including Micronesia etc., but excluding Indonesia. Furthermore it includes the Antarctic and other states of the pacific rim like Japan, Australia, New Zealand and South Korea.

LatinPlus is formed by the countries of Central- and South America and the Caribbean.

Subsahara is made up from all countries of the African continent, southward of a line drawn between Senegal and Kenya. Mali belongs to the next group.

MidEastPlus consists of the rest of (northern) Africa and the Middle East. Note: For reasons of trade balance, Iran belongs to SAFTAPlus.

ASEAN the Association of Southeast Asian Nations, consisting of Brunei, Cambodia, Indonesia, Laos, Malaysia, Myanmar, Philippines, Singapore, Thailand, Timor-Leste and Vietnam.

Aggregated data sets include population, GDP, energy use rates, and fossil carbon use rates for each region's countries. An overview about the database structure regarding those various types of energy is given in Fig. (2.5). Water includes hydroelectric, geothermal electric, and tidal electric power. New renewables includes wind electric, solar thermal electric, and photovoltaic power. Electrical energy production is converted to a fossil fuel thermal equivalent by dividing by a reference thermal to electric conversion efficiency of 0.38. Fig. (2.6) gives some examples of aggregated time series from the database.

2.2.3. Allowing for Inflation

Inflation is a reality in most economies. Especially when considering long term data sets, one has to deal with the fact that a dollar in 1930 bought much more than in the 1990s.

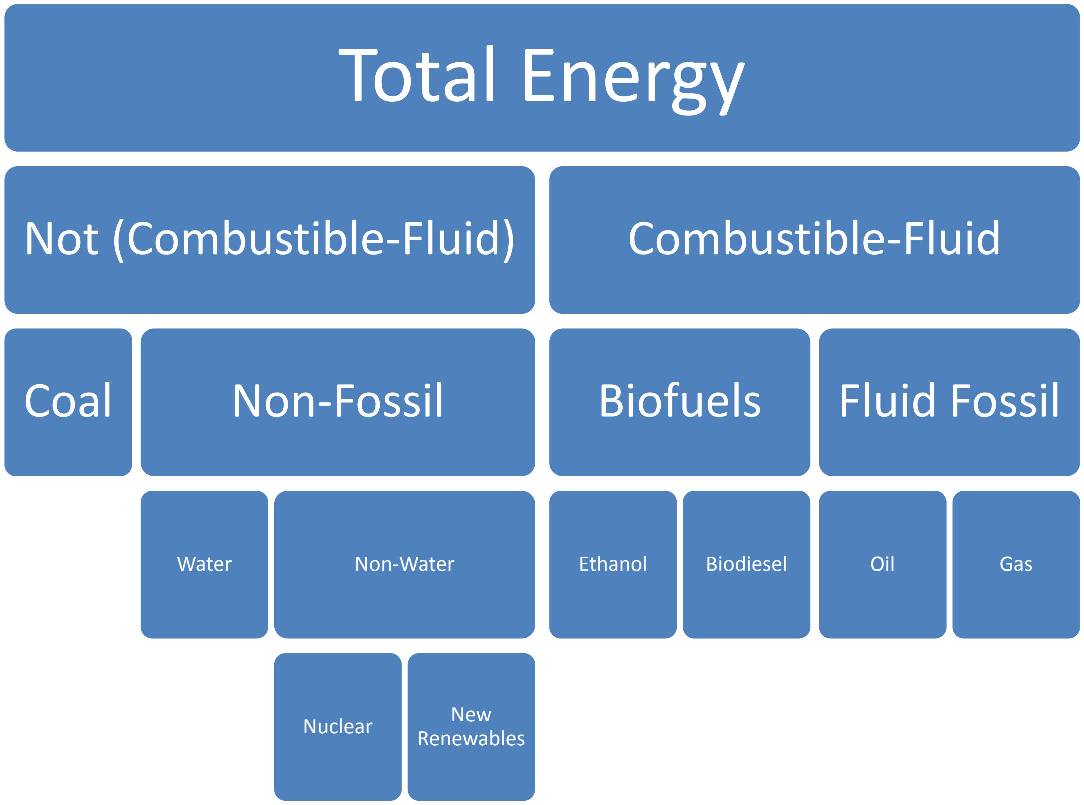


Figure 2.5. – Various types of energy production in the database

There are two methods for presenting economic estimates in such situations; using the actual number of dollars that will be spent or received in a given year, without adjusting for the fact that those dollars in the future will each buy less than a dollar does today (so-called "nominal" dollars), or adjusting for the effects of inflation so that all dollar values are quoted in the dollars of a particular year, and can be directly compared against each other (so-called "real" dollars).

Furthermore, monetary exchange rates are often floating, since they are sensitive to the vagaries of capital flows in the international market. A simple (and humorous) example is the so-called "Big Mac Price Index"[22], devised by *The Economist*, which uses a single good (the "Big Mac" sandwich) and its retail price all over the world to compute differences in purchasing power (at least for fast-food burgers). From that, a parity exchange rate can be derived under the assumption that every Big Mac around the world is the same

[22]In the words of its inventors "arguably the world's most accurate financial indicator to be based on a fast-food item"

as the one sold for US$3.99 (2009) in the United States. However there are many (and much more sophisticated) aggregation methods that produce so-called PPP currencies. In this thesis, the GDP figures of the time series data are given in terms of a particular type of PPP currency called 1990 Geary-Khamis dollars (c.f. [Ret05]). This approach is taken because the database used does not have direct PPP adjustments for every year of interest. However, in some cases it may be more instructive to present results in more recent prices, as the value of the US$ changed considerably in the last twenty years. In these instances, the 1990 Geary-Khamis dollars are adjusted by the United States consumer price index [U.S10] and referred to as inflation-adjusted dollars for other years. To view those figures in units they were actually calculated in, one has to divide the given price by the inflation index increase ratio.

2.2.4. Fitting

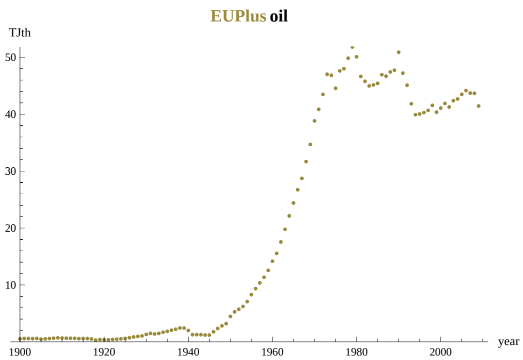
Firstly, the population increment data is fit by applying a logistic fitting function. Then, finite Fourier power spectrum analysis of the fit vs. the data is made to formulate first order periodic corrections. The power spectrum analysis is then repeated, and the resultant residuals analyzed. If those are still significant, the process is repeated until otherwise (c.f. [Ret05], Appendix F). Then solutions for GDP and energy use can successively be fit to the data, with periodic corrections applied at each step.

As noted above, the theory on business cycles dates back to the early 20th century, when Kondratiev observed that historical records of real[23] GDP, real income, employment and industrial production appeared to indicate a cyclic regularity of phases of gradual increases in values followed by phases of decline. Kondratiev judged the period of these apparent oscillations to be around 50 years. Later, more types of business cycles with shorter periodicity were discovered. Schumpeter and others proposed a typology according to the names of their discoverers or proposers: [Sch06]

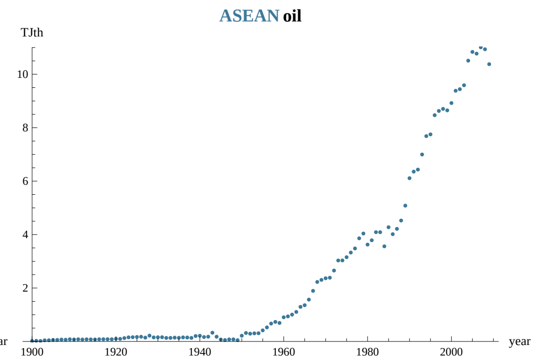
- the *short* or *Kitchin* cycle of 3–4 years
- the *Juglar* cycle of 7–9 years, which is often referred to by the generic term *business cycle*
- the *Kuznets* cycle of 15–25 years. Possibly a harmonic of the Kondratiev-Wave.
- and the *Kondratiev* wave or long technological cycle of 52–53 years [KT10]

Recent analysis of world GDP data has found Kondratiev waves to be present with a significance of 4 to 5%, and Kitchin as well as Juglar cycles with a still high significance of 2 to 3% [KT10]. Therefore, especially in light of the recent economic downturn, these periodic corrections can be considered a primary advantage of LOGICAL, especially as most other models ignore these. However, the time scale of Kondratiev waves is too long to be reflected

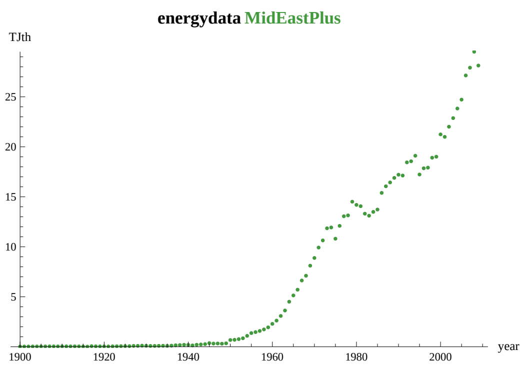
[23] in other words: inflation-adjusted



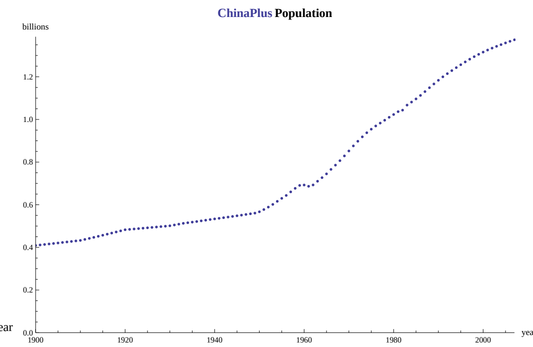
Oil Consumption in the *EUPlus* Region. The general decline from the oil crisis in the 1970s on can easily be observed.



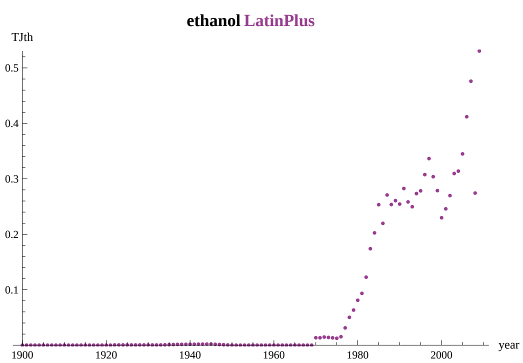
In contrast, the oil consumption is Asia has only recently dropped due to 2007 peaks in oil price.



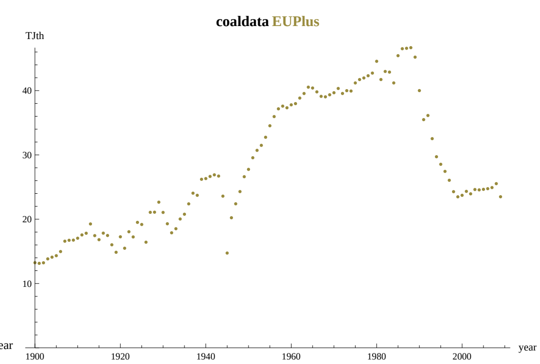
Total energy consumption in the *MidEast-Plus* region. Note the dip the Iranian revolution and the Iran-Iraq war causes (1979-88).



Population in *ChinaPlus*. Note the dent the “Big leap forward” (1958-1961) causes.



Comparatively large capacities for ethanol production have been developed in Latin America quite early.



The renovation of the former east bloc’s economy brought a significant decline in energy production from coal.

Figure 2.6. – Some exemplary data form the Database.

in the portion of the data base used here, but shorter period cycles are reflecting periodic variations in energy use and are included in the present analysis. Furthermore, accounting for longer period cycles remains challenging, particularly given the complicated turbulence of the first half of the twentieth century. One of possible shortcomings of the present approach is that it assigns even the slowest varying departures from the equation solutions to periodic effects without allowing for the possibility that these departures may in fact not be periodic. Thus, it should be kept in mind that results of long term extrapolations disregarding these cycles, might be substantially different in cases where long period variations of significant amplitudes result from the present analysis. In the following sections fits are given both with and without periodic corrections.

2.3. Results

2.3.1. Fits to the Population Growth Rate

The starting point for any top-down econometric modeling is usually a look into demographics or more specifically at population growth rates. As a first result, we see in Figure (2.7) a comparison between the fits with and without periodic corrections.

The year 1820 is used as a base year because it is the first for which annual population estimates are available from the database, which is built from Maddison's annual population reports [Mad06]. This choice also makes historical sense, as the dust of the Napoleonic Wars and the reorganizing of Europe by the Congress of Vienna has mostly settled by 1820. That date marks the beginning of a very long time of population growth and relative economic stability which lasted until the outbreak of World War I.

As discussed earlier, the population growth rate (Fig. (2.8)) can be used as a measure of the shortfall of a development index A, $Z = 1 - A$. In other words: population growth rate is roughly anti-proportional to A. So the population growth rate can be used to calibrate the development index, which evolves logistically. Therefore, the population growth rates are fit to Eq. (2.17) which we recall to be

$$A = 1 - \frac{e^{-\nu t}}{1 + e^{-\nu t}} \tag{2.17}$$

The results of the fits and their extrapolation can be observed in Figure (2.9). Note, that *USAPlus* has a relatively high population growth rate, which extrapolates to continue for several decades. In contrast, *EUPlus* is already saturated. However, the Chinese graph must be viewed with caution. As the country's one-child-policy has curbed population growth for decades, the numbers do not reflect the actual connection to the development index. In reality, China is probably nearer to its productivity factor inflection point than it seems to be the case when just looking at its demographics.

In the case of *USAPlus*, the United States' population in this region is the biggest by far. According to the model, this country's population could double to the end of the century. This has to be kept in mind when looking at the GDP plots later on. When viewed in

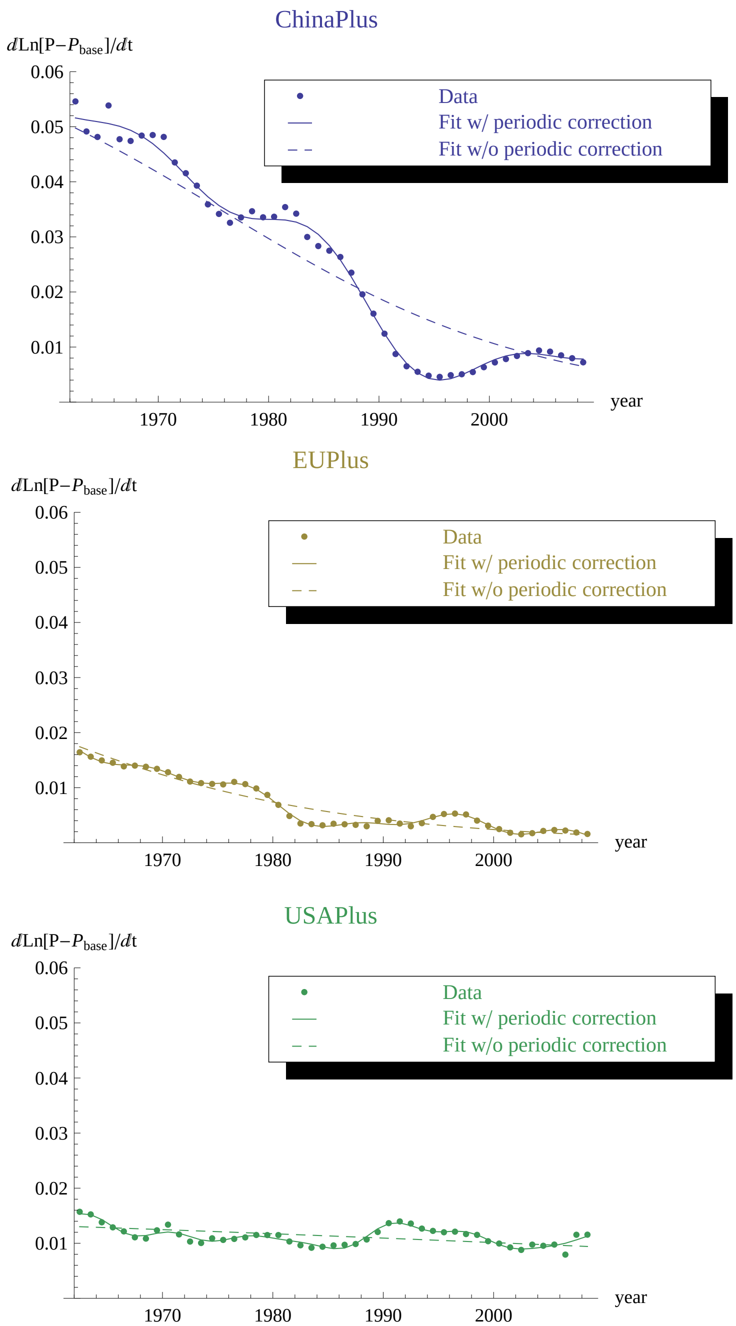


Figure 2.7. – Calibration of population growth rates. Data and Fits with periodic corrections, showing both the full result and the background trend with periodic corrections subtracted out of the population increase over a fixed base value in three regions. The population in the year 1820 is used as this base value.

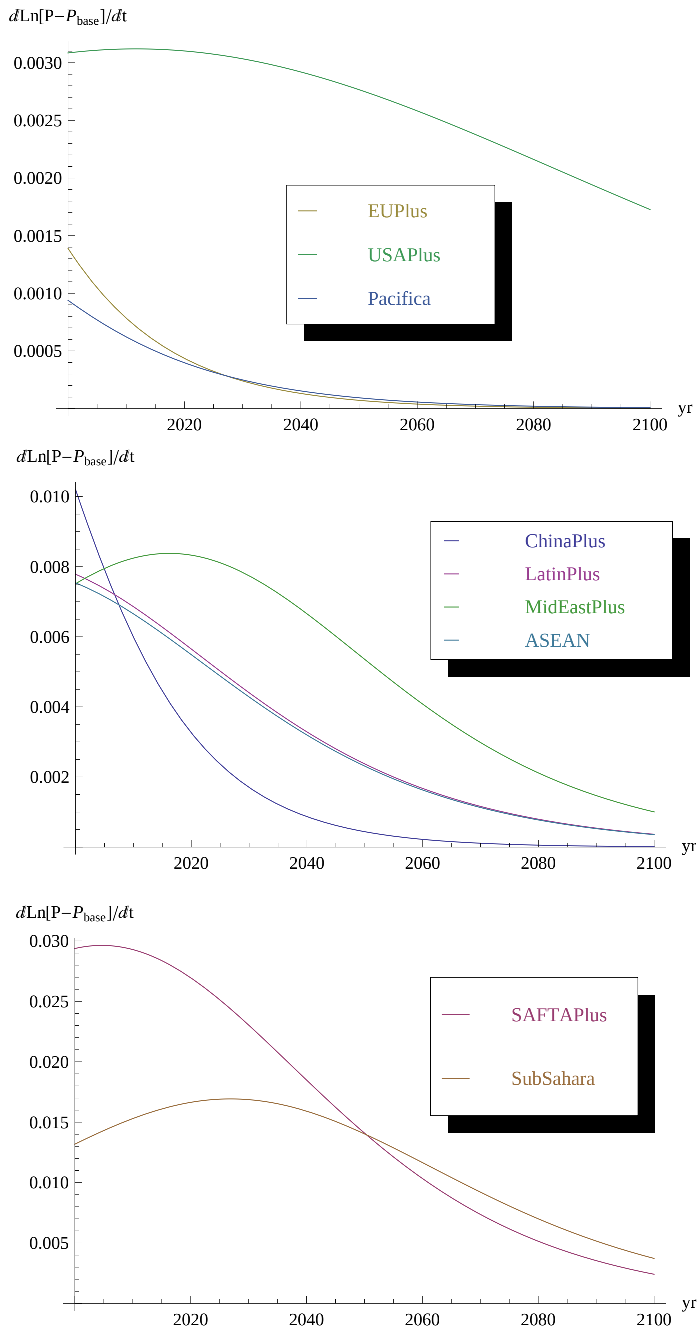


Figure 2.8. – Extrapolations of population growth rates in the various regions from 2000 to 2100. The periodic corrections have been omitted for better clarity. Note that approaching inflexion points of population growth are easily recognizable as points where the temporal derivative of the growth rate equals zero.

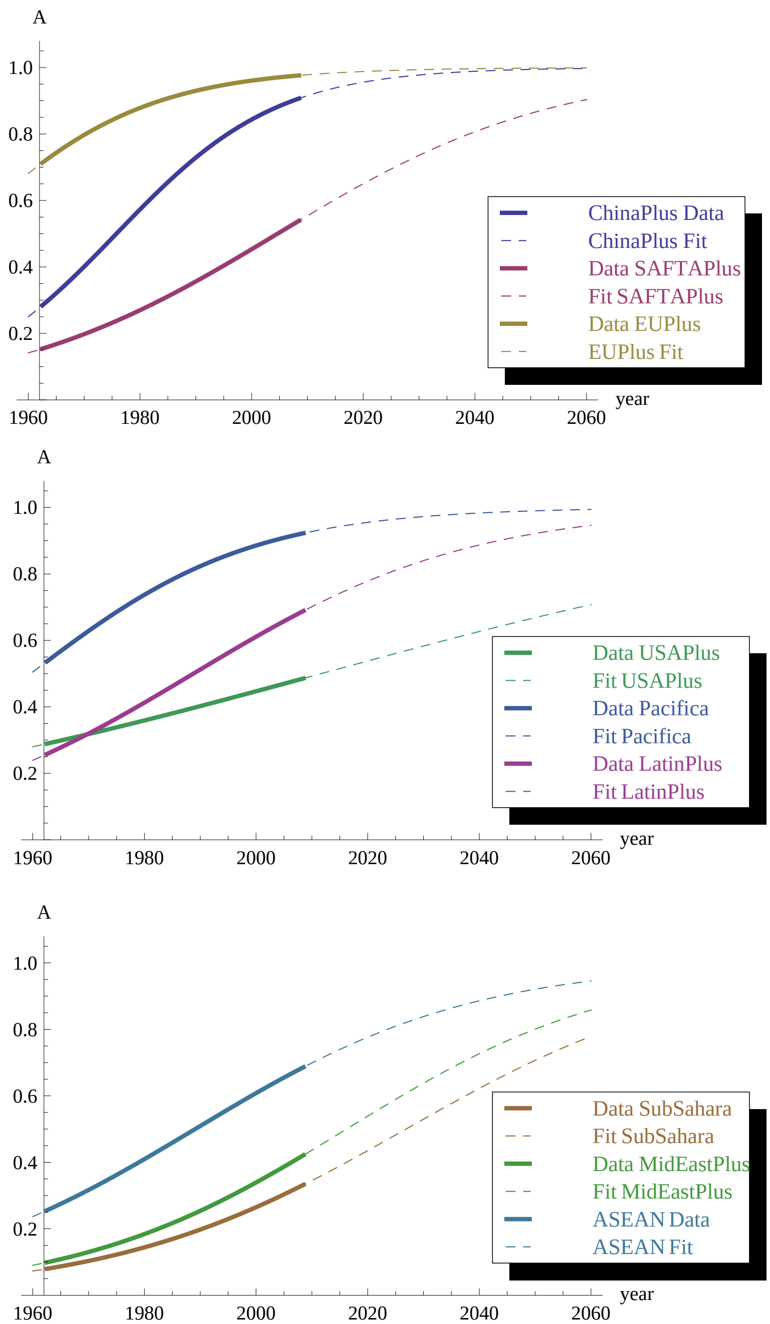


Figure 2.9. – Predictions for the Development Index, A, in the various regions from 1960 to 2060. The periodic corrections have been omitted for better clarity.

broader context, these findings translate to a world population of little over 10 billion people by 2100, up from 6.9 billion in 2010. The extrapolated world population with the base population added in is shown in Figure (2.10).

