# **ACDIS** Research Report

### **Passenger Prescreening Issues in Aviation Security**

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Research of the Program in Arms Control, Disarmament, and International Security

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#### PART ONE

#### **Introduction and Motivation**

On September 11, 2001, four commercial aircraft were hijacked and used as bombs to destroy the World Trade Center twin towers and inflict severe damage to the Pentagon. These violent events prompted efforts to improve aviation security, leading to widespread aviation security policy and operational changes throughout the nation's airports. Part of the response was promulgation of the Aviation and Transportation Security Act, which initiated a policy of 100 percent screening of checked bags by explosive detection technologies, with a deadline of December 2002. This deadline was later extended until December 2003. To meet this objective and to further improve aviation security procedures, the Transportation and Security Administration (TSA) must develop new security system paradigms to optimally utilize and simultaneously coordinate several security technologies and procedures. Illustrating the enormous scope of this effort is the fact that in 2000 there were over 600 million airline passengers, with forecasts of nearly one billion passengers by 2013.<sup>1</sup> However, developing strategies to effectively, yet efficiently, screen passengers, as well as determining the optimal allocation and utilization of screening of passengers and their checked and carry-on baggage before they enter an aircraft. Moreover, even after such systems are in place, it can be very difficult to measure their effectiveness. The primary objective of all these efforts is to improve security operations at the nation's airports.

The contribution of this paper is to illustrate systematic approaches for analyzing and designing enhanced aviation security strategies. The conclusions that can be inferred from the paper are mathematical in nature and affect the security operations in the nation's airports. Although aviation security touches on economic and political agendas, the questions of how much aviation security will cost and who should pay for it will not be answered here. Additionally, the concept of aviation security can be defined in several ways, encompassing different parts of the airport and the surrounding area. In this paper, the scope of aviation security is limited to the prescreening of passengers and their checked and carry-on baggage.

This paper is organized as follows. Part Two contains information regarding the history of aviation security. Part Three contains definitions and a description of the performance measures used to determine whether or not an aviation security strategy is effective. Parts Four and Five contain the partial baggage screening and multilevel screening problems, the two main areas of research that have been performed, and explain the mathematical framework used in this research. Finally, Part Six contains the concluding remarks.

The research presented in Parts Four, Five, and Six are mathematical in nature, but this paper was written to be accessible to readers with limited knowledge of mathematics. The mathematical description of these problems is contained in the Appendix.

#### PART TWO

#### The History of Aviation Security

This section summarizes the history of aviation security and describes the various technologies and procedures used for aviation security. It primarily focuses on aviation security operations within the United States from its beginnings in the 1970s until the most recent changes in aviation security since September 11, 2001. Aviation security operations in Europe and Israel are also briefly outlined.

#### **Initial Aviation Security**

Domestic hijackings were relatively rare until 1968 when seventeen hijackings occurred on US aircraft. On September 11, 1970, then President Nixon announced a program of deploying surveillance equipment to the nation's airports to deal with the increased numbers of hijackings. Furthermore, air carriers worked with the Departments of Defense and Transportation to determine whether x-ray devices and metal detectors could be integrated into airports to prescreen passengers and their carry-on baggage. On February 1, 1972, the FAA announced that all passengers were to be screened by at least one approved method, which included a behavioral profile, metal detector, identification check, and physical search. When hijackings persisted, the FAA adopted emergency rules on December 5, 1972 requiring air carriers to use screening procedures to prevent passengers from bringing weapons and explosives onto the aircraft. Figure 1 shows the rate of hijackings from 1968 to 1994.<sup>2</sup> There were at least twenty-five total hijackings during the years from 1969 to 1972, but the number of hijackings plummeted to two in 1973. There were relatively few hijacking attempts after 1972, with no more than five hijacking attempts in any single year since 1984.

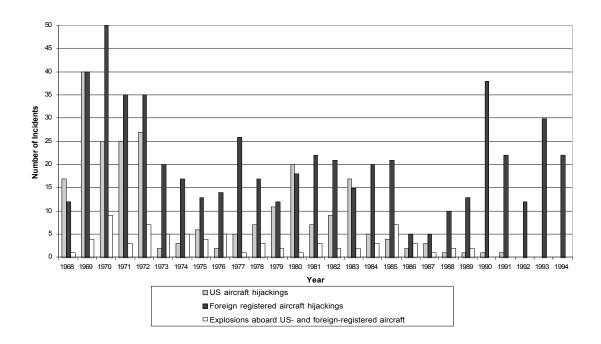


Figure 1: Foreign and Domestic Aircraft Incidents

#### Aviation Security 1996-2001

Aviation security operations at the nation's airports did not significantly change from December 1972 to 1996. However, the destruction of Pan Am Flight 103 over Lockerbie, Scotland on December 21, 1988 led to the eventual creation of the Commission on Aviation Safety and Security on July 25, 1996, headed by then Vice-President Al Gore. The Commission on Aviation Safety and Security recommended that the aviation industry improve security using existing explosive detection technologies, automated passenger prescreening, and passenger-baggage matching. An FAA administrator remarked that the FAA is in "...hot pursuit of equipment and procedures that can spot these [explosive] devices with high degrees of confidence for the nearly one billion pieces of luggage and 500 million passengers traveling annually on United States carriers."<sup>3</sup>

The Commission on Aviation Safety and Security developed the Computer-Aided Passenger Prescreening System (CAPPS) in conjunction with the FAA, Northwest Airlines, and the United States Department of Justice to