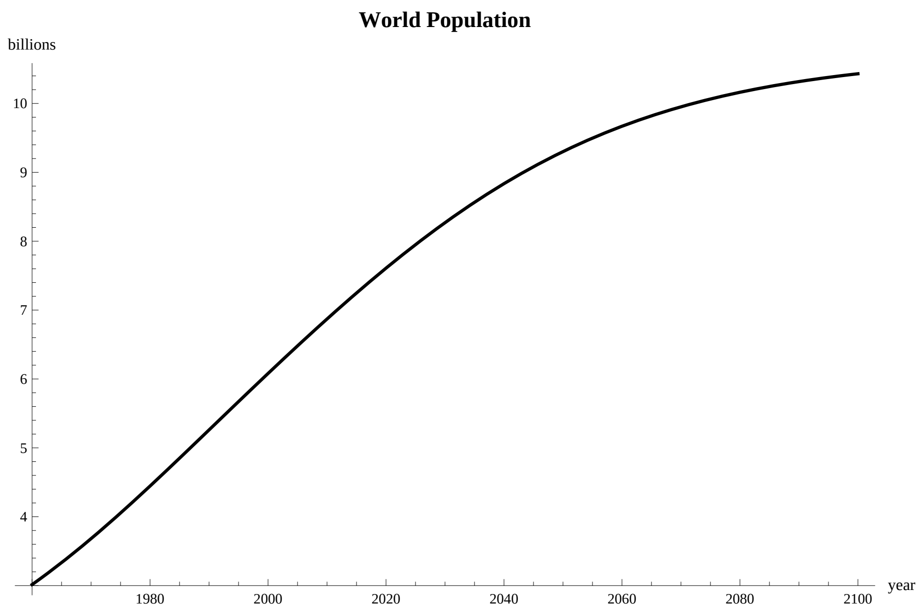


Figure 2.10. – Development of World population from 1960 to 2100.

2.3.2. Results for the Per-Capita GDP

GDP development is one of the prime indicators of every economy. To make the regions comparable, we look at GDP per-capita. When looking at Figure (2.11), the differences in scale are very noticeable. Most significantly is the GDP prediction of *USAPlus*, which is a direct result of its high population growth rate (Fig. (2.8)).

However, the problems with the capitalization lag, δ become apparent at this point. While δ does not influence the sustainability level in the distant future, a high value of δ flattens the curve while simultaneously moving the inflexion point to the right. Additional terms which have the most impact near the population curve inflection point are not included. This means, that extrapolated GDP's near that inflection point are slightly different from what would result from numerically solving the full welfare maximization equation. This is especially in *ChinaPlus*, and *EUPlus* the case. A list of the values for δ for all the different regions can be found in Table (2.2). Up to a value of about $\delta = 0.3$ there are no appreciable complications, beyond that terms of higher order of the δ-series-expansion start to become substantial. For such cases the graphs shown still give good empirical fits to the data, but should not be considered to accurately reflect the results of the underlying welfare maximization theory.

Region	δ
ChinaPlus	2.46
SAFTAPlus	0.37
EUPlus	2.71
USAPlus	0.33
Pacifica	1.28
LatinPlus	0.28
SubSahara	0.03
MidEastPlus	0.18
ASEANPlus	0.63

Table 2.2. – Various Values for δ

Below is a figure (2.12) of the world's main industrial regions. As already stated, the extraordinarily high per-capita GDP of *USAPlus* is a result of its high population growth rate. The contrary is true for *EUPlus*, which will not increase its population growth and therefore its per-capita GDP significantly. The case of *ChinaPlus* is more difficult, as China's economy has changed much in the 1990s and is still somewhat volatile. Fitting its GDP is problematic, as the slope of its log-linear growth path has changed after the country began its reforms. Moreover, thanks to state controlled fertility, its development index is misleading. Therefore, while *ChinaPlus*'s near term growth rate can be predicted with confidence, anything more distant than 2030 is difficult to forecast.

2.3.3. Energy Calibration and Projection

The log-linear energy production function w in LOGICAL-0 (see Fig. (2.13)) is defined as

$$w = pA^{\zeta} (kK)^{\alpha} (lL)^{\omega} \tag{2.21}$$

Here, p is taken to be the energy production efficiency and ζ is a coefficient, calibrated against data. Unfortunately the database for *ChinaPlus* is quite weak here, as earlier data is unusable because of economic disruptions. The same is true (but to a lesser extent) of *MidEastPlus*, which underwent a reorganization after the Iran-Iraq war. However, at least in the latter case the quality of the fit does not suffer too much. In *ChinaPlus* on the other hand, the periodic corrections are not usable at all, while the plain fit still raises doubts as well.

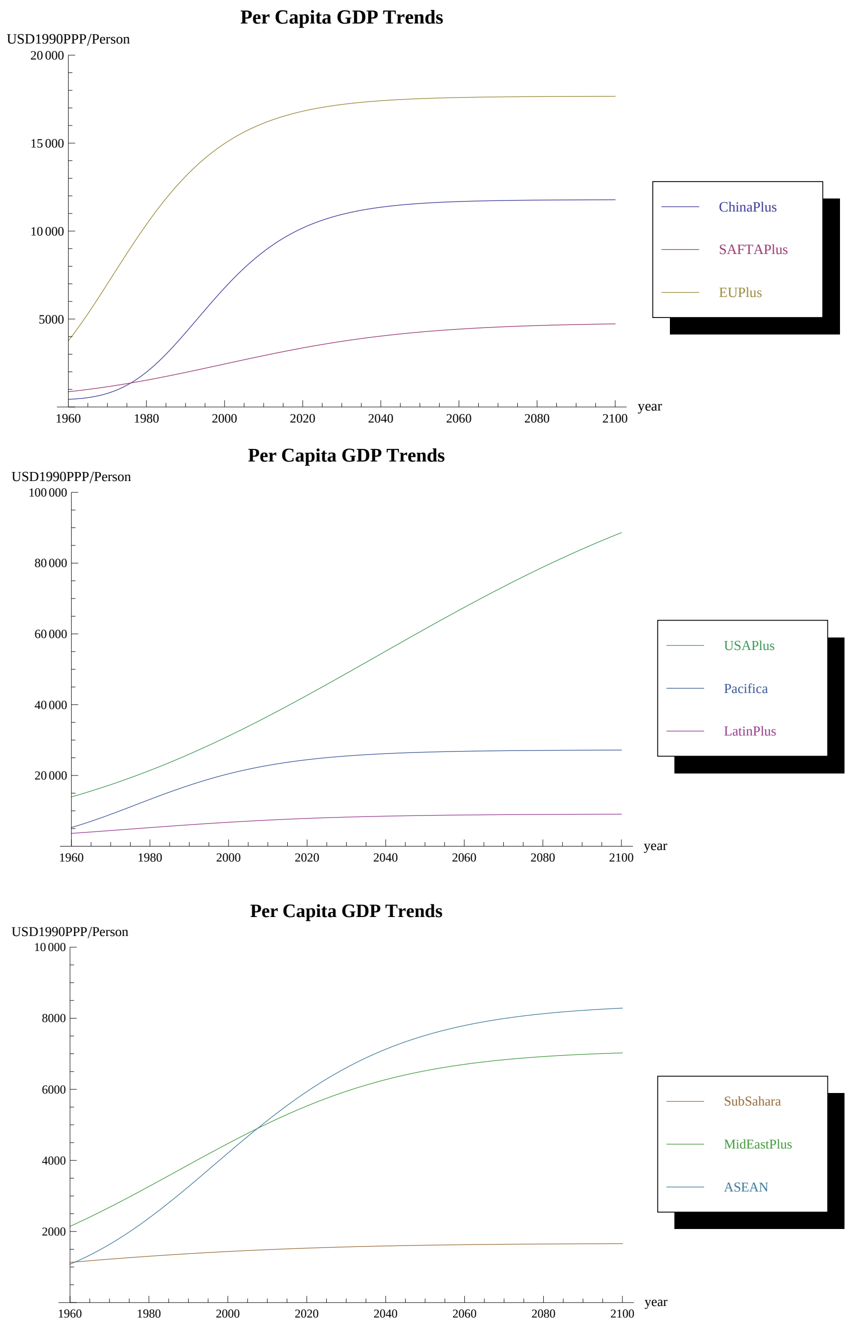


Figure 2.11. – Predictions for the Gross Domestic Product for all regions in 1990 US$. Note the variation in scale of the y-axis.

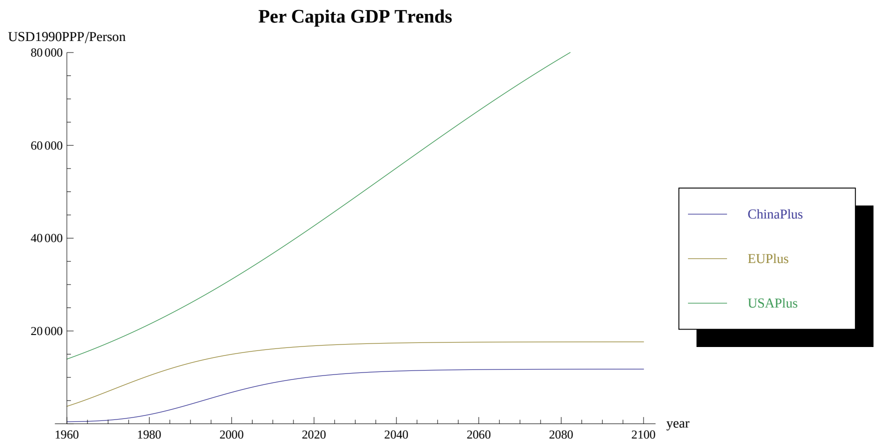


Figure 2.12. – GDP Predictions for *ChinaPlus*, *EUPlus* and *USAPlus*, again in 1990 US$.

2.4. Extensions of the model

2.4.1. Logical-2

In it second extension, the model is able to differentiate between "new" and "old" types of renewable fuels for energy production. Biofuels are considered "new" and are aggregated with other liquid fossil fuels. Non-fluid-fossil (mainly coal) as well as water, nuclear and wind are aggregated in the second group.

2.4.2. Fuel Fractions Model

Prior to being able to utilize the total diversification of energy LOGICAL-2 is supposed to offer throughout the modeling process, some of the results can be obtained by means of a Fuel Fractions Model (FFM). This has already been done for the two-region-scenario [Ret05, ST07]. The mathematics for dealing with nine regions is complete (attached in Appendix B.3), however the coding was not finished.

2.5. Critique of the model

As already stated often, the purpose of this work is to discover trends. That means by necessity to leave aside some substructures that would invariably complicate matters and, in the case of the database, to try to get reasonable and significant answers. For example, in several cases data points which are very inconsistent with the trend or mean average have been omitted in pairs (one above and one below the average of course) in trying to minimize the standard deviation. GDP figures from various sources (reported by the country itself or the United Nations (UN)) match not always exactly and have to be adjusted accordingly.

Moreover, the results are not exact. The fits and curves drawn do not include variations resulting from general volatility, wars or natural catastrophes. They rather resemble a "path", the world will more or less go. As described at the beginning of this chapter, the return to these paths after a perturbation however, is very likely.

Further systematic sources of error are the periodic corrections and the Fourier analysis as already stated in (2.2.4). If the available data is just not long enough to display a complete cycle, the system has obviously difficulties in determining wavelength and amplitude. This happens to very long cycles (Kondratiev), and also in cases where the usable data stream is very short (as in *ChinaPlus*).

The constraints makes forming the regions difficult, as it is almost impossible to define completely self-sufficient ensembles. For example the division on the North-American continent between *USAPlus* and *LatinPlus* is reasonable under many provisions, but in terms of migration it proves troublesome. While the "natural" population growth rate of the United States is not significantly higher than in most other industrial nations, it gets boosted significantly by immigration form *LatinPlus* (Fig (2.8)). Because of the development index being calibrated along the population growth rate, the model is led to predict a tripling of per-capita GDP in *USAPlus* (Fig. (2.12)) in contrast to a stagnating outlook in comparable regions. This example shows clearly, that calibrating the development index with the population growth rate, while reasonable and advantageous in many cases, can also be misleading in some.

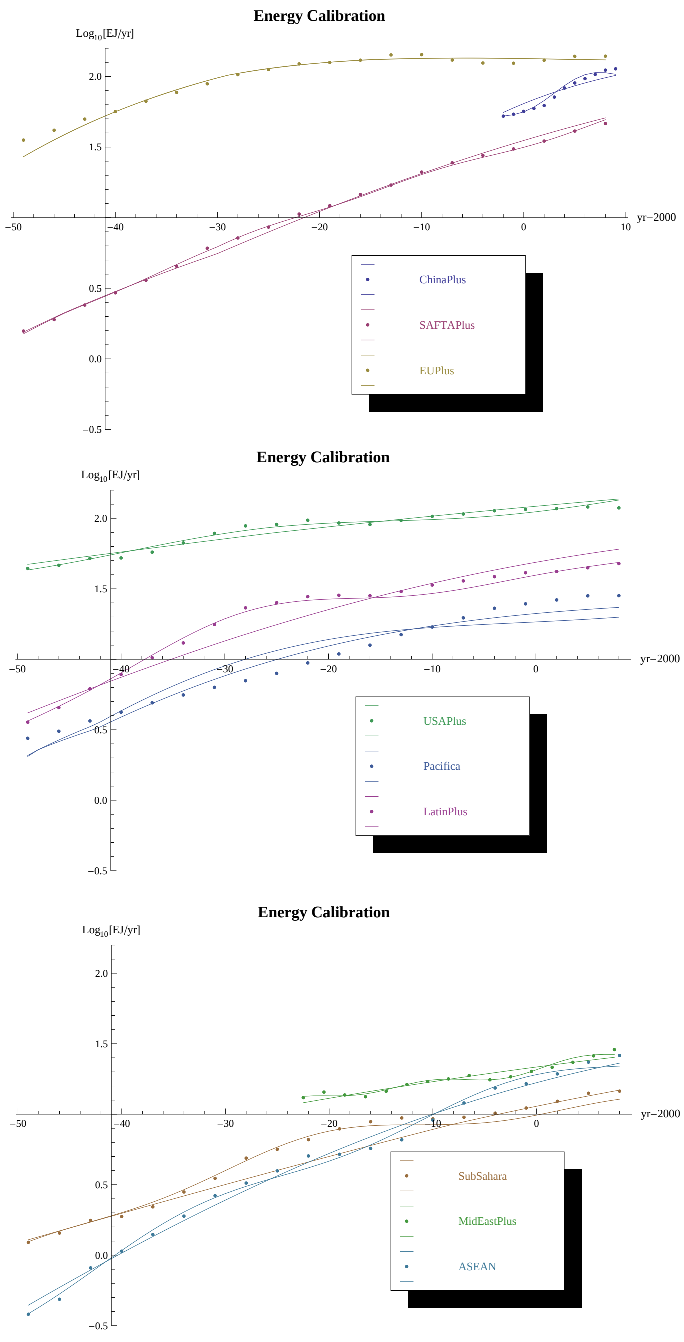


Figure 2.13. – Data and Fits (with and without periodic corrections) for energy use rates (in EJ/yr) in all nine regions

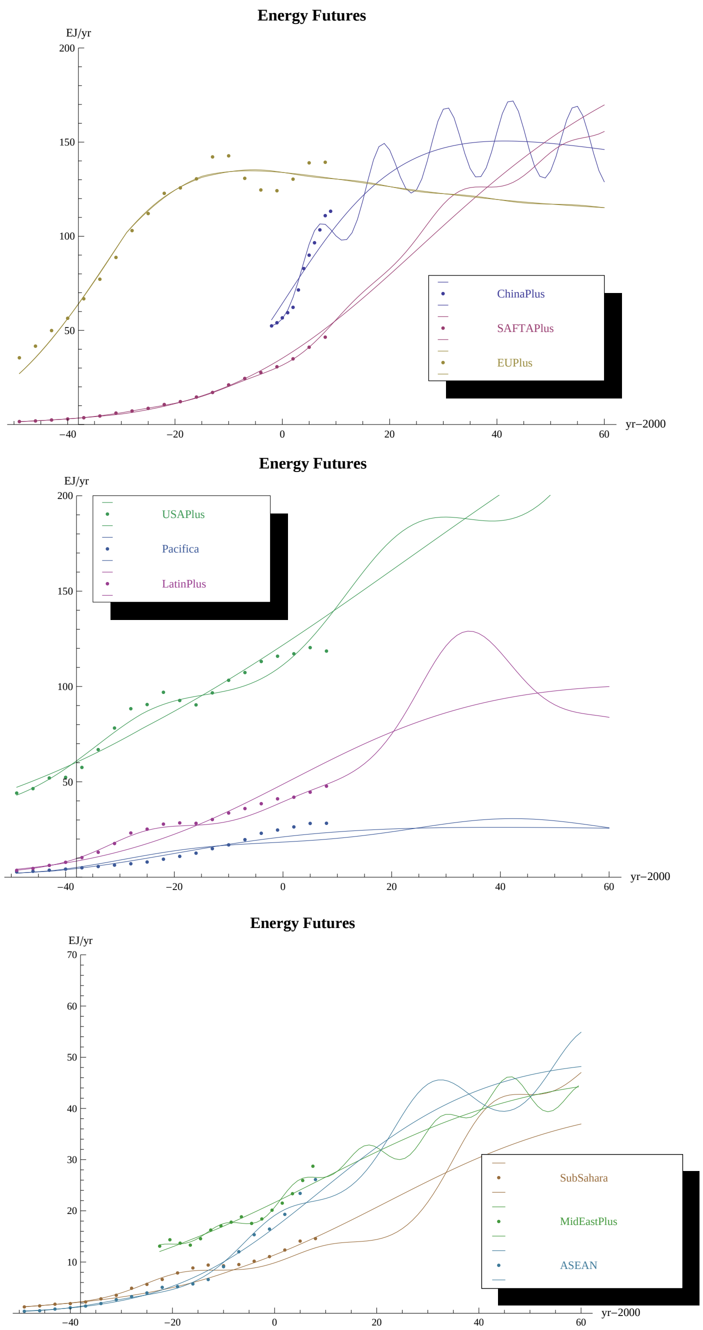


Figure 2.14. – Extrapolations of energy use rates for all regions

3. Budgets on CO_2 - Emissions

3.1. Introduction

Although methane, nitrous oxide, and various halocarbon compounds have a much larger effect per atmospheric molecule than CO_2, because of its larger amounts and its atmospheric longevity CO_2 is central to concerns about climate change. It has been shown that the scope of climate change is directly linked to the cumulative amount of anthropogenic carbon dioxide release [SQ+07]. For determining how long it takes atmospheric CO_2 to get reabsorbed, the IPCC uses a response function based on the revised version of the Bern-Carbon-Cycle-Model (*Bern2.5CC*, [JPS+01]), using a background CO_2 concentration of 378 ppm. Here, the decay of a pulse of CO_2 is given by

$$a_0 + \sum_{j=1}^{3} a_j \cdot e^{-\frac{t}{\tau_j}} \tag{3.1}$$

with time t and constants $a_0 = 21.7\%$, $a_1 = 25.9\%$, $a_2 = 33.8\%$, $a3 = 18.6\%$, $\tau_1 = 172.9$ years, $\tau_2 = 18.51$ years and $\tau_3 = 1.186$ years [SQ+07]. Here a_0 is approximated as constant when projecting forward for times of order $\tau_1 = 172.9$ or less because there can be a much longer impact on atmospheric CO_2 concentrations, on the order of a millennium [SPKF09]. So even after a complete halt of emissions, temperatures would only decline slowly. Therefore, it is only logical to use *cumulative* anthropogenic emissions as the ultimate measure when instituting limits to slow down or even stop global warming.

There has been much discussion of how to allot to various countries and regions the amount of CO_2 that could be emitted before a certain limit in atmospheric concentration (which corresponds to a certain increment in global mean temperature). The northern- and southern hemispheres are divided on this issue, as are the richer- and the poorer nations. Localities in every nation are contemporaneously contributors to and possibles victim in this process, albeit the levels of impact differ vastly. The problem is well defined in game-theoretical terms as a “social dilemma”. Here, actors prefer individual benefits to cooperative ones, and therefore inflict damage not only on everyone else, but also on themselves. In other words, “it is precisely the payoff utilities that lead the players to defect, while the other utilities - e. g. those connected with altruism, norms, and conscience - lead the players to cooperate”. By contrast, communication (with or without commitment), public disclosure, and moralizing enhance the understanding of these utilities and therefore have potential to increase cooperation [Daw80].

Applied to the world of international politics, there are three conditions for effective and mutually beneficial cooperative action on climate change: the relative payoff for cooperation and price of defection have to be increase and communication enhanced. As climate change in general is a relatively slow process with long reaction times, presumably some decades will pass until effective cooperation emerges.

As to how this dilemma may be solved, several proposals exist. Baer et. al. propose to set up an "Greenhouse Development Rights Framework (GDR)", in which emission rights are coupled to per-capita GDP so as to give underdeveloped countries some leverage to increase prosperity, while industrial nations have to reduce their emissions much more quickly [BAKKB08, CCD+09]. While it can be said that the proposed approach is not especially simple, its main disadvantage is probably that it might prove impossible to reach consensus on its grounds.

Another approach mainly represented by Meyer, and adopted by the United Nations Framework Convention on Climate Change (UNFCCC) is known as "Contraction & Convergence (C&C)". This refers to a reduction of total emissions (Contraction) and therefore a decline of per-capita emissions while the latter are converging between countries (Convergence) [Mey01, Mey07]. This calls for a high rate of reduction of release in some countries, although others can still increase their emissions. While this situation is certainly not avoidable per se, individual countries may face a much steeper slope of reduction as might be economically advisable, prompting resistance from those states.

Höhne et. al. refine the above approach by exempting non-Annex-I[1] countries from adhering to the reduction targets in the near future ("Common but Differentiated Convergence" (CDC)) [HDW06]. However, by treating developing countries preferentially, this approach is complicating the task of reaching a global consensus even more.

The approach presented in this chapter poses an alternative to the paths indicated above. It has also been put forward by Messner et. al. and is the standing policy guideline of both the German and the Indian government [MSRK10, SML+09]. The general idea is to found a consensus on universally accepted principles of fairness and equality. Firstly, this can only mean that every human being has the same amount of emissions to their disposal. Secondly, consensus has to be formed on what the limit (*guardrail*) should be in terms of *cumulative* emissions. Thirdly, as carbon dioxide is a natural component of the atmosphere, it has to be defined what *anthropogenic* emissions are, or better, from when on emissions by humanity became relevant to the earth's climate. Messner et. al. propose a guardrail of 2°C for this approach. For the purpose of this chapter however, it will be assumed that the time for a meaningful and committal arrangement will only have come when a doubling of preindustrial values of atmospheric CO_2 concentrations (which will result in an 2°C increment of global mean temperatures already by itself) is reached. The reason for this assumption is explicated below. Meanwhile, the guardrail assumed here, unlike the one Messner et. al. introduced, is not time-dependent. In this context, "guardrail" means simply a limit *never* to be exceeded. Key parameters of this hypothetical agreement would then

[1] Annex I countries of the Kyoto protocol include the industrialized countries that were members of the OECD in 1992, plus countries with economies in transition (the EIT Parties), including the Russian Federation, the Baltic States, and several Central and Eastern European States. [Uni10]

be:

- Preindustrial CO_2-concentration of 280 ppm [Bla10] as "starting point"
- 3.5°C increment or 762 ppm carbon dioxide concentration as time-independent "guard-rail"
- Even distribution of the amount of as yet unreleased emissions, equivalent to an increment of 202 ppm CO_2-concentration, between all human beings (introduction of emission rights)
- Under certain circumstances the possibility to trade emission rights

3.2. Modeling

The structure here is the same as in the last chapter. The regions analyzed are the ones defined in section (2.2.2) without *ASEANPlus*, which is integrated into *Pacifica*. The results from Chapter 2 are forming the starting point for theses calculations. The atmospheric model, already used in [SRA+08] is used to calculate atmospheric concentrations. Here, the fractional increase of atmospheric carbon concentration ΔC and global average temperature increment ΔT are [PHSB+99, SRA+08]

$$\Delta\dot{C} = \kappa\hat{E} - \gamma\Delta C \tag{3.2}$$

$$\Delta\dot{T} = \lambda\ln\frac{C}{C_0} - \sigma\Delta T \tag{3.3}$$

Here, $C_0 + \Delta C$ is the actual carbon concentration in [ppm] and $C_0 = 280$ppm [Bla10] represents its preindustrial base value. $\hat{E}$ represents the sum of emissions of elemental carbon in [Gt], and is a complex function of emissions rate and cumulative emissions. $\kappa = 0.47\frac{ppm}{Gt}$ converts elemental carbon emissions to atmospheric concentrations. $\gamma = 0.0449\text{yr}^{-1}$ stands for the rate of absorption and is a function of atmospheric carbon half-life (eq. (3.1)) and $\sigma = 0.0171\text{yr}^{-1}$ for the rate of cooling (heat sinks, black body radiation, etc.). While T_0 is the preindustrial average temperature, $T_0 + \Delta T$ stands for the current equivalent. The opacity effect coefficient $\lambda = 0.0979\frac{°C}{yr}$ is a measure of the greenhouse effect. As only a rough approximation is made here, only the greenhouse effect of fossil carbon emissions is taken to be secular [SRA+08]. For further information, c. f. [SRA+07, SRA+08].

All important is the energy production and use rate w from eq. (2.14) which we recall to be:

$$w = pB\left(kK\right)^{\alpha}\left(lL\right)^{\omega} \tag{3.4}$$

As a reminder: p is taken to be the energy production efficiency, which has a substructure, defined by $p = 1 + (h-1)f$. Here h is the ratio between the cost in capital and labor of all non-fossil vs. all-fossil energy production. For the examples shown herein, a value of $h = 2$ is used based on currently observed spectrum of energy prices in the United States.

f in turn is the (dimensionless and time dependent) carbon intensity of energy production, defined as a ratio of actual carbon intensity of energy use to that if only coal were used. f therefore ranges from 1 before the use of oil to 0 in the far future. Historically the value of f has been a monotonic non-increasing function of cumulative carbon use, u. A piecewise linear approximation to its dependence on u is used to calibrate f. Thus p ranges time-dependently from h to 1 as u goes from 0 to its long term asymptotic limit value.

We recall further the carbon balance constraint (2.15), expressing that the overall increase in cumulative carbon use is proportional to the product of carbon intensity (f) and energy use rate (w) with a region-dependent proportionality constant ϵ.

$$\dot{u} = \epsilon f w \tag{3.5}$$

With all that one can arrive at detailed predictions on the development of energy production (and therefore CO_2 emissions) for every region. With the population and energy consumption forecasts from chapter (2) one can easily arrive at per-capita figures, which will be presented in the next section.

3.3. Results and Interpretation

The first results are the ones for the development of carbon intensity of energy production f, or rather the ratio f/f_{Coal} with f_0 being the all-coal value. If extrapolating data on f/f_{Coal}, the results unfortunately tend to drop to 0 for some regions. To avoid this, a minimum f_s is introduced which is defined as follows:

$$f_s = \frac{C_{\text{Gas}}}{C_{\text{Coal}}}\,(1 - f_{\text{new}}) \tag{3.6}$$

Hereby f_{new} is the dimensionless carbon intensity of energy production for new forms of energy such as wind and solar, and C_{Gas} and C_{Coal} the dimensional carbon intensities of energy production for natural gas and coal respectively.

The next step is to combine the carbon intensity with energy production rates from chapter (2), to arrive at the rate of CO_2 production (both for the world *in toto* and for the eight regions) and, with previous amounts factored in, predictions on cumulative carbon emissions (Figs. (3.1) and (3.2)).

To measure each region's increment, the pre industrial concentration is estimated at 280 ppm [Bla10]. As in the last chapter, available data is fitted and the Laplace transformation solutions scaled to an estimate 387.35 ppm of global atmospheric CO2 concentration at year 2009 [SRA$^+$08, Bla10]. The model extrapolates to a doubling of preindustrial values of atmospheric CO_2 by 2106, which would result in a 2°C rise in global mean temperature soon thereafter. At the time when an agreement on limiting further emissions is concluded, *EUPlus* and *USAPlus* have obviously already overused their quota, while *ChinaPlus* is quite close to that point. Moreover, the reduction of carbon intensity of energy production is expensive in economic terms. Therefore this rate can only be moderate, if it is to be

[*ppm*] and [Gt]	*ChinaPlus*	*SAFTAPlus*	*EUPlus*	*USAPlus*	*Pacifica*	*LatinPlus*	*SubSahara*	*MidEastPlus*
Fraction of World Population	14%	25%	8%	7%	10%	8%	17%	11%
CO_2 quota for 761ppm	*69*	*123*	*38*	*32*	*49*	*39*	*85*	*55*
CO_2 already injected	*65*	*21*	*43*	*71*	*33*	*24*	*6*	*18*
CO_2 quota remaining	*4*	*102*	*-5*	*-39*	*15*	*15*	*79*	*38*
C quota for 761ppm	365	110	299	397	177	135	34	100
C already injected	400	709	221	185	282	223	488	319
C quota remaining	34	599	-78	-212	105	89	454	220
C increment per year	3.0	1.1	1.3	3.9	1.9	1.1	0.3	0.8
additional cumulative CO_2 @ 3.5% reduction rate	87	32	36	111	55	33	9	25
Total Balance	-51	567	-114	-324	50	55	445	195

Table 3.1. – Situation when global emissions reach double the preindustrial value (2106). Quotas to restrain global warming to 3.5°C. The upper set of quotas (in *italic*) are atmospheric concentrations in [*ppm*], everything below these are amounts of carbon in [Gtonnes Carbon].

sustainable and not cause economic disruptions. For the purpose of this chapter an annual reduction of in the carbon intensity of energy use of 3.5% is assumed. This results of course in a prolongation of the time of higher emissions from regions *ChinaPlus*, *EUPlus* and *USAPlus* and causes them to overextend their quota even more (see table (3.1)).

On the other hand however, other regions have still considerable amounts left in their budgets, especially regions *SAFTAPlus* and *MidEastPlus*. The advantages of striking a bargain in this situation are obvious. How such a deal could be worked out and which modalities it would contain is of course highly speculative. It should nevertheless be noted that some form of compensation for *SAFTAPlus* and *MidEastPlus* would be necessary in the present context, as these regions have to be deterred from using their full quota without regard to implications for the earth's climate.

In figures (3.3) and (3.4) the effects of the reduction agreement and the emission-quota trade can be observed. Because the slope chosen at 3.5% annually is relativly gentle, it roughly takes 130 years to cut the rate of emissions by half. In effect, the outcome depends on the course of *SAFTAPlus* and *MidEastPlus* in the far future if the "guardrail" is touched or not. In the calculations presented here this is not the case. While an amount of 2782 Gt in cumulative worldwide carbon emissions would be necessary to raise global temperatures by 3.5°C, only 2358 Gt will have been emitted by 2313 according to the model.

3.5°C in comparison to 2°C as a guardrail

Already global temperatures have risen 0.8°C from preindustrial values [SQ+07]. The Greenland ice cap is melting on average [SHM+07], which causes sea level to rise [RCC+07].

This process will accelerate according to rising atmospheric temperatures, resulting in an elevation of 70-170 cm (in comparison to preindustrial times) by 2100 [RCC+07]. This will cause some islands and even densely populated coastal areas to become uninhabitable. While the implications of a 2°C increment are barely manageable, an increase by 3.5°C is nothing less than short of catastrophic failure of the earth's ecologic systems. The latter scenario would probably include the loss of 30% of biodiversity [SQ+07] and would transform vast parts of the globe; climate conditions in many regions would resemble circumstances not present on earth for millions of years [MSRK10]. Socio-economic consequences would be very severe and could present a serious threat of conlict set off by water, land and food shortages, or evironmental migration [Sch09]. Even extreme repercussions such as deglaciation of Greenland or a collapse of the great oceanic currents can not be ruled out completely.

In light of these consequences it is of course much more preferable to stay below a 2°C limit. Global politics are however an unwieldy tool, and to reach consensus amongst all the necessary parties will likely be difficult. The present analysis investigates a situation where only severe ecologic and socioeconomic pressure can force a compromise, which is why the doubling of preindustrial concentration values is adopted here as the trigger for such an agreement. Hopes remain that this outlook may be a too pessimistic one, but there is a distinct possibility that these hopes will not be fulfilled.

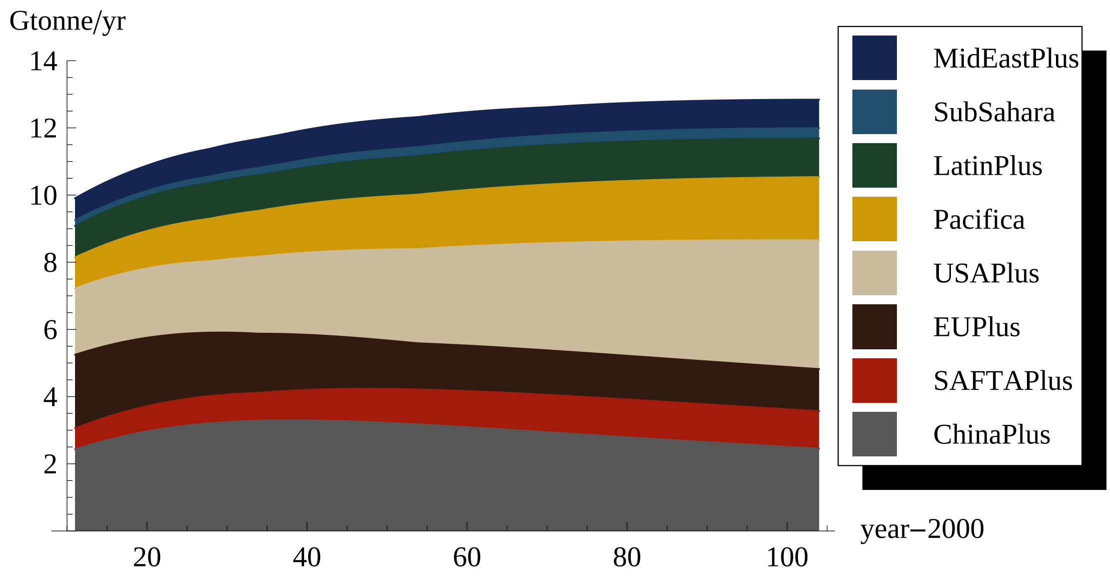


Figure 3.1. – Development of the rate of carbon emissions in all eight regions in Gt/yr. Results for *ChinaPlus* are doubtful after 2030 (see section (2.2.1)).

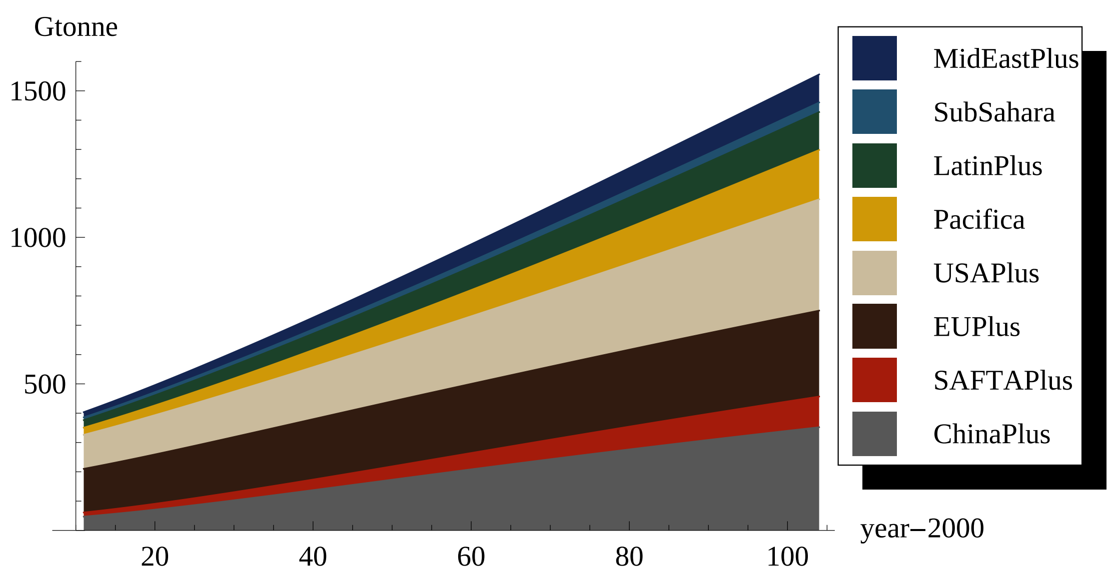


Figure 3.2. – Development of the cumulative amount of carbon emissions in all eight regions in Gt.

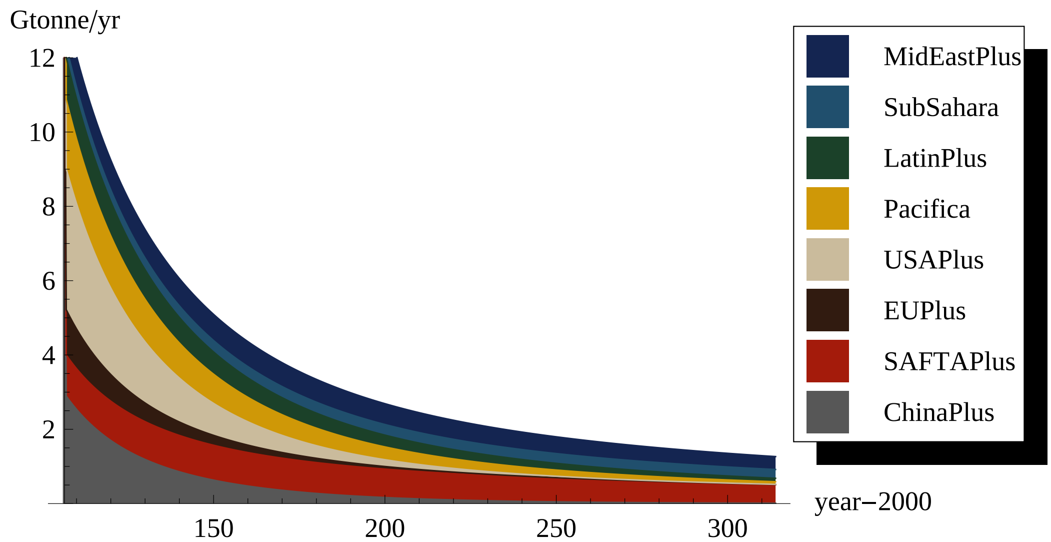


Figure 3.3. – Long-term development of the rate of carbon emissions in all eight regions after conclusion of a reduction agreement in [Gt/yr].

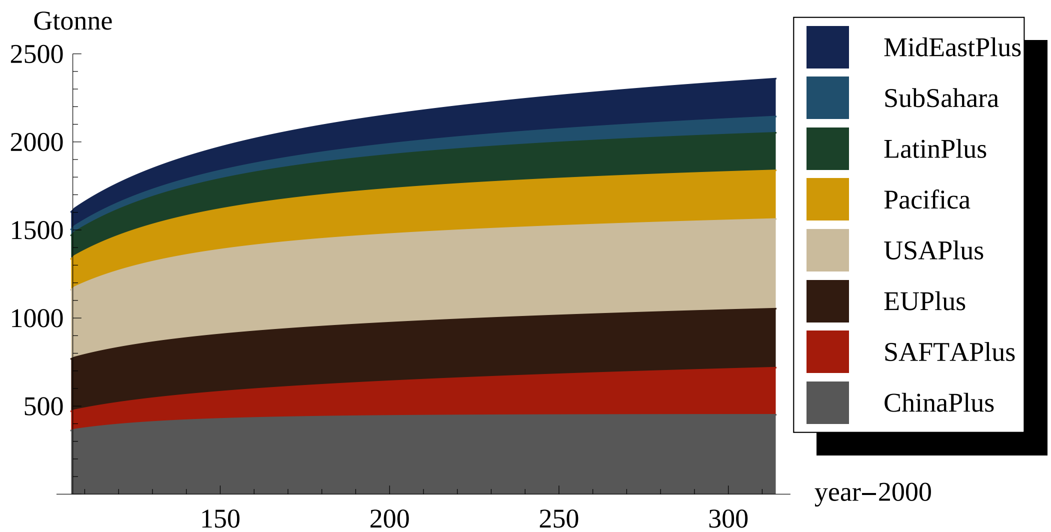


Figure 3.4. – Long-term development of the cumulative amounts of carbon emissions after the conclusion of a reduction agreement in all eight regions in [Gt]. *SAFTAPlus*, *SubSahara* and *MidEastPlus* have still margins left in their quota at the time of the agreement.

4. Uranium Resources and Price of Uranium[1]

4.1. Introduction

Worldwide, 27 countries are using nuclear power. As of 2006, there were 435 reactors on line with a total generating capacity of about 370 GW_e. On average, each reactor consumes an annual fuel load of roughly 25 tonnes of enriched uranium each year. That amounts to a current annual consumption of 10000 tonnes of natural uranium (at an average enrichment of 3.5%). On the back end of the cycle, this material is being treated as waste by most, but not all countires. However, reprocessing spent fuel is a complex and expensive process. Bunn and Fetter have shown that at least presently, reprocessing is highly uneconomic [BFHvdZ03]. Unfortunately the most prominent alternative, indefinite storage in a repository, has its own disadvantages, particularly with the difficulty of actually getting repositories licensed. For various reasons, no country has commissioned such a repository as of yet. However, it has been proposed to mix these two approaches: storage of the spent fuel, until either the price for fresh uranium has risen sufficiently to make reprocessing economically viable, or until concerns about the increasing ease of unauthorized use of ever less radioactive spent fuel lead make its continued storage without re-use purportedly undesirable [DC09]. While there are other applications for nuclear power than electricity generation (like submarines and ships), they represent a very small fraction and will therefore not be taken into account.

Long term uranium price extrapolations hinge mainly on estimating the recoverable amount of ore of a certain richness at various production costs. Various works have been published on that issue, with quite diverse conclusions. There are even voices that claim that the current consumption is not sustainable and that uranium reserves will run out in the next decade Dittmar [Dit09]. But most sources estimate that uranium resources will last at least another 80 years [PB02, Mac04].

The price of uranium reached its all-time peak in the 1970s, driven by a combination of military requirements and growth of civilian nuclear power. After this peak, prices rapidly dropped and continued a steady decline over the next 20 years driven in large part by slower than expected growth in nuclear power, a result of the Three Mile Island and Chernobyl accidents, and a surplus supply that resulted in the build-up of large inventories. The price hit a historic low in 2000 and began a rebound that continued through 2005, as the market adjusted to the reality of potential near to mid-term supply shortfalls in the absence of the opening of new mines.

[1] As this chapter is in some parts analog to a joint article by Clifford Singer and Hermann von Brevern [SvB10], Sections (4.4.2) - (4.6) draw on a draft by Clifford Singer.

4.2. Red Book Estimates

Since 1965, the OECD Nuclear Energy Agency (NEA) and the IAEA have jointly prepared periodical updates (currently every two years) on world uranium resources, production, and demand. These updates have been published by the OECD/NEA in what is commonly known as issues of the "*Red Book*" series after the color of their covers. Uranium resources are reported in categories of confidence level and production cost. In terms of cost they are distinguished into:

- < 40 US$/kgU
- < 80 US$/kgU
- < 130 US$/kgU

Unfortunately from the point of view of making inflation-adjusted price comparisons, these prices are always reported in current US$, which complicates the comparison between different editions of the book and the identification of trends. That the price category thresholds have not been changed since 1977 is obscuring the situation.

In terms of those, Red Book estimates discriminate between

Reasonably Assured Resources (RAR) are resources of the highest confidence level. Extent and ore grade are known. They are reportet in all three price brackets, which is to say that they are recoverable at a cost of less than 130US$/kgU with present technology.

Inferred Resources previously known as Estimated Additional Resource Category I (EAR-I), these resources are being expected with high confidence, based on direct geological evidence. Usually they are extensions to deposits categorized as RAR. Inferred Resources and RAR together comprise the *Identified Resources* category.

Prognosticated Resources refer to those expected to occur in known uranium provinces, generally supported by some direct evidence but with less confidence in their scope than *Inferred Resources*. Previosly known as Estimated Additional Resource Category II (EAR-II).

Speculative Resources refer to those expected to occur in geological provinces that may host uranium deposits. They are usually based on geological trend extrapolations of *Prognosticated Resources*.

In both the case of *Prognosticated* and *Speculative Resources*, significant amounts of exploration are required before their existence can be confirmed and grades and amounts can be defined.

[2] also including 2 973 300 tU in Speculative Resources of unassigned cost range

Category	Cost ranges			All
	< 40 US$/kgU	< 80 US$/kgU	< 130 US$/kgU	
Reasonably Assured Resources	1 766 400	2 598 000	3 338 300	7 702 700
Inferred Resources	1 203 600	1 858 400	2 130 600	5 192 600
Identified Resources	2 970 000	4 456 400	5 468 900	12 895 300
Prognosticated Resources	NA	1 946 200	2 769 000	4 715 200
Speculative Resources	NA	NA	4 797 800	4 797 800
Total	2 970 000	6 402 600	13 035 700	25 381 600[2]

Table 4.1. – Uranium resources reported by the Red Book 2007 [NI08]. Amounts as of 1 January 2007, in tonnes of U and rounded to nearest 100 tonnes.

However, Red Book estimates are an understatement at best.[3] For probably economic reasons, not every country is reporting their resources in full. I.e. Australia, currently the second largest producer of natural uranium, does not state RAR and Inferred resources above 80$ per kg, and does not report at all on prognosticated- or Speculative Resources. In general, reports are limited to locations on earth where already some exploration has taken place. Due to the inherent cost of exploration, countries and companies limit themselves to exploring just enough resources to assuage short- and mid-term needs. No assumption whatsoever is presented, on how much uranium could possibly be found if demand and therefore exploration incentive would rise significantly. To address directly whether Red Book figures are likely to be underestimates of actual resources, Singer included the following question amongst those posed in a session scheduled for that purpose at one of the technical committee meetings on uranium Resources, Production and Demand which included representatives from twenty-five countries, mostly uranium producing countries: "Have 'conservative' biases towards underestimation which are referenced in some of the older literature been removed by recent changes in reporting procedures and coverage, or have they perhaps been overcompensated for" (For a complete list of the questions posed, see the article by Singer) [Sin01]. With the possible exception of some central Asian countries interested at the time in increasing uranium production and exports, the general response from the assembled experts was, that the estimates reported for compilation in the Red Books were conservative. A stated reason for this was quite simply that responsible parties reporting back on successive occasions to their countries would prefer to bring back "good news" that estimates had increased rather than "bad news" that previous estimates were too high. It can therefore be concluded that the results of the approach taken by

[3]Speculative estimates "do not represent a complete account of world undiscovered conventional resources." [NI08]

the authors of the Red Book, while very useful when looking at the status quo of “nuclear economics” is far from comprehensive when examining long term development.

4.3. The cost of Nuclear Energy

Most long-term forecasts and simulations of the development of nuclear energy and its supply situation base their calculations on a growing scarcity of uranium under the condition of significant growth of nuclear power generation. However, before looking into projections of the future uranium price, one should consider its relevance to the governing question of the cost of nuclear energy. Bunn and Fetter have suggested a very simple algorithm to demonstrate that even a fivefold increase in uranium prices would influence the price of electric energy generated in a nuclear power plant only marginally [BFHvdZ03]. For the moment we will assume a price of US$ 70 per kg of natural uranium. At a desired enrichment level of 4%, and a tails assay of 0.2%, roughly 8kg of NU are needed in order to produce 1kg LEU. If we then assume a burnup of 50 $\mathrm{MW_{th}}$d/kg, and 32% efficiency when transforming thermal into electrical energy, we arrive at a cost of US$1.46 per KWh for the raw material (see Eq. (4.1)) [BF06].

$$\left(\frac{\$70}{kg_{\mathrm{U}}}\right)\left(\frac{8kg_{\mathrm{U}}}{kg_{\mathrm{LEU}}}\right)\left(\frac{kg_{\mathrm{LEU}}}{50MW_{\mathrm{th}}d}\right)\left(\frac{MW_{\mathrm{th}}}{0.32MW_{\mathrm{el}}}\right)\left(\frac{d}{24h}\right) = 1.46\frac{\$}{MW_{\mathrm{el}}} \tag{4.1}$$

For comparison: Bunn and Fetter estimate the production cost of electricity in an Light water moderated Reactor (LWR) at approximately 70$ per MWh. [BFHvdZ03]

4.3.1. Price and availability of resources

Concerns about the earth’s limited resources are widespread, based on the clear fact that mined mineral resources are limited and not renewable. However, it can be shown in most cases that limits to the supply of resources are so far away that concerns have little practical meaning. Only in a few cases such as oil, where prices and sophisticated projections may now be indicating that proven reserves probably are indeed beginning to run out, can such limits be seen as substantial. Because of these examples, concerns about resource depletion nevertheless deserve careful examination.

As already stated above, predictions of scarcity based on published mineral reserve figures (like the Red Book) are often unsatisfactory. Regardless of the mentioned currency conversion issues, they structurally fail to take account of key “resource-expanding factors”:

1. Gains in mineralogical knowledge and discovery capabilities.
2. Productivity increase in production, i.e. allowing for progress in mining and processing technologies used to recover mineral deposits.

3. Productivity increase and substitution in consumption. As technology moves forward, less raw material is needed in production or substitution starts if these get to be too expensive.

4.3.2. The Simon-Ehrlich wager

An illuminating example concerning the points above is a prominent wager between the economist Julian Simon and Paul R. Ehrlich, an ecologist, in 1980. Simon, being an cornucopianist[4], proposed that Ehrlich was to select five raw materials and a time-frame of anything over a year. He (Simon) was adamant that the commodity prices would remain constant or would even decline, based on his theory about human ingenuity and unlimited resources in the long run. Ehrlich on the contrary, based on his Malthusian theory of increasing scarcity (he was the author of a widely noticed 1967 book, called "The Population Bomb" [Ehr68]) believed those prices to go up. He chose chromium, copper, nickel, tin, and tungsten, and bought on paper \$200 worth of each using 1980 prices. They agreed to "sell" these on September 29, 1990, 10 years later and that the loser should pay the winner the difference to the 1980 prices (adjusted for inflation). As a result, in October 1990, Paul Ehrlich mailed Julian Simon a check for \$576.07 to settle the wager in Simon's favor.

Simon was not sure if he would win the bet, he just expressed that the chance of the prices to fall would always be greater than that of their rising. In the short term however, commodity prices may very well increase. Considering copper prices at the time of the bet for example, the prices were indeed be expected to increase in the absence of any new technologies, as growing economies demanded more copper to meet the needs of expanding communications networks and plumbing infrastructure. Technological changes mitigated much of this expected demand as copper wire networks were replaced by fiber optics, and various plastics replaced the once ubiquitous copper pipes throughout the construction industry.

4.4. A Comprehensive Approach

4.4.1. A simple assumption

Disregarding for once that uranium resources are in fact limited (which is legitimate because there are plenty of them as will be shown) the question arises as to how exactly the supply reacts to rising prices. Choosing for a starting point identified resources of the Red Book, we currently have at our hands about 2.97 million tonnes of U at a price up to roughly

[4]from *cornucopia*, the "horn of plenty" of Greek mythology. The theory, that earth's resources are sufficient to provide for approximately 10 billion people in 2050. Simon explains: "More people, and increased income, cause resources to become more scarce in the short run. Heightened scarcity causes prices to rise. The higher prices present opportunity, and prompt inventors and entrepreneurs to search for solutions. Many fail in the search, at cost to themselves. But in a free society, solutions are eventually found. And in the long run the new developments leave us better off than if the problems had not arisen. That is, prices eventually become lower than before the increased scarcity occurred." [Sim95]

$40/kg. In Eq. (4.2), p symbolizes the market price and R the amount of available resources [BF06].

$$R = 2.97 \left(\frac{p}{40} \right)^{\epsilon} \tag{4.2}$$

The interesting parameter now is ϵ, which is the long term elasticity of supply. If doubled prices really would create a tenfold increase in measured resources as Hore-Lacy believes [Hor99], then $\epsilon = 3.3$. As presented in the following work, calculations for this thesis have yielded a value for ϵ as between 2.2 and 2.5 which translates into an 5-6 fold increase of resources should prices double.

4.4.2. Long term elasticity

To circumvent the drawbacks of an approach relying on accounting present amounts, price-quantity driven models use estimates of ore concentration in various geological formations and assess their occurrence around the globe. Inherently they are not taking into account transients and natural price fluctuations, but aim to forecast long-term trends. Classic works are Deffeyes' and MacGregor's on uranium content and concentration in the earth's crust, and its interpretation by Schneider and Sailor [DM80, SS08a]. Schneider and Sailor concentrate primarily on log-linear estimates of the form $\ln q = s_0 + s_1 \ln p$, where q is the cumulative historical global extraction of native uranium and p is inflation-adjusted production cost. Values of s_1 (which is another form of expressing ϵ) in the various approaches reviewed by Schneider and Sailor range from 0.5 to 3.5. The value 0.5 results from considering Red Book estimates. The value $s_1 = 3.5$ is the slope drawn by Schneider and Sailor of the high-ore-grade portion of a curve drawn by Deffeyes and MacGregor on a log-log scale of the increments in earth crustal amounts of uranium between various uranium ore concentrations (see Fig. (4.1)). As an estimate of the values of s_0 to accompany each value of s_1, Schneider and Sailor quote a reference point (p_0, q_0) from the 2003 Red Book report of ($40/kgU, 2.523 Mtonne) [NI04] for cost per kilogram and millions of metric tons of elemental uranium.

4.4.3. Quantity vs. Ore/Uranium Ratio

To re-examine the logic behind using various choices of the quantity-cost relations described by Schneider and Sailor, we ask two questions. First, what slope results from a systematic fit to the underlying estimates of amounts of various uranium ore grades given by Deffeyes and MacGregor on a log-log scale?

Second, how do the type of estimates given by Deffeyes and MacGregor translate into mining practice? In the discussions denoted (1) and (2) below, these questions are addressed in the context of extraction of uranium from the earth's crust up to the point where the alternative of extraction of uranium from seawater might become economically competitive, so only the rising portion of a fit to the Deffeyes and MacGregor estimates is relevant.

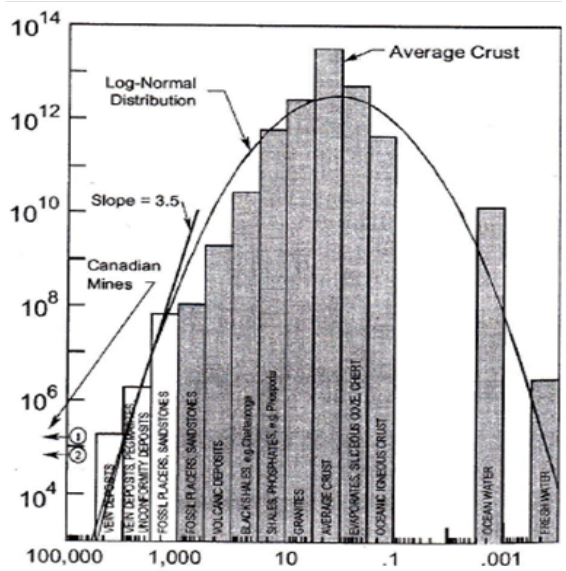


Figure 4.1. – Distribution of uranium in the earth's crust. On this log-log plot, the slope of the leading edge of the parabola represents s_1. Here, the total amount of uranium in the earth's crust is estimated at around 80 trillion tonnes. (after [DM80, SS08a]

(1) For the ore concentrations of interest here, a fit to the rising portion of Deffeyes and MacGregor's estimates is given in Fig. (4.2). The variable on the abscissa is the ore/uranium (ore/U) ratio r, which is the inverse of the ore grade. This choice of ore/U for the abscissa variable in Fig. (4.2) makes the slope $\frac{d \ln q}{d \ln r}$ positive. The least squares fit slope is $s = 2.08$. For the rest of this discussion, the value of s_1 will be rounded from 2.08 to 2 for mathematical convenience, since greater precision is not justified by the approximate nature of the input information.
(2) If the cost of uranium were proportional to the ore/U ratio *and* miners responded to depletion of higher grade ores only by moving on to lower grade ores, then a cumulative mined quantity v vs. r relation of the form $\ln v = s_0 + s_1 \ln r$ would be appropriate. However, in fact the response appears to have been both to use lower grade ores and to go after higher grade ores that are less readily accessible.

The dots in Fig. (4.3) show the fraction of uranium mined up to a given ore/U ratio as a function of the ore/U ratio normalized to the largest value thereof in the plotted data set [Wor10]. If mining progressively moved to lower ore grades with higher ore/U ratios, the data shown in Fig. (4.3) should hug the zero line until the normalized ore/U ratio nears 1.0 and then abruptly rise. Evidently this is not the case.

The curve fit shown in Fig. (4.3) was made as follows: Assume that the lowest cost ores are successively extracted, where the cost is $c = br + d^k$ as a function of the difficulty d of extracting a given amount of ore and of the ore/uranium ratio r. For example, if the cost

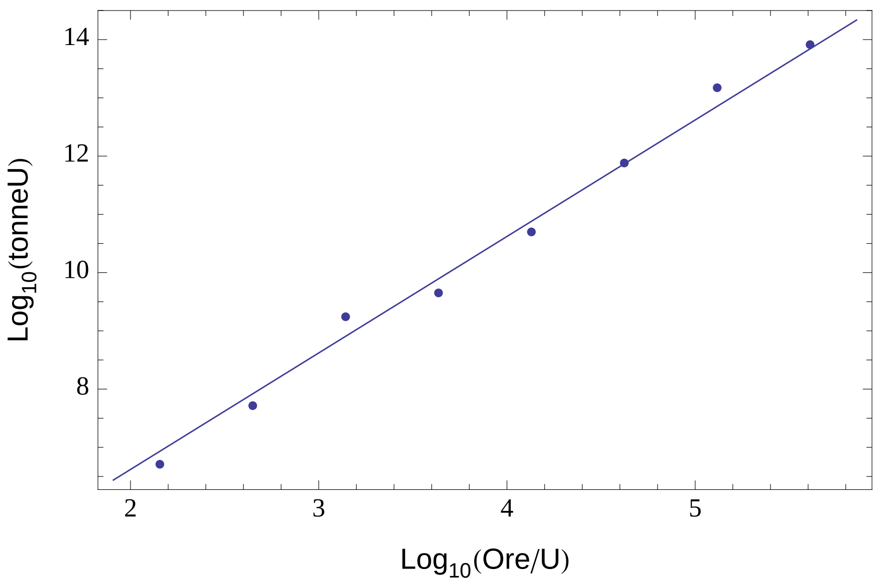


Figure 4.2. – Least squares "power law" fit to Deffeyes and MacGregor's estimates of cumulative crustal amount of uranium amounts in the top 1 km of the earth's crust, on the ordinate, up to the ore/uranium content ratios on the abscissa.

of extraction a given grade of ore increased linearly with d, e.g. due to the need to dig ever deeper, then we would have the exponent $k = 1$.

Let

$$v_r(c, r) = v_{\text{norm}} \int_{r_1}^{c/b} r(c - br)^m r^{(s_1 - 1)} dr \tag{4.3}$$

where solving $c = br + d^k$ gives $(c - br)^m = d$ if $m = 1/k$.

Here v_{norm} is a scale factor that will be calibrated to historical experience, and r_1 is the lowest ore/U ratio available. The total uranium extracted historically up to the point where the extraction cost is c, is $v(c) = v_r(c, c/b)$. The fit shown on Fig. (4.3) is for $\frac{\partial v_r/\partial c}{dv/dc}$, which is the incremental extraction up to ore grade r divided by the total incremental extraction dv as c increases by dc, in the limit of small dc. Working out the math gives the result

$$\frac{\partial v_r/\partial c}{dv/dc} = 1 - (1 + mx)(1 - x)^2 \tag{4.4}$$

with $x = r/r_{max}$ and r_{max} the maximum ore/uranium ratio for the mines in the data set. The least squares result for the adjustable fitting parameter m is $m = 2.47$.

Mines where uranium is extracted with substantial amounts of other minerals can have much higher ore/U ratios because the economics are influenced by the value of th co-product,

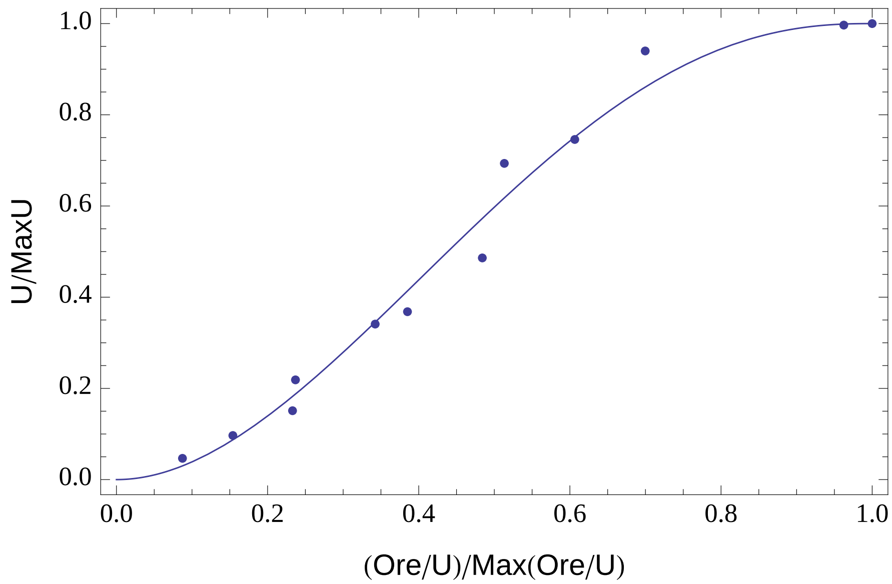


Figure 4.3. – Cumulative amounts of uranium estimated to be mined annually from information on 12 mines with actual or planned operations from 2008–2012 for mines up to the ore/uranium ratios on the abscissa, with both the data on the ordinate and abscissa divided by that for the upper right-most point.

and are thus excluded from Fig. (4.3). Also omitted from the data and fit shown in Fig. (4.3) are an equal number of mines with the largest and smallest ore/U ratios. The number of largest and lowest ore ratios so dropped was chosen to minimize the standard deviation at 0.052 of the residuals between the fit and the data shown in Fig. (4.3). In the complete data set there are clear anomalies from a set of Canadian mines producing a small fraction of global output with very high ore/U ratios and thus not appropriate to include as separate data points, as well as some unusually large ore/U ratios at the high ore/U end of the distribution.

4.5. Uranium Price Oscillation

For all of the reasons noted in section (4.2), it was chosen to calibrate and extrapolate uranium prices based on actual historical prices and cumulative amounts mined, rather than on numbers in Red Book resource categories. Schneider and Sailor in their work have discounted price variations. However, as historical uranium prices have by no means increased monotonically with cumulative mining, it is essential to account for price variations around an underlying trend. The fit shown in Fig. (4.4) does this using the formula

$$\ln\left(\frac{p}{p_0}\right) = \frac{\ln(v/v_0)}{s} + A\sin\left[\frac{2\pi(t-\tau_2)}{\tau_1}\right] \tag{4.5}$$

where $s = m + 2$.

The logic behind the term $\frac{\ln(v/v_0)}{s}$ is described in the next section. The points on Fig. (4.4) represent delivered United States contract prices from von Hippel [vH09], except for 2009 [Wor09], and before 1973 [NI06], for which spot market prices were scaled to match the nearest available year's contract price. All prices in this chapter are inflation adjusted to $US2007 as described in Section (2.2.3). The following discussion concentrates on the underlying price trend without the oscillating correction depicted in Fig. (4.4), but it should be kept in mind that even if the oscillations are eventually damped they could be of substantial amplitude for many decades if the type of behavior illustrated here continues well into the future.[5]

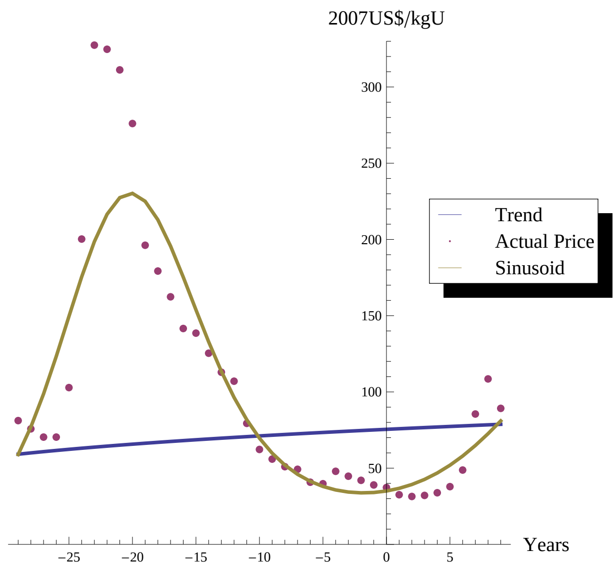


Figure 4.4. – Uranium prices background trend and damped sinusoidal oscillation of the log of the price. 0 on the abscissa corresponds to the year 2000.

The price oscillation in Fig. (4.4) reflects periods of uranium overproduction compared to consumption rates before 1980 and underproduction during the 1990s. This production pattern reflected both the failure of nuclear energy use to grow as anticipated earlier during the 1980s, and the interaction between civilian and military uses of nuclear energy during, and for some time after the cold war. Both the technology of uranium mining and the stringency of occupational safety and environmental regulation of uranium mining have evolved considerably as well. There remains considerable uncertainty about how these various influences on the temporal variation of uranium prices will work out in the future.

[5] The functional form of Eq. (4.5) corrects the underlying production cost trend derived from the fits in Figs. (4.2) and (4.3). For the $m = 5/2$ case, the resulting damped sinusoidal correction has a phase shift τ_2=Julian year 1971.4, oscillation period $\tau_1 = 36.8$ years, and a natural logarithm amplitude of 0.89.

The point here is that inflation-adjusted uranium prices appear on the average to have been larger than $40/kgU, suggesting a possible disconnect between the cost categories as used in the recent Red Books and the various mechanisms that determine actual market prices. Another implication of the information shown in Fig. (4.4) is that it could well be several decades before the possible influence of uranium resource depletion can be clearly seen in the temporal history of uranium prices. Caution is thus in order concerning long-term capital investments in technologies aimed at reducing the amount of mined uranium used per unit of nuclear energy production on the assumption that very recent price data reflects a strong underlying cost escalation.

4.6. Uranium Cost Trend vs. Cumulative Uranium Use

When extrapolating the above results to much larger quantities of cumulative uranium mined, eventually a limit will be reached on the lengths to which miners will go to reach the highest concentration ores. Currently the deepest shaft and open pit mines are $\ell = 4$ times deeper than the uranium shaft and uranium open pit mines respectively.[6] Allowing for such limitation on the lengths miners will go to recover the highest concentration ores, Appendix B.4 shows that the relationship between the amount v of uranium that can be recovered up to a cost and that cost c itself is given, with $s = m + 2$, by

$$\frac{c}{c_m} = \left(\frac{v}{v_m}\right)^{\frac{1}{s}} \quad \text{for} \quad v \le v_m \tag{4.6}$$

$$\frac{c}{c_m} = \left[1 + \sqrt{1 + \left(2\frac{v}{v_m} - mn\right)\frac{n}{sm^2}}\,\right]\frac{m}{n} \quad \text{for} \quad v \ge v_m \tag{4.7}$$

Here $n = m + 1$, $c_m = c_0\ell^{1/m}$, and $v_m = v_0\ell^{s/m}$.

The difficulty of extraction of uranium is not, of course, just a function of mine depth. The type of materials in the overburden and the ore also impact costs. Dittmar points out, that the size of deposits also affects costs [Dit09]. However, initially only the small fraction of the inaccessible ores that have the highest uranium concentrations are excluded when the "depth" limit is encountered, so the results are not very sensitive to the value chosen for this parameter until after much larger amounts of cumulative uranium mining.

A plot of the cost trend c as a function of cumulative uranium use v is given in Fig. (4.5). This uses year 2007 reference values of v_0=2.37 Mtonne and c_0 = 80$/kgU as the underlying trend cost. The red curve in Fig. (4.5) shows the result from Eqs. (4.6) and (4.7). The blue curve in Fig. (4.5) shows the continuation of the formula $c = c_m(v/v_m)^{1/s}$, i.e. without the limitation accessing the highest concentration ores imposed for $v > v_m$.

[6] While drilling techniques are used for jet washing of high grade ores and for in situ leaching, these are not the dominant methods of uranium recovery and are thus not considered here for this reference value of ℓ. However, it should be kept in mind that such methods provide another avenue for extraction of resources not economically accessible by shaft and open pit mining.

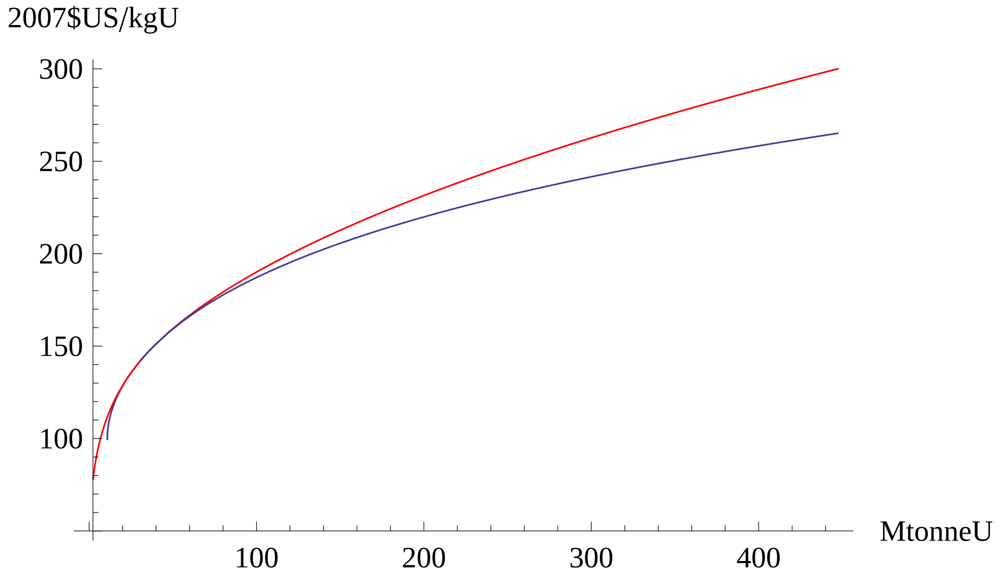


Figure 4.5. – Uranium cost trend as a function of cumulative uranium use with (red) and without (blue) "depth" limit correction included.

In order to put the above graph into perspective: The *cumulative* amount of uranium used by the entire world was *2.234 MtU* up to 2007 [NI08]. This shows, that there is plenty of uranium to be mined and at very moderate prices as well. Furthermore, any differences in calculations with and without "depth-limit" are only apparent in the far future, hence this detail will be left aside in further calculations.

4.7. Comparison to SRES-Scenarios

The IPCC has also incorporated in its SRES-Scenarios storylines for the development of power generation. These are:

A1 High growth scenario, with ample supply in oil and gas.

A1C High growth scenario, with a shift in energy production towards coal. It shows the highest cumulative use of uranium nevertheless, for reasons not understood so far.

A2 High growth scenario, with a phaseout of fossil fuels. Interestingly lacks significant growth in nuclear energy production and relies on true renewables instead.

B1 Middle course scenario, that relies on gains in efficiency on all fields.

The bundle of SRES-Scenarios include none in which nuclear energy is favored. However, a good number of conclusions can be derived from the ones at hand. On further details about Special Report on Emission Scenarios (SRES) see section (2.1.5). Table (4.2) lists the results of several scenarios (run by the MESSAGE algorithm) for the development in nuclear energy production.

Year	**1990**	**2000**	**2010**	**2020**	**2030**	**2040**	**2050**	**2060**	**2070**	**2080**	**2090**	**2100**
A1 MESSAGE	7	9	12	19	34	55	84	126	185	247	306	358
A1C MESSAGE	7	8	11	21	41	79	127	191	265	348	415	432
A2 MESSAGE	7	9	13	17	22	34	47	62	79	97	116	136
B1 MESSAGE	7	8	11	15	20	27	36	42	47	48	46	41

Table 4.2. – Primary Nuclear Energy Consumption in four IPCC-Scenarios. Units in Exa-Joules. [NS00]

Every data set is interpolated and then normalized to 2007 uranium usage figures taken from the 2007 Red Book [NI08] so as to both convert EJ into Mt and set a common starting point.[7] Then the amount of previously used uranium (before 2007) is added in order to get realistic figures for world wide cumulative uranium use. Table (4.6) shows the results. As these seem to indicate a cumulative use less than that where the "depth limit" should occur even in the most intensive, the A1C-Scenario, the graph for cost trend development has been produced with eq. (4.6), i.e. again without the limitation accessing the highest concentration ores imposed for $v > v_m$.

[7] Unfortunately, several scenarios differ considerably in their starting points (in general, however not in the scenarios picked above), while no information about the reason could be obtained.

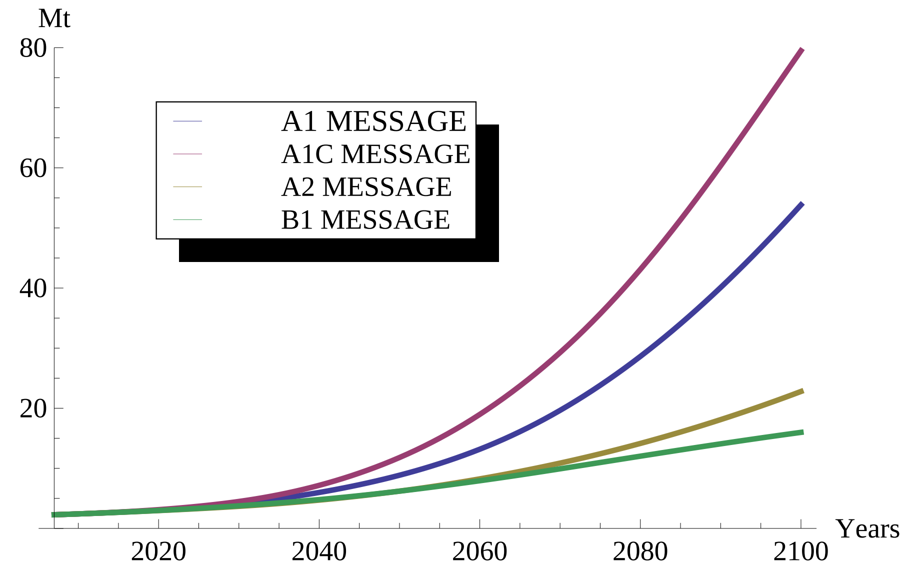


Figure 4.6. – Cumulative use of uranium in four IPCC-Scenarios until the year 2100.

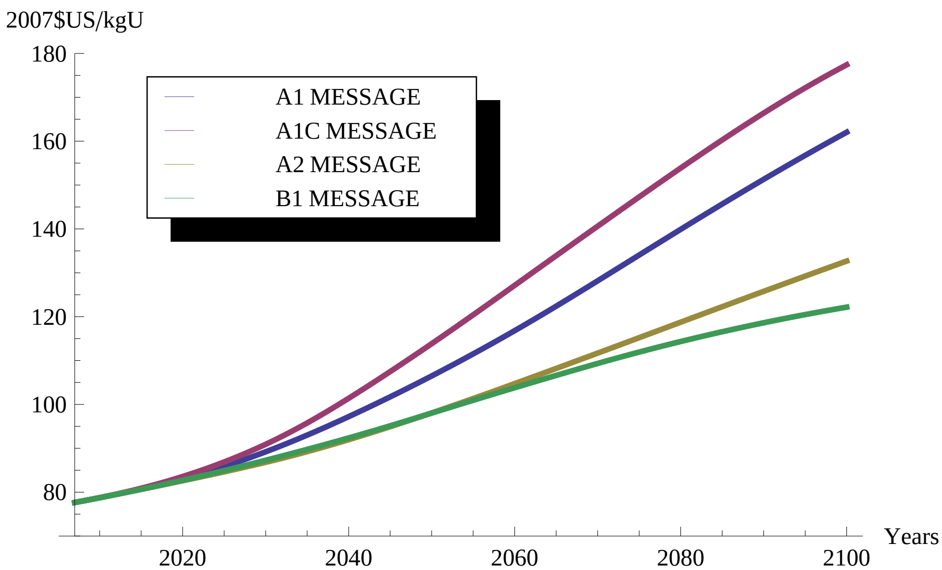


Figure 4.7. – Uranium cost trend in four IPCC-Scenarios until the year 2100.

5. Possible Sources of Error

As many sources of errors have been mentioned in the previous chapters, mainly a short synopsis is provided here. When looking at the results of chapter 2, the most palpable problems are the ones with *ChinaPlus*. Because of state controlled fertility, China's development index is overstated. The *scope* of this overstatement is, however, uncertain. LOGICAL-0 is not equipped to handle a lag between population growth rate and development index. In effect, the model is predicting China to be farther away from its inflection point regarding population growth. This causes the algorithm to predict a reduced economic growth in the future, which will begin to level off in the 2030s. This also influences forecast for energy consumption and GHG emissions of course. This results in a few cases in calibrating results rather empirically than fully theory-based, and in the case of *ChinaPlus* underestimating GDP, energy, and carbon use rates in the forecast.

Having a closer look on the topmost graph of Fig. (2.8), one can see the drop the installment of the one-child policy (starting in 1978) caused. Today, less than 40% of the Chinese population is subject to this policy [Chi07], which explains the bounce upwards in the late 1990s. If one draws an imaginary curve from the growth rates of the 1970s, one can see that the growth rate would probably be in reality about 1% higher, which makes a huge difference to the model. Moreover, the question arises what would happen if the Chinese government would release its policy some day; would the population revert to its old growth-path or stick to the new one? In any case, the next installment of LOGICAL will come up with a new way to tackle the problem. Until then the forecasts for this region have to be taken with a grain of salt.

Systemic sources of error are the periodic corrections as already discussed in chapter 2. Sometimes usable data streams are too short for the wavelength of a cycle to be detected with reasonable precision. If the data is even shorter, the task of differentiating between an oscillation of some sort and a simple parabola becomes quite difficult. In addition, even if enough data points are available to identify wavelength and initial amplitude, discerning a forcing or dampening of the amplitude is sometimes a challenge. However, all this applies only to a small number of cases, mainly the region of *ChinaPlus* (as seen in the topmost graph of fig. (2.13)) and, to a certain extent, *MidEastPlus*. All other regions have been stable enough to yield enough information, at least since World War II.

A further system immanent limitation to results is that they are simply obtained by extrapolating historic trends. That leaves no room of course for the consequences of dramatic changes in policy or technology. As examples might serve a sudden consensus to ban coal-fired power plants or a breakthrough in electrical storage capability.

Chapter 3 inherits most of the possible sources of errors from the preceding chapter. The gravest uncertainties, however, lie in the nature of politics. The model is based on the striking of a bargain when global CO_2 concentrations reach a double of the preindustrial value that restricts global warming to a maximum of 3.5°C. As already discussed, this assumption might be overly pessimistic. In any case, the predicted time at which this bargain negotiated, 2106, is far off. This again is related to the China-Problem discussed above.

Another source of error here is the limitation to CO_2 emissions from energy production. Emissions for various other sources, like agriculture, or the production of concrete, are left aside.

In addition to the uncertainties in *ChinaPlus*'s energy future, the accuracy of chapter 4 suffers from volatility in policy issues, namely the acceptance of nuclear power, and institutional inertia. For example, while Germany wants to terminate its use of nuclear power in the next thirty years, the United States and other countries are working on its renaissance. While their contrasting motivations are not a subject here, their probable impact on the development of global use of nuclear power must be considered potentially extensive. Because of that, no time dependent estimate of the use of uranium was derived from the results, for it would have been too speculative. To demonstrate this capability, IPCC scenarios were thus was used.

Another problem is the obscurity of prices in the nuclear sector. This results partly from intransparent cost structures in this part of the economy, as many actors are government owned or government funded at least which often leads to unrealistic price figures. Furthermore, companies are very wary of industrial espionage and the history in national security of this sector is long; both reasons hamper the flow of information greatly. This results e.g. in the use of spot-market prices for uranium for some years, which are very volatile and as often as not express an average price level.

6. Conclusion

The purpose of this study was to acquaint the reader with mathematical modeling of socioeconomic and climate problems, and their application in several cases. After laying some ground by explaining the development of growth theory, the capabilities of the LOGICAL-1 system were demonstrated. Because the algorithms could not be completed in time, the results were obtained with the older LOGICAL-0 system. The implications have been detailed previously.

The results are very useful in giving insights as to how the global economy may develop. Especially the consequences for the industrialized regions of a high population growth rate or the lack thereof can clearly be seen. This is especially instructive when looking at the energy futures (fig. 2.14). While energy use in *USAPlus* and *EUPlus* is now roughly at par, it will be decreasing in the latter because of efficiency gains, but in the former, it will be soaring in spite of that.

Proceeding to the question how to deal with the emissions this usage will cause, chapter 3 outlines an approach to handle this question. An important assumption is that no binding agreement on the emission (and reduction) of GHG's will take place until average atmospheric CO_2 concentration reaches double the value of preindustrial times. This guarantees an increment in global mean temperatures of 2°C even before any measures can take hold. To avoid too abrupt changes in world economy, a global warming of 3.5°C on average will then probably be set as "guardrail" together with an annual reduction in emissions of 3.5%. When it comes to distributing remaining emission quotas up to this limit, only a per-capita approach can be considered fair and sustainable. As some regions will already have overused their quota by then (*EUPlus*, *USAPlus* and to a certain extend *ChinaPlus*), while others have vast amounts left (*SAFTAPlus* and *SubSahara*), the introduction of some sort of emission rights trading system seems compelling. Chapter 3 shows that under these circumstances exceeding the 3.5° limit can be avoided.

One of the many options to reduce carbon intensity of energy production is to increase the usage of nuclear energy. Appendix A investigates the extent to which this can be effective and concludes that nuclear energy can have a very small "carbon footprint".

Meanwhile there is dissent if this is at all possible because of allegedly scarce uranium resources. The famous "Red Book" lists amounts that would last some decades at the present rate of consumption. It can, however, be doubted if figures in this book are useful for making long-term predictions. It is shown in chapter 4, that these doubts have valid foundations and that the amount of uranium that could be discovered if appropriate incentives existed would considerably exceed known quantities. The modeling shows in effect that a doubling of current prices should occur after a cumulative use of about 80 MtU, barring speculative and other short-term variations. As the world cumulatively used only 2.234 MtU so far (2007) this point is quite far off even when considering a vast increase in production of nuclear energy. Even then, prices would be less than half of what would make other technologies for uranium recovery like reprocessing[1] or extraction from seawater[2] economically competitive. This has obviously vast implications:

- A once-through nuclear cycle is sufficient to supply nuclear energy production for several hundred years.
- Reprocessing only for strategic reasons and/or to reduce the volume of wastes.
- Fast reactor technology is not needed for general power production.

Trying to predict the share of nuclear in future energy production is somewhat speculative, as several subjective factors are concurring with the pure economic ones. Therefore, it is not part of this work to form a separate forecast. However, in order to provide the reader with some measure as to how the results of cumulative use vs. price of uranium can be interpreted in terms of time vs. price, usage predictions of several IPCC scenarios were drawn on for that purpose. Fig. (4.7) shows the results. The bottom line here is, that none of the sampled scenarios inferred a uranium price beyond 180 $/kgU by 2100, which supports the conclusions drawn in the previous paragraph.

Outlook

As soon as the LOGICAL-1 coding is complete, much better forecasts for *ChinaPlus* should be available. This will also improve the accuracy of extrapolations of carbon emissions considerably. The implementation of the fuel fractions model detailed in appendix B.3 for the nine region model will allow extrapolating usage rates for various forms of energy production (not only nuclear but a considerable number of others as well) without relying on IPCC-scenarios. The long-term goal is to implement the LOGICAL-2 specifications and therefore to integrate the fuel fractions model into all aspects of the system in order to be comparable to the latest MERGE model. Finally, it might be beneficent to devise a trade model in order to be able to freely choose the countries that make up a region. Then, bodies like OECD or G-20 could be modeled.

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[1] Bunn and Fetter have estimated that at a price of 1000 $/kgHM, reprocessing and recycling plutonium in LWR's will be more expensive than direct disposal, until the uranium price exceeds 360 $/kgU (conservative estimate). [BFHvdZ03]

[2] Uranium is abundant in seawater: about 4.5 bn tonnes at a concentration of 3-4 μg per Liter. Currently, the extraction costs are estimated at about 1000 $/kgU, while it might be reasonably achievable to lower them to 330 $/kgU. [SSS+04]

A. The Carbon Footprint of Nuclear Power

A.1. Fuel Cycle

A.1.1. Scope

There are basically two types of fuel cycles. They are commonly referred to as "once-through" and "closed" (albeit the former is not a cycle per se, the term is used nevertheless). The closed fuel-cycle incorporates reprocessing and reuse, which includes the use of Plutonium as a fuel. We will limit ourselves to the "once-through" cycle, as it is the one employed by the United States. Here the fuel is only used once, and stored as waste afterwards. Although there are experiments concerning the Thorium fuel-cycle, the vast majority of power generation nuclear plants use uranium as fuel. We will therefore disregard Thorium.

A.1.2. Mining

Usually, uranium deposits are found in form of various minerals, immersed in different types of rock. To assess the energy needed to extract the uranium exactly, it is certainly necessary to know the composition of the ores. The world's richest deposits contain up to 10% uranium though these high-grade deposits are of course the rarest, but concentrations of under 0.1% are still feasible for extraction. Each ton of seawater for example, contains about 3mg uranium (0.000001%). uranium is extracted either by open pit mining (30%), underground excavation (38%), in situ leaching (21%), or as a byproduct in other mining operations (11%) [Len08]. Procedures to recover uranium from seawater are under investigation [SSS+04]. When calculating the energy requirement and recovery rate for uranium mining, it is important to consider the composition of the ores. If, for example, like in Australia's Olympic Dam mine, uranium is extracted as a byproduct of Copper [BHP04], the energy requirements of the products must be disaggregated (for example by mass).[1]

From an input output based hybrid life cycle assessment for the USA [RPR75] (Table 1), detailed data on the energy requirements of uranium mining is available.

[1] BHP Billiton [BHP04] states that "It is correct to say, for Olympic Dam, that copper, gold, uranium and silver are extracted from one and the same rock body in a simultaneous operation. In the case of the Olympic Dam orebody, we can apportion the energy cost for mining the orebody amongst the four metals based on their relative mass contribution. Once the orebody reaches the surface, energy costs can also be apportioned for grinding. Once the ore then enters the processing circuit the calculation then becomes very process specific - i.e., at Olympic Dam a lot of the copper goes through flotation, smelting and refining, whereas uranium goes through none of these processes, so the flowsheet needs to be well understood in order to make a complex calculation."

Rock	MWh/t ore	MWh/t U 0.3%	MWh/t U 0.2%	MWh/t U 0.1%	MWh/t U 0.01%
Ore	0.38	182	273	545	5,450
Shale	30.11	53	79	158	1,581

Table A.1. – Total energy requirements of uranium mining [RPR75]

A.1.3. Milling

Milling is undertaken usually adjacent to mining operations, to avoid transportation of vast amounts of unprocessed rock. The ore is milled to powder and treated with various chemical processes to extract the uranium-Oxide (U3O8) from it. The product is the so-called "yellowcake", a yellow powder containing more than 80% uranium [Wor05].

Again, from an input output based hybrid life cycle assessment for the USA [RPR75] (Table 2), detailed data on the energy requirements of uranium milling is available.

Rock	MWh/t ore	MWh/t U 0.3%	MWh/t U 0.2%	MWh/t U 0.1%	MWh/t U 0.01%
Ore	0.37	178	267	533	5,334
Shale	0.30	143	214	428	4,229

Table A.2. – Total energy requirements of uranium milling [RPR75]

A.1.4. Conversion

To further process the fuel, the uranium has to be converted into a gaseous state. First, it is converted to uranium Dioxide, then by adding hydrogen fluoride (HF) and gaseous fluoride(F), first to form uranium-tetrafluoride (UF_4) and then into uranium-hexafluoride (UF_6).

The energy requirements differ in literature: Weis et. al. state only 7 MWh_{th}/tU for the wet process (which differs from the description above) [WKH90], the Australian Coal Association's figures are 21 MWh_{el}/tU and 155 MWh_{th}/tU [Len08], and Rotty et. al. state requirements of 14.6 MWh_{el}/tU and 396 MWh_{th}/tU respectively [RPR75], with most of the energy needed in the form of natural gas. Their figure is also the highest in Storm van Leeuwen and Smith's literature review [SS08b].

A.1.5. Enrichment

Natural uranium is composed of two isotopes, U-235 and U-238. The half-life of U-235 is about 0.7 billion years. In comparison, the half-life of the other natural occurring isotope, U-238, is 4.5 billion years, which is just about the age of earth itself. Because of this

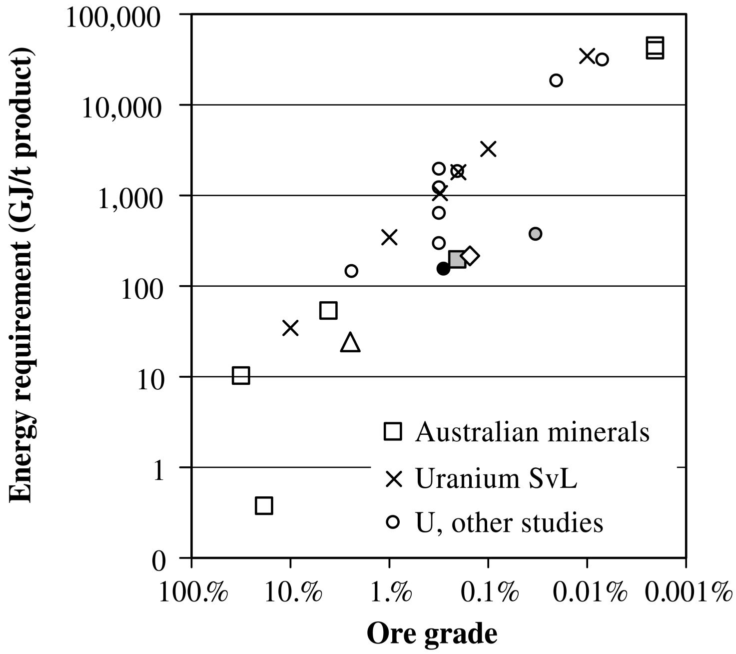

Figure A.1. – Energy intensities for metal ore mining and milling (from [Len08]). Australian minerals are uranium, iron ore, mineral sands, silver–lead–zinc ores, and gold. The outliers are the Rössing mine in Namibia (●), the Ranger mine in the Northern Territory (■), and the Beverley mine in South Australia (◇). The triangle (△) represents Olympic Dam. In-situ leaching is shown to require less energy than conventional mining (●) [Len08]

discrepancy in half-lives, the erstwhile fraction in uranium of 30% U-235, has reduced itself to about 0.7% today. Considering fission, the important difference between those two isotopes are their respective fission cross-sections, essentially measuring how likely a chain reaction in a material takes place. Because U-235 has an about 2000-times higher cross-section than the heavier isotope, it is far better suited for usage as reactor fuel. While there are reactor types which can use natural uranium, the vast majority has to have the fraction of U-235 in its fuel boosted. This process is called enrichment. Typical enrichment boosts the fraction of U-235 to 3-5%. However, naval reactors (not considered here) for example, need higher than 90% enriched fuel.

There are two basic enrichment methods, gaseous diffusion and centrifugation. Worldwide use is roughly split 40/60 on those two methods. There are several other methods (Laser, etc.) as well, however their impacts are statistically insignificant. The effort made to enrich uranium is usually expressed in Separative Work Units (kgSWU[2]. A typical LWR (1.3 GW)

[2]Enrichment is measured in Separative Work Units (SWF's), the amount of separation done by an enrichment process. Separative work is *not* energy. The work W_{SWU} necessary to separate a mass F

Year	**Type**	$\frac{kWh_{el}}{kgSWU}$	**Comments**
1997	C	170	converted using 3.5 SWU per kg 3%–U
2006	C	50	
2006	C	62.3	URENCO, UK.; includes "infrastructure and capital works"
2006	C	250	
1997	C	282	including investment in plant
2006	C	75	URENCO, Europe and TENEX, Russia
1997	C	40	converted using 3.5 SWU per kg 3%–U
2006	D	2860	
1997	D	2330–2737	
2006	D	2100–3100	
1997	D	2500	
2006	D	2420	
1997	D	≈2520	including capital
2006	D	2810	
1997	D	30502	including plant construction, fossil fuels and process materials
2006	D	3080	
1997	D	2400	
2006	D	2400	Eurodif Tricastin, France
1997	D	2600	USEC Paducah, USA

Table A.3. – Total energy requirements of uranium enrichment [RPR75], differentiated by method (C = Centrifuge enrichment and D = Diffusion enrichment

uses about 25 tonnes of 3.5% enriched uranium per year. This takes about 210 tonnes of Natural Uranium (NU) using about 120 kgSWU. We will use those numbers in our later assessment of the actual carbon footprint of enrichment.

A.1.6. Fuel Fabrication

After the uranium is enriched, it has to be brought in a state useful for fueling a reactor. Therefore the fluorine is stripped and solid uranium is fabricated in cylindrical shaped "pellets", usually 10x15mm in size. They are stacked in long tubes, 11ft long, which are made of Zirconium alloy, the "cladding". 179-264 of these fuel rods, together with control rods in the middle, are then in turn combined into a fuel assembly, ready for use in a reactor. The Australian Coal Association states an energy requirement of 52.7 MWh_{el}/tU and 32.7 MWh_{th}/tU respectively for this process [Len08].

of feed of assay x_f into a mass P of product assay x_p, and tails of mass T and assay x_t is given by the expression $W_{SWU} = P \cdot V(x_p) + T \cdot V(x_t) + F \cdot V(x_f)$, where V is the *Value Function*, defined as $V(x) = (1-2x) \cdot ln\left(\frac{1-x}{x}\right)$.

A.1.7. Repository

Spent fuel is considered high level radioactive (>10^4Ci/m^3). However, most of the fission fragments are short lived, so that before transferring this type of waste to a final storage site they are kept in ponds in the vicinity of the reactor to allow the radiation levels to decrease. In these ponds, the water absorbs the heat and shields the radiation. Used fuel is held in such pools for several months to several years [Wor05].

According to Rotty et. al., the majority of the energy requirement in this stage is for materials such as concrete and steel for storage canisters and structures [RPR75]. They report values of about 167 MWh_{el} of and 1800 MWh_{th} annually, for storing the spent fuel from a 1 GW LWR. However, it is unclear if this includes construction of the repository. Studies by the Australian Coal Association yield figures of about 80 MWh_{el}/t fuel and 600 MWh_{th}/t fuel [Len08]. Based on the assumption of the 1 GW LWR this corresponds to 1997 MWh_{el} and 14,733 MWh_{th} respectively, which is significantly more.

Lenzen [Len08] mentions also an assessment by White and Kulcinski [WK00]. Applied to operational waste only, their estimate amounts to 400 MWh_{th}/t of radioactive material, which is roughly comparable to the Australian Coal Association's numbers. Sovacool [Sov08], mentions high end estimates of 40.75 g CO_2e/kWh and low end estimates of 0.4 while the mean is about 9.2 gCO_2e/kWh [SSS+04].

A.2. Operation, Construction and Decommissioning [3]

A.2.1. Operation

In all nuclear power plants, a fission chain reaction must be maintained in order to generate electricity. Inside a light water nuclear reactor, a neutron is absorbed by the nucleus of an U-235 atom and then becomes unstable and it fissions into two or more fission fragments, releasing more neutrons that will be used to create a fission chain reaction. On average, about 2.4 neutrons will be released from the fission of U-235, but the vast majority of these neutrons will be at energy levels that are too high for subsequent U-235 atoms to have a high probability of absorbing the neutron and continuing the chain reaction. A moderator is used to slow down the neutrons, which will give the U-235 atom a higher probability of absorbing the neutron. In light water reactors, the moderator is water, which also serves as the coolant. Control rods are used to absorb neutrons and allow the operators to control the fission reaction; usually they are made out of zirconium. The fission fragments and neutrons collide with the fuel rods and the moderator losing their kinetic energy and generating heat. The fuel rods and control rods constitutes the main components of the core of the reactor. In pressurized light water reactors, the core is housed inside a reactor vessel in which cool water enters from the bottom of the core and is heated up as it passes to the top. The water then passes through the pressurizer and then into the steam generator. Here the water in the primary loop is cooled and then pumped back into the core. The water from

[3] this section A.2 courtesy of William Matisiak

the secondary loop passes through the steam generator and evaporates. The steam passes through a turbine which creates a large amount of mechanical torque that is used to turn a generator to create electricity. The steam from the turbine is returned to the condenser where it is cooled and condensed before it is pumped into the steam generator again. The condenser is cooled by water from the cooling towers and/or from a nearby river or lake.

In a boiling water reactor steam is generated directly inside the reactor vessel which is then passed through a turbine. The steam returns to the condenser where it is cooled and condensed again before it is pumped back into the reactor. Again as in the PWR, the condenser is cooled by water from a cooling tower and/or nearby lake or river. According to a report by Rotty, Perry, and Reister, the inputs of dies, chemicals, hardware, and maintenance for the operation of a light water reactor require about 8.5 GWh_{el} of electricity and 80 GWh_{th} of thermal energy for the annual operation [Len08]. The operation of the pumps, backup systems, repairs, periodic maintenance on the facility, fuel reloading and operation of the control support systems, all require energy. Some of this energy releases carbon dioxide and other green house gasses. Accurately accounting for all these green house gas emissions is difficult and few studies have sufficiently accounted for these emissions. According to the 2007 report on nuclear fuel cycles from Storm Van Leeuwen and Smith, the total estimate of reactor operation and maintenance was 24.4 gCO_2/kWh [SS08b]. This estimate is higher than the mean from other qualified studies for the nuclear fuel cycle, which is 11.58 g/kWh_{el} [Sov08].

A.2.2. Construction

The construction of a nuclear power plant is a very time consuming and costly endeavor. It can take over eight years for a nuclear power plant to go from planning and reactor licensing to final operational status, since there are many factors that inhibit the process. The construction phase alone will take a several years to complete. The major factors involved in the construction phase includes the transportation and manufacture of material to create generators, turbines, pipes, cooling towers, control rooms, pumps, and other necessary infrastructure needed for safe and reliable operation of a nuclear power plant.

In a normal nuclear power plant there are over 50 miles of piping, which are welded some 25 thousand times, 900 miles of electrical cables, and thousands of electric motors, conduits, batteries, relays, switches, transforms, and fuses [Sov08]. Other necessary components for a reactor facility include firewalls, radiation shields, spent nuclear fuel storage facility, emergency backup generators; as well as sensors for temperature, pressure, power levels, radiation levels, and coolant water, in addition to chemistry monitoring systems. An estimate from a 1995 report from White, calculated that a 1000 MW PWR requires on average 170,000 tons of concrete, 32,000 tons of steel, 1,363 tons of copper, and around 2,100 tons of miscellaneous materials [Whi95]. These materials can be quite carbon intensive, with many studies varying on estimates for the CO_2 emission from the construction phase, ranging from a minimum of 0.27 g CO_2e/kWh to 35 gCO_2e/kWh, with a mean value of 8.20 gCO_2e/kWh [Sov08].

A.2.3. Decommissioning

Decommissioning a nuclear facility is the final step in the nuclear life cycle. It involves the dismantling of the reactor and reclamation of the uranium mine site. It may take as long as 50 to 100 years for the reactors to cool off, before it can be dismantled and placed into concrete blocks for final disposal [Sov08]. The expected operating lifetime for nuclear power plants is 40 years, although there are reactor facilities applying for extensions for up to 60 years. The expected decommissioning will be at least in the 60 year time frame, depending on technique and reactor type, and the total energy needed for decommissioning could be as high as 50% of the energy needed for the original construction [Sov08]. The estimate of the cost to decommission a commercial nuclear power plant is in the range of 250-500 million U.S. dollars, according to the IAEA. The decommissioning step in the nuclear life cycle has an estimated Carbon emission that range from 0.01 g CO_2e/kWh to 54.5g CO_2e/kWh with a mean of 12.01 gCO_2e/kWh [Sov08].

A.3. Conclusion

A.3.1. Conversion factors

To assess the GHG emissions from various forms of energy consumed, we first of all transfer them to electrical energy by multiplying the thermal energy with a conversion factor of 0.38 (correlating to a general overall efficiency of 38% in that conversion) [SRA+07]. Then we can assign a CO_2e value to it, according to the type of power generation used (Table A.4).

A.3.2. Carbon Footprint of Nuclear Power

When bringing it all together, one has to be careful about the units. We chose to express amounts in the text in regard to its context (i.e. a ton before enrichment does not equal a ton after enrichment). This makes sense when reading through the paragraphs. However, when putting the pieces together this has to be assessed. So we put all the accumulated values (and often their mean from several studies) in the following table (Table A.5) in units of natural uranium (NU), so to be consistent within the table. Multiplying the above values with some factor for CO_2e (Table 4), yields the emissions for one ton of NU as a preliminary number. If we perhaps estimate the bulk of the energy has been created using fossil fuels we would use an estimate of 850 gCO_2e/MWh. But if the grid would be fully de-carbonized, this number would be much lower, perhaps about 30. Therefore the impact of the surrounding energy concept on the carbon footprint of nuclear power becomes clear. Furthermore, there are emissions that cannot be assessed so easily in terms of energy, but rather in terms of emissions (i.e. concrete). In our assessment, the means mentioned above

[4] Wind, hydroelectric, biogas, solar thermal, biomass, and geothermal estimates taken from Pehnt [Peh06]. Diesel, heavy oil, coal with scrubbing, coal without scrubbing, natural gas, and fuel cell estimates taken from Gagnon et. al. [GBU02]. Solar PV estimates taken from Fthenakis [FKA08]. Nuclear is computed here. Estimates have been rounded to the nearest whole number.

Technology	Capacity / Configuration / Fuel	Estimate gCO_2e/kWh
Wind	2.5 MW, offshore	9
Hydro	3.1 MW, reservoir	10
Wind	1.5 MW, onshore	10
Biogas	Anaerobic digestion	11
Solar Thermal	80 MW, parabolic through	13
Biomass	Forest wood steam turbine	22
Biomass	Waste wood steam turbine	31
Nuclear	Various reactor types, lowest estimate	32
Solar PV	Polycrystalline silicone	32
Biomass	Short rotation forestry steam turbine	35
Geothermal	80 MW, hot dry rock	38
Nuclear	Various reactor types, highest estimate	65
Natural gas	Various combined cycle turbines	443
Fuel cell	Hydrogen from gas reforming	664
Diesel	Various generator and turbine types	778
Heavy Oil	Various generator and turbine types	778
Coal	Various generator with scrubbing	960
Coal	Various generator without scrubbing	1050

Table A.4. – Lifecycle estimates for electricity generators[4]

in the text are used (31.79 gCO_2e/MWh cumulative) and added to the values obtained from the energy-usage-calculation.

As mentioned earlier, a typical LWR (1.3GWel) uses about 25t of 3.5% enriched uranium per Year. This takes about 210 t of NU to fabricate, using about 120 kgSWU. This amounts to an energy-output of 54.000 MWh_{el}/tNU. All that combined, we can identify four different scenarios (Table A.6): Nuclear power is touted as a carbon dioxide emission free source of electricity, but clearly this statement is not entirely true. While nuclear power is much cleaner than fossil fuels, roughly by the factor 20, it still emits at least three times more CO_2e than a wind turbine or a hydro-plant. The main part of the emissions of nuclear power, come from constructing and decommissioning the plants. The second big contributor is enrichment. But already on those two issues, the complexity of the issue can clearly be seen: i.e. if the Aluminum- or enrichment plant uses nuclear power instead of power generated from fossil fuels the numbers begin to drop significantly.

But even if the numbers are inaccurate, they clearly point to nuclear energy as one of

Purpose	MWh_{el}/tNU
Mining @2%	200
Milling	230
Conversion (mean)	122
Enrichment (Centrifuge, mean)	43
Enrichment (Gaseous Diffusion, mean)	1371
Operation	185
Repository (mean)	1.5
Total (Centrifuge / Diffusion)	782 / 2111

Table A.5. – Energy Usage for Steps in the Nuclear Fuel Cycle

	Diffusion enrichment	Centrifuge enrichment
Energy from fossil sources	65.20	44.10
Energy from renewable sources	32.96	32.22

Table A.6. – End Results in gCO_2e/MWh

the low-emission technologies when it comes to producing power. Nuclear power is roughly as clean as Solar and much more reliable. In fact, its reliability and low emissions make nuclear energy the ideal contributor for the base load of our futures energy-mix.

B. Mathematics of the Model [1]

B.1. Solution of LOGICAL-1 Equations

This appendix outlines of expansion methods for solving the necessary Euler-Lagrange equations for utility optimization described in chapter (2). In general a more detailed explanation can be found in Rethinaraj [Ret05], although the equations differ in several points as there are some new concepts introduced in LOGICAL-1. To make the formulas more readable, the abbreviation for time derivatives as generally known in physics is used as described below.

$$\dot{x} \triangleq \frac{dx}{dt}$$

B.1.1. Model Setup

For better comparison, LOGICAL-1 is supposed to have the capability to emulate the behavior of the RICE-Model [NB00]. This function is not fully implemented yet, there is however already a parameter included in some equations called s to switch between the two modes.

Because the begin of the collection of the data is different for various countries, we define time by $t = \tau - t$; where τ is the time from some uniform reference time (which nevertheless may vary from region to region). To ease the complexity of the notation we will drop the subscripts relating to the region until they are needed when adding the global carbon use. Therefore we maximize the welfare function separately for each region by adjusting total capital $K(t)$, the capital in energy sectors $\beta k(t)K(t)$ by adjusting $k(t)$, and the labor in each energy sector $\beta l(t)L(t)$ by adjusting $l(t)$. The welfare function can be written as:

$$W = \int_{t_1}^{t_2} \frac{L\left((C/L)^{1-\vartheta} - 1\right)}{(1-\vartheta)} e^{-\rho t} dt \tag{B.1}$$

We will apply boundary conditions at $\{t_1, t_2\}$ while the borders may or may not be finite. The consumption can be expressed as

$$C = \frac{1}{\alpha} Y - rK - \dot{K} \tag{B.2}$$

[1] The equations in this chapter were formulated by Clifford Singer and proofed by the author.

Where $\alpha^{-1}Y$ is the GDP with

$$Y = A\left[\left((1-\beta k)\,K\right)^{\alpha}\left((1-\beta l)\,L\right)^{\omega}\right]^{\varphi} w^{\beta} \tag{B.3}$$

Where $\omega = 1-\alpha$ and $\varphi = 1-\beta$. Energy per unit time is

$$w = pB\,(kK)^{\alpha}\,(lL)^{\omega} \tag{B.4}$$

The exogenous productivity factors for GDP and energy satisfy the equations

$$\dot{A}\big/A = \eta_A \nu Z \tag{B.5}$$

$$\dot{B}\big/B = \eta_B \nu Z \tag{B.6}$$

$$\text{where } Z = \frac{e^{-\nu t}}{1+se^{-\nu t}} \tag{B.7}$$

with s as a switch which is set to 0 for RICE and 1 elsewhere. Likewise, we note that

$$\dot{L}\big/L = \lambda \nu y \tag{B.8}$$

$$\text{with} \qquad y = \frac{e^{-R\nu t}}{1+s_L e^{-R\nu t}} = \frac{e^{-R\nu(t-t_2)}}{1+e^{-R\nu(t-t_2)}} \tag{B.9}$$

when $s_L = e^{R\nu t_2}$ and t_2 is a "development transition delay time". y can be expressed in terms of Z using

$$e^{-\nu t} = Z + sZe^{-\nu t} \Leftrightarrow e^{-\nu t} = \frac{Z}{1-sZ} \tag{B.10}$$

to give

$$y = \frac{Z^R}{(1-sZ)^R + s_L Z^R} \tag{B.11}$$

The endogenous, fossil-fuel-dependent production efficiency factor p has a different functional form depending on whether one wants to reproduce the logistic independent variables calibrated vs. historical data (LOGICAL), or the RICE model to lowest order in the energy fraction of productivity, β.

B.1.2. Carbon Balance

The impact of cumulative carbon use on production efficiencies, p_J, can depend on cumulative use summed over any sets of regions, globally (as in RICE) or by region individually (as in LOGICAL). In addition, RICE and LOGICAL formulate p differently. In the latter case,

$$p = 1 + (h-1)f(a) \tag{B.12}$$

where f is a piecewise linear function of u, the cumulative quantity of carbon used for each region. More generally, for LOGICAL-1 we can recover this and other cases with carbon per energy f_J for each region J using

$$p_J = p_J(f_j) \tag{B.13a}$$

$$f_J = f_J \sum_j \Delta_{Jj} u_j \tag{B.13b}$$

where $p_J(f_J)$ are specified functions (e.g., but not necessarily linear) and $\Delta_{Jj} = 1$ if f_J depends on cumulative carbon use in region J and $\Delta_{Jj} = 0$ otherwise.

For RICE, to lowest order in β, we will show below that

$$p_J = \frac{Y_{0J}}{B_J C_\beta} \tag{B.14}$$

$$\text{where} \qquad Y_{0J} = A_J K_J^\alpha L_J^\omega \tag{B.15}$$

and where $C_\beta \left(\sum_J U_j \right)$ is a function of global cumulative use.

The RICE form for p_J follows, by using $k = l \approx 1$ for $\varepsilon \ll 1$ for each region (as shown further below), from comparing the LOGICAL formulas above with the RICE formula:

$$Q = \frac{1}{\alpha} Y_{RICE} \tag{B.16}$$

$$Y_{RICE} = A_{RICE} K^\gamma L^{1-\gamma-\beta} (EZ)^\beta - \beta C_\beta ZE \tag{B.17}$$

where Z is an exogenous carbon intensity of energy production.

$$C_\beta = \frac{1}{\beta} \left[\xi_0 + \xi_4 \left(\sum_j U_j \right)^4 \right] \tag{B.18}$$

With subscripts on C_β omitted and ξ_0, ξ_4 depending on the units chosen. [Looking at the numbers in vBP should show $C_\beta \sim 1$ and in $\beta C_\beta \sim \beta$. Let $\gamma = \alpha\varphi$ and get from B.17

$$Y_{RICE} = A_{RICE} K^{\alpha\varphi} L^{\omega\varphi} (EZ)^\beta - \beta C_\beta ZE \tag{B.19}$$

Expanding to first order in β yields

$$Y_{RICE} = A_{RICE} K^{\alpha\varphi} L^{\omega\varphi} + \beta A_{RICE} K^{\alpha\beta} L^{\omega\varphi} \ln(EZ)^\beta - \beta C_\beta ZE \tag{B.20}$$

In LOGICAL, this operation gives (from B.3)

$$Y = AK^\alpha L^\omega + \beta AK^\alpha L^\omega \ln(pBk) - \beta AK^\alpha L^\omega k \tag{B.21}$$

Where for each region the carbon balance constraint is $\dot{U} = \varepsilon f w$. Comparing B.20 with B.21 we receive

$$EZ = pBk \tag{B.22}$$

and

$$AK^{\alpha}L^{\omega}k = C_{\beta}ZE = C_{\beta}pBk \tag{B.23}$$

so we receive for RICE

$$p = \frac{AK^{\alpha}L^{\omega}}{C_{\beta}B} = \frac{Y_0}{C_{\beta}B} \tag{B.24}$$

B.1.3. Euler-Lagrange Equations

The effects of cumulative carbon depletion on energy production efficiency are taken to be coupled only within certain groups of regions, i.e. because transport of energy occurs primarily, to an adequate approximation, only within those. Therefore we group regions (index J) into suitably chosen metaregions with index M. [When breaking down energy into fluids and non-fluids for the LOGICAL-2 generalization, it could be useful to introduce a model of international trade in fluid fuels.]

For each region J in metaregion M, let

$$\mathcal{L}_J = \underbrace{\frac{1}{1-\vartheta} L_J^{\vartheta} C_J^{1-\vartheta} e^{-\rho t}}_{\text{Integrand of the Welfare - Function}} + \overbrace{\beta_J L_J^{\vartheta} C_J^{-\vartheta} e^{-\rho t} \kappa_J \left(\left(\sum_{j \in M} \varepsilon_j f_j w_j \right) - \dot{U}_M \right)}^{\text{Constraint}} \tag{B.25}$$

For each metaregion with N_M regions, there are $2N_M + 1$ coupled Euler-Lagrange equations for U_M and $\{k_J, l_J, K_J, \kappa_J\}$:

$$0 = \frac{\partial \mathcal{L}_J}{\partial k_J} = \frac{\partial \mathcal{L}_J}{\partial K_J} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}_J}{\partial \dot{K}_J}\right) = \frac{\partial \mathcal{L}}{\partial U_M} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{U}_M}\right) \tag{B.26}$$

$$\text{and} \qquad \dot{U}_M = \sum_{j \in M} \varepsilon_j f_j w_j \tag{B.27}$$

B.1.4. *k=l*

We will first show that $k_J = l_J$ for each J. From B.27 we get

$$0 = L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \frac{\partial \mathcal{L}_J}{\partial k_J} = L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \frac{\partial \mathcal{L}_J}{\partial l_J} \tag{B.28}$$

$$\Rightarrow \qquad = \frac{\partial C_J}{\partial k_J} + \beta_J \kappa_J \frac{\partial w_J}{\partial k_J} \varepsilon_J f_J = \frac{\partial C_J}{\partial l_J} + \beta_J \kappa_J \frac{\partial w_J}{\partial l_J} \varepsilon_J f_J \tag{B.29}$$

$$\Rightarrow \qquad = \frac{1}{\alpha} \frac{\partial Y_J}{\partial k_J} + \beta_J \kappa_J \frac{\partial w_J}{\partial k_J} \varepsilon_J f_J = \frac{1}{\alpha} \frac{\partial Y_J}{\partial l_J} + \beta_J \kappa_J \frac{\partial w_J}{\partial l_J} \varepsilon_J f_J \tag{B.30}$$

or

$$0 = \frac{1}{\alpha} Y_J \frac{\partial \ln Y_J}{\partial k_J} + \beta_J \kappa_J w_J \frac{\partial \ln w_J}{\partial k_J} \varepsilon_J f_J$$
$$0 = \frac{1}{\alpha} Y_J \frac{\partial \ln Y_J}{\partial l_J} + \beta_J \kappa_J w_J \frac{\partial \ln w_J}{\partial l_J} \varepsilon_J f_J \tag{B.31}$$

Thus

$$0 = \frac{1}{\alpha} \left[\frac{1}{\alpha} Y_J \left(\frac{-\alpha \beta_J \varphi_J}{1 - \beta k_J} + \frac{\alpha \beta_J}{k_J} \right) + \frac{\beta_J \varepsilon_J \kappa_J \alpha w_J}{k_J} \right]$$
$$0 = \frac{1}{\omega} \left[\frac{1}{\alpha} Y_J \left(\frac{-\omega \beta_J \varphi_J}{1 - \beta l_J} + \frac{\omega \beta_J}{l_J} \right) + \frac{\beta_J \varepsilon_J \kappa_J \omega w_J}{l_J} \right] \tag{B.32}$$

This shows, after the divisions by α and ω respectively, that the equations for k_J and l_J are identical. Therefore

$$k_J = l_J \tag{B.33}$$

Thus

$$Y_J = A_J K_J^\alpha L_J^\omega \left(1 - \beta_J k_J\right)^{\varphi_J} \left(p_J B_J k_J\right)^{\beta_J} \tag{B.34}$$

or

$$Y_J = D_J K_J^\alpha L_J^\omega \left(1 - \beta_J k_J\right)^{\varphi_J} \left(p_J k_J\right)^{\beta_J} \tag{B.35}$$

$$\text{with} \quad D_J = A_J B_J^{\beta_J} \quad \text{and} \quad \frac{\dot{D}}{D} = \frac{\dot{A}}{A} + \beta \frac{\dot{B}}{B} = \left(\eta_A + \beta \eta_B\right) \nu z \tag{B.36}$$

Let

$$\eta_J = \eta_{AJ} + \beta_J \eta_{BJ} \tag{B.37}$$
$$\Rightarrow \qquad \frac{\dot{D}_J}{D_J} = \eta_J \nu z_J \tag{B.38}$$

B.1.5. Expansion in ε_J

Next, B.32 is multiplied by $\beta_J^{-1} \left(1 - \beta_J k_J\right) k_J$ to give

$$0 = \frac{1}{\alpha} \left[Y_J \left(1 - \beta k_J - k_J + \beta k_J\right) + \varepsilon_J \kappa_J w_J \left(1 - \beta k_J\right) \right] \tag{B.39}$$
$$= \frac{1}{\alpha} \left[Y_J \left(1 - k_J\right) + \varepsilon_J \kappa_J w_J \left(1 - \beta k_J\right) \right] \tag{B.40}$$

Which results in $k_J \approx 1$ when expanding ε_J in lowest order. The Lagrange multipliers κ_J embody the foreshadowing of future cumulative carbon use on energy production efficiency. We recall from B.25 that

$$\mathcal{L}_J = \frac{1}{1 - \vartheta} L_J^\vartheta C_J^{1-\vartheta} e^{-\rho t} + \beta_J L_J^\vartheta C_J^{-\vartheta} e^{-\rho t} \kappa_J \left(\left(\sum_{J' \in M} \varepsilon_{J'} f_{J'} w_{J'} \right) - \dot{U}_M \right) \tag{B.41}$$

Using the result of B.40 and plugging it into B.4 gives for the above

$$w = p_J B K_J^{\alpha} L_J^{\omega} \tag{B.42}$$

This is used in B.27 and we are multiplying the result for reasons of convenience by $\beta_J^{-1} L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t}$ (for C_J see B.2), which yields

$$0 = \beta_J^{-1} L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \left(\frac{\partial \mathcal{L}}{\partial U_M} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{U}_M} \right) \right) \tag{B.43}$$

We compute the left derivative as follows (Note: $f = f(U_M)$, $w = w(U_M)$)

$$\beta_J^{-1} L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \frac{\partial \mathcal{L}}{\partial U_M} = \beta_J^{-1} \frac{\partial C_J}{\partial U_M} + C_J^{\vartheta} \frac{\partial \left(C_J^{-\vartheta} \kappa_J \left(\left(\sum_{j \in M} \varepsilon_j f_j w_j \right) - \dot{u}_M \right) \right)}{\partial U_M} \tag{B.44}$$

$$= \beta_J^{-1} \alpha^{-1} Y_J \frac{\partial \ln Y_J}{\partial U_M} - 0 + \kappa_J \left(\sum_{j \in M} \varepsilon_j \frac{\partial \ln (f_j w_j)}{\partial U_M} \right) \tag{B.45}$$

And the right derivative from B.43:

$$-\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{U}_M} \right) = L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \frac{d \left(L_J^{\vartheta} C_J^{-\vartheta} e^{-\rho t} \kappa_J \right)}{dt} = \kappa_J \left(\vartheta \frac{\dot{L}_J}{L_J} - \vartheta \frac{\dot{C}_J}{C_J} - \rho \right) + \dot{\kappa}_J \tag{B.46}$$

Using the two previous results in B.43 we get

$$0 = \beta_J^{-1} \alpha^{-1} Y_J \frac{\partial \ln Y_J}{\partial U_M} + \kappa_J \left(\sum_{j \in M} \varepsilon_{j'} \frac{\partial \ln (f_j w_j)}{\partial U_M} \right) + \kappa_J \left(\vartheta \frac{\dot{L}_J}{L_J} - \vartheta \frac{\dot{C}_J}{C_J} - \rho \right) + \dot{\kappa}_J \tag{B.47}$$

We note that

$$\frac{\partial \ln Y_J}{\partial U_M} = \beta_J \frac{\partial \ln p_J}{\partial U_M} \tag{B.48}$$

To lowest order in ε_J:

$$0 = \alpha^{-1} Y_J \frac{\partial \ln p_J}{\partial y} + \kappa_J \left(\vartheta \frac{\dot{L}_J}{L_J} - \vartheta \frac{\dot{C}_J}{C_J} - \rho \right) + \dot{\kappa}_J \tag{B.49}$$

$$k_J \approx 1 \tag{B.50}$$

$$\dot{u}_M = \sum_{j \in M} (\varepsilon_j f_j w_j) \tag{B.51}$$

By also expanding in β_J, in B.42 the K_J can be eliminated. The coupled equations for the carbon balance U_J can be solved to get $U_J(t)$ and hence $U_M = \sum_j U_j$. The result for the U_J

in each metaregion M can then be inserted into the equations for κ_J, which can be reduced to quadrature using an integrating factor. If f_j and p_j are simple enough expressions of $\{U_J, Z_J\}$, the carbon balance equations may integrate analytically. Alternatively, given $K_J(t)$ to lowest order in β_J, the equations for κ_J and U_J can together be numerically integrated.

B.1.6. Expansion in β_J

The key to expansion in β_J is the equation (with B.25, B.2, B.3, B.4)

$$0 = L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \left(\frac{\partial \mathcal{L}}{\partial K_J} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{K}_J} \right) \right) \tag{B.52}$$

For the left derivative we get

$$L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \frac{\partial \mathcal{L}}{\partial K_J} = \underbrace{\frac{\partial C_J}{\partial K_J}}_{\frac{Y_J}{K_J} - r} + \beta_J \kappa_J \varepsilon_J f_J \alpha \frac{w_J}{K_J} \tag{B.53}$$

And for the right one

$$\begin{aligned} -L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{K}_J} \right) &= L_J^{-\vartheta} C_J^{\vartheta} e^{\rho t} \frac{d}{dt} \left(L_J^{\vartheta} C_J^{-\vartheta} e^{-\rho t} \right) \\ &= \vartheta \frac{\dot{L}}{L} - \vartheta \frac{\dot{C}}{C} - \rho \end{aligned} \tag{B.54}$$

Thus, by choosing units of time so that $r + \rho = 1$, we receive

$$0 = \frac{Y_J}{K_J} + \beta_J \kappa_J \varepsilon_J f_J \alpha \frac{w_J}{K_J} + \vartheta \frac{\dot{L}}{L} - \vartheta \frac{\dot{C}}{C} - 1 \tag{B.55}$$

We define

$$\begin{aligned} F_J^{-1} &= \frac{Y_J}{K_J} \\ \left(\alpha^{-1} - r \right) G_J^{-1} &= \frac{C_J}{K_J} \end{aligned} \tag{B.56}$$

$$\begin{aligned} \text{Then} \qquad \frac{\dot{K}_J}{K_J} - \frac{\dot{F}_J}{F_J} &= \frac{\dot{Y}_J}{Y_J} \\ \text{and} \qquad \frac{\dot{K}_J}{K_J} - \frac{\dot{G}_J}{G_J} &= \frac{\dot{C}_J}{C_J} \end{aligned} \tag{B.57}$$

Now we know from B.36 that

$$\frac{\dot{Y}_J}{Y_J} = \frac{\dot{D}_J}{D_J} + \alpha \frac{\dot{K}_J}{K_J} + \omega \frac{\dot{L}_J}{L_J} + \beta_J \varphi_J \frac{\dot{k}_J}{1 - \beta_J k_J} + \beta_J \frac{\dot{k}_J}{k_J} + \beta_J \frac{\dot{p}_J}{p_J} \tag{B.58}$$

And since

$$
\begin{aligned}
-\frac{\varphi_J}{1-\beta_J k_J} + \frac{1}{k_J} &= -\frac{k_J\left(1-\beta_J\right)}{k_J\left(1-\beta_J k_J\right)} + \frac{1-\beta_J k_J}{k_J\left(1-\beta_J k_J\right)} \\
&= \frac{-k_J+\beta_J k_J}{k_J\left(1-\beta_J k_J\right)} + \frac{1-\beta_J k_J}{k_J\left(1-\beta_J k_J\right)} \\
&= \frac{1-k_J}{k_J\left(1-\beta_J k_J\right)}
\end{aligned}
\tag{B.59}
$$

We get, solving for $\frac{\dot{K}_J}{K_J}$ (and remembering that $\omega = 1-\alpha$)

$$
\frac{\dot{K}_J}{K_J} = \omega^{-1}\frac{\dot{D}_J}{D_J} + \omega^{-1}\frac{\dot{F}_J}{F_J} + \frac{\dot{L}_J}{L_J} + \beta_J\omega^{-1}\frac{1-k_J}{\left(1-\beta_J k_J\right)}\frac{\dot{k}_J}{k_J} + \beta_J\omega^{-1}\frac{\dot{p}_J}{p_J}
\tag{B.60}
$$

From B.55 we get with B.56, B.57 and the above

$$
\begin{aligned}
0 &= F_J^{-1} - 1 + \vartheta\frac{\dot{L}_J}{L_J} - \vartheta\frac{\dot{G}_J}{G_J} - \vartheta\frac{\dot{K}_J}{K_J} + \beta_J\kappa_J\varepsilon_J f_J\alpha\frac{w_J}{K_J} \\
&= F_J^{-1} - 1 + \vartheta\frac{\dot{L}_J}{L_J} - \vartheta\frac{\dot{G}_J}{G_J} - \\
&- \frac{\vartheta}{\omega}\left(\frac{\dot{D}_J}{D_J} + \frac{\dot{F}_J}{F_J} + \omega\frac{\dot{L}_J}{L_J} + \beta_J\frac{1-k_J}{\left(1-\beta_J k_J\right)}\frac{\dot{k}_J}{k_J} + \beta_J\frac{\dot{p}_J}{p_J}\right) + \beta_J\kappa_J\varepsilon_J f_J\alpha\frac{w_J}{K_J} \\
&= F_J^{-1} - 1 - \vartheta\frac{\dot{G}_J}{G_J} - \frac{\vartheta}{\omega}\frac{\dot{D}_J}{D_J} - \frac{\vartheta}{\omega}\frac{\dot{F}_J}{F_J} + \beta_J\left(-\frac{\vartheta}{\omega}\frac{1-k_J}{\left(1-\beta_J k_J\right)}\frac{\dot{k}_J}{k_J} - \frac{\vartheta}{\omega}\frac{\dot{p}_J}{p_J} + \kappa_J\varepsilon_J f_J\alpha\frac{w_J}{K_J}\right)
\end{aligned}
\tag{B.61}
$$

And from B.2

$$
C_J = \frac{1}{\alpha}Y_J - rK_J - \dot{K}_J \quad \Rightarrow \quad 0 = \frac{C_J}{K_J} - \frac{1}{\alpha}\frac{Y_J}{K_J} + r + \frac{\dot{K}_J}{K_J}
\tag{B.62}
$$

We get with B.60

$$
0 = \frac{C_J}{K_J} - \frac{1}{\alpha}\frac{Y_J}{K_J} + r + \omega^{-1}\frac{\dot{D}_J}{D_J} + \omega^{-1}\frac{\dot{F}_J}{F_J} + \frac{\dot{L}_J}{L_J} + \beta_J\omega^{-1}\left(\frac{1-k_J}{\left(1-\beta_J k_J\right)}\frac{\dot{k}_J}{k_J} + \frac{\dot{p}_J}{p_J}\right)
\tag{B.63}
$$

So the correction terms of the following uncoupled (because of that we will drop the index J) equations for each region are all of the order $\beta\varepsilon \ll \beta$ and $\beta\varepsilon \ll \varepsilon$ only.

B.2. Global and Regional Parameters

Parameter	Value	Meaning
α	0.325	capital share
γ	0.0449	CO_2 relaxation coefficient, [yr^{-1}]
h	2	ratio between the cost in capital and labor of all non-fossil vs. all-fossil energy production
κ	0.47	converts Gtonne elemental carbon to ppm atmospheric CO2 [ppm/Gt]
λ	0.0979	opacity effect coefficient [°C/yr]
r	0.107	depreciation rate [yr^{-1}]
σ	0.0171	thermal relaxation coefficient, [yr^{-1}]
ϑ	1.34	intertemporal substitutability of consumption
$\bar{t}$	7.76	capitalization time [yr]

Table B.1. – Global parameters

Region	δ	$\epsilon = \bar{m}\bar{t}\bar{E}$	$\bar{E}$ $\left[\frac{EJ}{yr}\right]$	$\bar{m}$ $\left[\frac{1}{EJ}\right]$	ζ	η
ChinaPlus	2.46	.029	146	.000026	6.14	3.39
SAFTAPlus	.37	0	170	0	2.17	.87
EUPlus	2.71	.067	115	.000075	3.21	4.27
USAPlus	.33	.031	237	.000017	1.56	1.73
Pacifica	1.28	.033	26	.000164	2.92	2.43
LatinPlus	.28	.078	100	.000100	2.00	.66
SubSahara	.03	.134	37	.000466	1.33	.08
MidEastPlus	.18	.039	44	.000114	.84	.41
ASEANPlus	.63	0	48	0	2.91	1.51

Table B.2. – Region dependent parameters. All parameters are used with Subscript J to indicate regional dependence.

B.3. Fuel Fractions Model

$$
\begin{aligned}
z &= 1 - a \\
a &= 1/(1 + e^{-\nu t}) \\
u_c &= a\psi f p F_1^{\alpha/\omega} \\
p &= 1 + (h-1)f \\
f &= f_u - m_k u_c \\
F_1 &= ((1 + a\delta)/(1+\delta))^{\alpha/\omega}
\end{aligned}
$$

Balances:

new bio:

$$\frac{dx_1}{du} = c_{11}\left(1 - e^{-\frac{x_1}{c_{12}}}\right) \quad \Rightarrow \quad \frac{dx_1}{da} = \left(\frac{du}{da}\right)^{-1} c_{11}\left(1 - e^{-\frac{x1}{c_{12}}}\right) \tag{B.64}$$

fluid burn:

$$\frac{dx_2}{du_1} = c_{21}\left(1 - e^{-\frac{x_2}{c_{22}}}\right) - \frac{x_2}{c_{23}} \tag{B.65}$$

coal:

$$\begin{aligned} dx_3/du_2 &= dx_3/da \;/\; (du/da - dx_1/da - dx_2/da - dx_3/da) \\ &= \begin{cases} 1 & \text{for } x_3 < x_4 \\ c_{31}(1 - e^{-x_3/c_{32}}) & \text{for } x_s < x_3 < x_t \\ c_{31}(1 - e^{-x_t/c_{32}}) & \text{for } x_t < x_3 < x_u \\ c_{31}(1 - e^{-x_3/c_{33}})(1 - \alpha_f/c_{32})/(1 - x_u/c_{33}) & \text{for } x_u < x_3 \end{cases} \end{aligned} \tag{B.66}$$

non-water:

$$\begin{aligned} dx_4/du_3 &= (1 - dx_4/da) \;/\; (du/da - dx_1/da - dx_2/da - dx_3/da) \\ &= c_{41}(1 - e^{-u_4/c_{42}}) + c_3 dx_4/dt \qquad \text{(B.67)} \\ &= c_{41}(1 - e^{-(u_3 - x_4)/c_{42}}) + \nu\, z\, a\, c_{43}\, dx_4/da \\ dx_4/da &= 1 - [c_{41}(1 - e^{-(u_3 - x_4)/c_{42}}) + \nu\, z\, a\, c_{43}\, dx_4/da](du/da - dx_1/da - dx_2/da - dx_3/da) \end{aligned} \tag{B.68}$$

new electric:

$$dx_5/du_4 = c_{51}(1 - e^{-x_5/c_{52}}) + c_{53}P_u \qquad \text{where } P_u = P_0 + (x_6/u_s)^{1/n_6} \tag{B.69}$$

$$dx_5/da = \left(\tfrac{du}{da} - \tfrac{dx_1}{da} - \tfrac{dx_2}{da} - \tfrac{dx_3}{da} - \tfrac{dx_4}{da}\right)[c_{51}(1 - e^{-x_5/c_{52}}) + c_{53}P_0 + (x_6/u_s)^{1/n_6}] \tag{B.70}$$

otherU:

$$dx_6/du_5 = (1 - P_u/P_{max})[1 - c_{61}(1 - e^{-x_6/c_{62}}) - c_{63}P_u] \tag{B.71}$$

$$= (dx_6/da)/\left(\tfrac{du}{da} - \tfrac{dx_1}{da} - \tfrac{dx_2}{da} - \tfrac{dx_3}{da} - \tfrac{dx_4}{da} - \tfrac{dx_5}{da}\right) \tag{B.72}$$

Algebra for otherU

$$\dot{u_6} = \dot{u_5}(1 - P_u/P_{max}) - \dot{x_6} \qquad \text{where } (1 - P_u/P_{max}) \triangleq \frac{\text{energy out}}{\text{U-235}} \text{ at min burnup} \tag{B.73}$$

old bio:

$$du_7/dx_2 = dx_2/du_1 - c_{71}(1 - e^{-x_7/c_{72}}) + c_{73}(dx_7/dt)/c_{ret} \tag{B.74}$$

$$\Rightarrow\ du_7/da = (du/da - dx_1/da)\left(\frac{dx_2/da}{\frac{du}{da} - \frac{dx_1}{da}} - c_{71}(1 - e^{-x_7/c_{72}}) + c_{73}\,\nu\,z\,a\,dx_7/da\,/c_{ret}\right) \tag{B.75}$$

$$= \frac{dx_2}{da} - \frac{dx_7}{da} \tag{B.76}$$

natural gas, e.g. for $u_g = 3$:

$$(c_{83}^{-1}\ dx_8/du_7)^{n_g} = (u_7 - x_6)/c_{82} - x_8/c_{81} \tag{B.77}$$

$$dx_8/du_7 = c_{83}((u_7 - x_6)/c_{82} - x_8/c_{81})^{1/n_g} \tag{B.78}$$

$$= (dx_8/da)/(du_7/da) \tag{B.79}$$

$$= (dx_8/da)/(dx_2/da - dx_7/da) \tag{B.80}$$

solar:

$$dx_9/dx_5 = f_{90} - c_{91}e^{-x_9/c_{92}} - (f_{90} - c_{91} - f_{91})e^{-u_9/c_{93}} \tag{B.81}$$

$$\Rightarrow\quad dx_9/da = (dx_5/da)[f_{90} - c_{91}e^{-x_9/c_{92}} - (f_{90} - c_{91} - f_{91})e^{-(x_5-x_9)}] \tag{B.82}$$

B.4. Uranium Price vs. Cumulative Use

Denote cumulative uranium extracted up to a cost c by v where, to a good approximation with $s = 2$, we have $dq/dr \propto r$. Then

$$v/v_{\text{norm}} = \int_0^{c/b} (c - br)^m, \ell c_0^m]rdr \tag{B.83}$$

The lower limit of the integral is set to 0 for mathematical convenience, since for the parameters of interest here this makes negligible difference from using the lowest naturally occurring values of $r_1 \sim 5$. Let $y = br/c$. For $c \le c_m$ with $c_m = c_0\ell^{1/m}$, the result is

$$v/v_{\text{norm}} = c^m \int_0^1 \text{Min}[(1-y)^m(c/b)^2 ydy = c^{m+2}/(b^2(m+1)(m+2)) \tag{B.84}$$

i.e. $v \propto c^{m+2}$ For $c \ge c_m$, the result is

$$v/v_{\text{norm}} = \int_0^{r_m} d_m r dr + \int_{r_m}^1 (c - br)^m r dr \tag{B.85}$$

where $r_m = (c - c_m)/b$. Letting $z = 1 - rb/c$ gives

$$v/v_{\text{norm}} = d_m^2 r_m^2/2 + c^m(c/b)^2 \int_0^{c_m/c} z^m(1-z)dz \tag{B.86}$$

Defining v_{norm} such that $v_m/v_{\text{norm}} = c_m^m(c_m/b)^2/((m+1)(m+2))$, substituting in $d_m = c_m^m$ and $r_m = (c - c_m)/b$, and dividing by $(m+1)(m+2)(v_m/v_1)/2$ gives

$$2(v/v_m)/((m+1)(m+2)) = ((c/c_m) - 1)^2 + 2(c/c_m)/(m+1) - 2/(m+2) \tag{B.87}$$

Solving this quadratic equation for c/c_m gives Eq. (4.7) in Section (4.6) above.

Bibliography

[BAKKB08] P. Baer, T. Athanasiou, S. Kartha, and E. Kemp-Benedict. *The greenhouse development rights framework: The right to development in a climate constrained world*. Heinrich Böll Foundation (Berlin), Christian Aid, EcoEquity and the Stockholm Environment Institute, Nov 2008.

[BF06] Matthew Bunn and Steve Fetter. Uranium resources for the long-term. Technical report, Harvard University, Aug 2006. `http://phys4.harvard.edu/~wilson/energypmp/2006_Fetter_and_Bunn.doc`, Accessed 05/26/2010.

[BFHvdZ03] Matthew Bunn, Steve Fetter, John P. Holdren, and Bob van der Zwaan. The economics of reprocessing vs. direct disposal of spent nuclear fuel. Technical report, J. F. Kennedy School of Government, Harvard University, 2003.

[BHP04] BHP Billiton. Olympic dam operations environmental data. `http://hsecreport.bhpbilliton.com/wmc/2004/performance/odo/data/index.htm`, Accessed 11/09/2009, 2004.

[Bla10] T. J. Blasing. Recent greenhouse gas concentrations. `http://cdiac.ornl.gov/pns/current_ghg.html`, Accessed 09/10/2010, Sep 2010. Carbon Dioxide Information Analysis Center, Oak Ridge, TN, USA.

[BS04] Robert J. Barro and Xavier Sala-i-Martin. *Economic Growth*. The MIT Press, Cambridge, MA, USA, 2004.

[CCD+09] S. Chakravarty, A. Chikkatur, H. De Coninck, S. Pacala, R. Socolow, and M. Tavoni. Sharing global CO2 emission reductions among one billion high emitters. *Proceedings of the National Academy of Sciences*, 106(29):11884–11888, 2009.

[CH03a] Ian Castles and David Henderson. Economics, emissions scenarios and the work of the IPCC. *Energy & Environment*, 14(4):415–435, July 2003.

[CH03b] Ian Castles and David Henderson. The IPCC emission scenarios: An economic-statistical critique. *Energy & Environment*, 14(2-3):159–185, May 2003.

[Chi07] China Daily Newspaper. Most people free to have more child. `http://www2.chinadaily.com.cn/china/2007-07/11/content_5432238.htm`, Accessed 09/10/2010, Jul 2007.

[Daw80] Robyn M. Dawes. Social dilemmas. *Annual Reviews in Psychology*, 31, 1980.

[DC09] Guillermo D. Del Cul and Emory D. Collins. Near complete recycling: A path forward for sustainable nuclear energy. Oak Ridge National Laboratory, Oak Ridge, Tennessee, 37831-6224, delculgd@ornl.gov, 2009.

[Dit09] Michael Dittmar. The future of nuclear energy: Facts and fiction. chapter iii: How (un)reliable are the red book uranium resource data? Institute of Particle Physics, ETH, Zürich, Switzerland, September 2009.

[DM80] K. S. Deffeyes and I. D. MacGregor. World uranium resources. *Scientific American*, 242, Jan 1980.

[Ehr68] Paul R. Ehrlich. *The Population Bomb*. Ballantine Books, USA, 1968.

[F+74] S. David Freeman et al. A time to choose: America's energy future. Technical report, Ford Foundation, 1974.

[FKA08] V. M. Fthenakis, H. C. Kim, and M. Alsema. Emissions from photovoltaic life. *Environmental Science and Technology*, 42, 2008.

[GBU02] Luc Gagnon, Camille Belanger, and Yohji Uchiyama. Lifecycle assessment of electricity generation options: the status of research in year 2001. *Energy Policy*, 30, 2002.

[HDW06] Niklas Höhne, Michel Den Elzen, and Martin Weiss. Common but differentiated convergence(CDC): A new conceptual approach to long-term climate policy. *Climate Policy*, 6(2):181–199, 2006.

[Hor99] Ian Hore-Lacy. *Nuclear Electricity*. Uranium information Centre Ltd., 5th edition, Feb 1999. available from `www.uic.com.au`.

[HS95] Douglas Holtz-Eakin and Thomas M. Selden. Stoking the fires? CO_2 emissions and economic growth. *Journal of Public Economics*, 57:85–101, 1995.

[HS06] Joel D. Haines and Nawaz M. Sharif. A framework for managing the sophistication of the components of technology for global competition. *Competitiveness Review: An International Business Journal incorporating Journal of Global Competitiveness*, 16(2):106–121, 2006.

[Jon95] Charles I. Jones. Time series tests of endogenous growth models. *The Quarterly Journal of Economics*, 110(2):pp. 495–525, 1995.

[JPS+01] F. Joos, I. C. Prentice, S. Sitch, R. Meyer, G. Hooss, G. K. Plattner, S. Gerber, and K. Hasselmann. Global warming feedbacks on terrestrial carbon uptake under the Intergovernmental Panel on Climate Change (IPCC) emission scenarios. *Global Biogeochemical Cycles*, 15(4):891–908, 2001.

[Key36] John Maynard Keynes. *The General Theory of Employment, Interest and Money*. Palgrave Macmillan, Basingstoke, reprinted edition, 2007/1936.

[KT10] Andrey V. Korotayev and Sergey V. Tsirel. A spectral analysis of world gdp dynamics: Kondratieff waves, kuznets swings, juglar and kitchin cycles in global economic development, and the 2008/2009 economic crisis. *Structure and Dynamics*, 4(1), 2010.

[L+83] Georg Löser et al. *Grundsatzprogramm Energie, BUND Energiekonzept*, volume 22 of *BUND-information*. Bund für Umwelt und Naturschutz Deutschland e.V., 1983.

[Len08] Manfred Lenzen. Life cycle energy and greenhouse gas emissions of nuclear energy: A review. *Energy Conversion and Management*, 49(8), 2008.

[Mac04] Colin MacDonald. Uranium: Sustainable resource or limit to growth? In *Proceedings of World Nuclear Association Annual Symposium 3-5 September 2003 - London*, 2004.

[Mad06] Angus Maddison. *The World Economy*, volume 2: Historical Statistics. OECD Publishing, Sep 2006.

[Mar71] Karl Marx. *Das Kapital*, volume 2. Verlag Dietz, Berlin, 4th edition, 1971. Reprint of 2nd Edition, ed. Friedrich Engels, Hamburg, 1893.

[Mey01] Aubrey Meyer. *Contraction & Convergence: The Global Solution to Climate Change*, volume 5 of *Schumacher Briefings*. Green Books, Feb 2001.

[Mey07] Aubrey Meyer. Contraction & convergence: The proportionate response to climate change. `http://www.gci.org.uk/Climate%20action%20book_2007_C&C_.pdf`, Accessed 09/26/2010, 2007.

[MR92] Alan S. Manne and Richard G. Richels. *Buying Greenhouse Insurance: The Economic Costs of CO2 Emissions Limits*. he MIT Press, Cambridge, MA, USA, 1992.

[MR04] Alan S. Manne and Richard G. Richels. MERGE: An integrated assessment model for global climate change. `http://www.stanford.edu/group/MERGE/GERAD1.pdf`, Accessed 06/04/2010, June 2004.

[MRE05] Alan S. Manne, Richard G. Richels, and James A. Edmonds. Market exchange rates or purchasing power parity: Does the choice make a difference to the climate debate? *Climatic Change*, 71(1-2):1–8, July 2005.

[MS95] S. Messner and M. Strubegger. *User's Guide for MESSAGE III*. International Institute for Applied Systems Analysis, Laxenburg, Austria, 1995.

http://www.iiasa.ac.at/Research/ECS/docs/MESSAGE_man018.pdf, Accessed 06/04/2010.

[MS00] S. Messner and L. Schrattenholzer. MESSAGE-MACRO: Linking an energy supply model with a macroeconomic model and solving it interactively. *Energy*, 25:267–282, 2000.

[MSRK10] D. Messner, J. Schellnhuber, S. Rahmstorf, and D. Klingenfeld. The budget approach: A framework for a global transformation toward a low-carbon economy. *Journal of Renewable and Sustainable Energy*, 2:031003, 2010.

[Nak00] Nebojsa Nakicenovic. Global greenhouse gas emissions scenarios: Integrated modeling approaches. *Technological Forecasting and Social Change*, 63(2-3), 2000.

[NB00] William D. Nordhaus and J. Boyer. *Warming the World: Economic Models of Global Warming*. MIT Press, Cambridge, 2000.

[NI04] NEA/OECD and IAEA. *Uranium 2003: Resources, Production and Demand*. OECD/NEA, 2004.

[NI06] NEA/OECD and IAEA. *Forty Years of Uranium Resources, Production and Demand in Perspective*. OECD, 2006.

[NI08] NEA/OECD and IAEA. *Uranium 2007: Resources, Production and Demand*. OECD/NEA, 2008.

[Nor04] William D. Nordhaus. *Managing the Global Commons: The Economics of Climate Change*. MIT Press, Cambridge, 2004.

[NS00] Nebojsa Nakicenovic and Rob Swart, editors. *Special Report on Emission Scenarios: a special report of Working Group III of the Intergovernmental Panel on Climate Change*. Cambridge University Press, Cambridge, England, 2000. http://www.ipcc.ch/ipccreports/sres/emission/index.php?idp=0, Accessed 05/26/2010.

[PB02] Robert Price and Jean René Blaise. Nuclear fuel resources: Enough to last? *NEA News*, 20(2), 2002.

[Peh06] Marin Pehnt. Dynamic lifecycle assessment of renewable energy technologies. *Renewable Energy*, 2006.

[PHSB+99] G. Petschel-Held, H.J. Schellnhuber, T. Bruckner, F.L. Toth, and K. Hasselmann. The tolerable windows approach: theoretical and methodological foundations. *Climatic Change*, 41(3):303–331, 1999.

[RCC+07] S. Rahmstorf, A. Cazenave, J.A. Church, J.E. Hansen, R.F. Keeling, D.E. Parker, and R.C.J. Somerville. Recent climate observations compared to projections. *Science*, 316(5825):709, 2007.

[Ret05] T. S. Gopi Rethinaraj. *Modeling Global and Regional Energy Futures*. PhD thesis, University of Illinois at Urbana-Champaign, 2005. `http://acdis.illinois.edu/assets/docs/358/ModelingGlobalandRegionalEnergyFutures.pdf`, Accessed 05/26/2010.

[RG92] L. Rosen and R. Glasser, editors. *Climate change and energy policy*. Los Alamos National Laboratory, American Institute of Physics, New York, NY, USA, 1992. International Conference on Global Climate Change: It's Mitigation through Improved Production and Use of Energy, October 21-24, 1991.

[Rom90] P. M. Romer. Endogenous technological change. *Journal of political Economy*, 98(S5), 1990.

[RPR75] R. M. Rotty, A. M. Perry, and D. B. Reister. Net energy from nuclear power. Technical report, Institute for Energy Analysis, 1975.

[RR00a] K. Riahi and R. A. Roehrl. Energy technology strategies for carbon dioxide mitigation and sustainable development. *Environmental Economics and Policy Studies*, 3(2):89–124, 2000.

[RR00b] K. Riahi and R. A. Roehrl. Greenhouse gas emissions in a dynamics-as-usual scenarios of economic and energy development. *Technological Forecasting & Social Change*, 63:175–206, 2000.

[Sch76] Joseph Alois Schumpeter. *Capitalism, Socialism and Democracy*. Harper and Brothers, New York, 5th ed.: george allen and unwin, london edition, 1976.

[Sch82] Joseph Alois Schumpeter. *Business cycles*, volume 1. Porcupine Press, Philadelphia, reprinted edition, 1982. Orig. publ.: 1. ed., New York: McGraw-Hill, 1939.

[Sch06] Joseph Alois Schumpeter. *Theorie der wirtschaftlichen Entwicklung*. Duncker&Humblot, Berlin, reprint of 1912 1st edition, 2006.

[Sch09] Jürgen Scheffran. The gathering storm: Is climate change a security threat? *Security Index*, 87(2):21–31, 2009. `http://acdis.illinois.edu/publications/articles.html`, Accessed 09/20/2010.

[SHM+07] J. Stroeve, M.M. Holland, W. Meier, T. Scambos, and M. Serreze. Arctic sea ice decline: Faster than forecast. *Geophysical Research Letters*, 34(9):9501, 2007.

[Sim95] Julian L. Simon. *The State of Humanity.* Blackwell, Malden, MA, USA, 1995.

[Sin01] Clifford E. Singer. An analytic uranium sources model. In *Assessment of Uranium Deposit Types and Resources - A Worldwide Perspective (Vienna, 10-13 June, 1997)*, IAEA-TECDOC-1258, page 27. NEA/OECD and IAEA, Dec 2001.

[Smi04] Adam Smith. *An Inquiry into the Nature and Causes of the Wealth of Nations.* Methuen&Co., Ltd., London, 5th edition, 1904. First Pub. Year 1776.

[SML+09] Hans-Joachim Schellnhuber, Dirk Messner, Claus Leggewie, Reinhold Leinfelder, Nebojsa Nakicenovic, Stefan Rahmstorf, Sabine Schlacke, Jürgen Schmid, and Renate Schubert. Solving the climate dilemma: The budget approach. Technical report, German Advisory Council on Global Change (WBGU), 2009.

[Sol56] Robert M. Solow. A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 70(1), 1956.

[Sov08] Benjamin K. Sovacool. Valuing the greenhouse gas emissions from nuclear power: A critical survey. *Energy Policy*, 36, 2008.

[SPKF09] Susan Solomon, Gian-Kasperer Plattner, Reto Knutti, and Pierre Friedlingstein. Irreversible climate change due to carbon dioxide emissions. *PNAS*, 106(6), 2009.

[SQ+07] S. Solomon, D. Qin, et al., editors. *Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press, Cambridge, England, 2007. `http://www.ipcc.ch/publications_and_data/ar4/wg1/en/contents.html`, Accessed 05/26/2010.

[SRA+07] Clifford E. Singer, T. S. Gopi Rethinaraj, Samuel Addy, David Durham, Murat Isik, Madhu Khanna, Brandon Kuehl, Jianding Luo, Wilma Quimio, Kothavari Rajendran, Donna Ramirez, Ji Qiang, Jürgen Scheffran, T. Nedjla Tiouririne, and Junli Zhang. Probability distributions for carbon emissions and atmospheric response: Results and methods. Technical Report 309, ACDIS, University of Illinois at Urbana-Champaign, September 2007. `http://acdis.illinois.edu/publications/RR.html`, Accessed 06/20/2010.

[SRA+08] Clifford E. Singer, T. S. Gopi Rethinaraj, Samuel Addy, David Durham, Murat Isik, Madhu Khanna, Brandon Kuehl, Jianding Luo, Wilma Quimio, Kothavari Rajendran, Donna Ramirez, Ji Qiang, Jürgen Scheffran, T. Nedjla Tiouririne, and Junli Zhang. Probability distributions for carbon emissions and atmospheric response. *Climatic Change*, 88(309), 2008.

[SS08a] Erich A. Schneider and William C. Sailor. Long term uranium supply estimates. *Nuclear Technology*, 162, Jun 2008.

[SS08b] J. W. Storm van Leeuwen and P. Smith. Nuclear power – the energy balance. http://www.stormsmith.nl/, Accessed 11/09/2009, Feb 2008.

[SSS+04] T. Sugo, T. Seguchi, T. Shimizu, M. Uotani, and R. Kashima. Evaluation of cost of seawater uranium recovery and technical problems towards implementation. *Nuclear technologies and non-proliferation policies*, 13(6), 2004.

[ST07] Clifford E. Singer and J'Tia Taylor. Nuclear's role in 21st century pacific rim energy use: Results and methods. Technical report, ACDIS, University of Illinois at Urbana-Champaign, Sep 2007.

[SvB10] Clifford E. Singer and Hermann von Brevern. Uranium price vs. cumulative use. *Nuclear Technology*, 2010. submitted for publication.

[SW05] Peter B. Sørensen and Hans J. Whitta-Jacobson. *Introducing Advanced Macroeconomics*. McGraw-Hill, London, UK, 2005.

[TLH04] P. Thomond, F. Lettice, and T. Herzberg. Disruptive innovation: Enabling practitioners to tackle the "innovators dilemma" with graphical techniques - a focus on resource allocation. In *Proceedings Design to Deliver: The Challenge of International Integration - The 11th International Product Development Management Conference, Dublin, Ireland*, 2004.

[Tol04] Richard S. J. Tol. Exchange rates and climate change: An application of fund. Working Papers FNU–45, Research Unit Sustainability and Global Change, Hamburg University, Germany, June 2004.

[Uni10] United Nations. United nations framework convention on climate change website. http://unfccc.int/2860.php, Accessed 09/26/2010, 2010.

[U.S10] U.S. Department of Labor Bureau of Labor Statistics. Consumer price index. ftp://ftp.bls.gov/pub/special.requests/cpi/cpiai.txt, Accessed 09/10/2010, Feb 2010.

[vH09] Frank von Hippel. Why reprocessing persists in some countries and not in others: Why reprocessing persists in some countries and not in others. Technical report, Non-proliferation Education Center, 2009. http://www.npolicy.org/files/vonhippel-TheCostsandBenefits_0.pdf, Accessed 09/10/2010.

[vW+09] Ernst von Weizsäcker et al. *Factor Five - Transforming the Global Economy through 80% Improvements in Resource Productivity*. Earthscan, London, Dec 2009.

[Whi95] Scott W. White. Energy balance and lifetime emissions from fusion, fission, and coal generated electricity. Master's thesis, Fusion Technology Institute, University of Wisconsin, Madison, WI, 1995. `http://rigel.neep.wisc.edu/pdf/fdm993.pdf`, Accessed 06/20/2010.

[WK00] S. W. White and G. L. Kulcinski. Birth to death analysis of the energy payback ratio and CO2 gas emission rates from coal, fission, wind, and dt–fusion electrical power plants. *Fusion Eng Des*, 48, 2000.

[WKH90] M. Weis, F. Kienle, and W. Hortmann. Atomic energy and co_2: energy expense and co_2-emissions in fuel extraction. *Elektrizitätswirtschaft*, 89, 1990.

[Wor05] World Nuclear Association. The nuclear fuel cycle. `http://www.world-nuclear.org/info/inf03.htm`, Accessed 11/09/2009, 2005.

[Wor09] World Information Service on Energy. Uranium mining issues: 2009 review. `http://www.wise-uranium.org/uissr09.html`, Accessed 09/10/2010, Jan 2009.

[Wor10] World Nuclear Association. World uranium mining. `http://www.world-nuclear.org/info/inf23.html`, Accessed 09/11/2010, May 2010.

Nomenclature

ASEAN Association of Southeast Asian Nations

BUND Bund für Umwelt- und Naturschutz, Deutschland

CANDU Canadian Deuterium Uranium Reactor (Trademark)

CIS Commonwealth of Independent States

CO2 Carbon Dioxide

DICE Dynamic Integrated Model of Climate and the Economy (Nordhaus)

EAR-I Estimated Additional Resource Category I

EAR-II Estimated Additional Resource Category II

EJ Exa-Joules, 1×10^{18} J

FFM Fuel Fractions Model

GDP Gross Domestic Product

GHG Green House Gas(es)

GWe Giga-Watt, 1×10^{9} W, electrical (energy)

GWth Giga-Watt, 1×10^{9} W, thermal (energy)

IAEA International Atomic Energy Agency

IIASA International Institute for Applied Systems Analysis

IPCC Intergovernmental Panel on Climate Change

kg Kilo-Gram

kgHM Kilo-Gram of Heavy Metal

kgU Kilo-Gram of Uranium

LOGICAL logistic independent variables calibrated vs. historical data

LWR Light water moderated Reactor

MACRO Macroeconomic Model

MARKAL Market Allocation Model

MERGE Model for Estimating the Regional and Global Effects of GHG reductions

MESSAGE Model for Energy Supply Strategy Alternatives and their General Environmental impact

Mt Mega-tonne, metric, 1×10^9 g

NEA Nuclear Energy Agency

NOAA National Oceanic and Atmospheric Association

NU Natural Uranium

OECD Organisation for Economic Co-operation and Development

ppm parts per million

RAR Reasonably Assured Resources

RES Reference Energy System

RICE Regional aggregated Version of the DICE-Model (von Below/Persson)

SAFTA South Asian Free Trade Association

SRES Special Report on Emission Scenarios

UN United Nations

UNFCCC United Nations Framework Convention on Climate Change

USSR Union of Soviet Socialist Republics

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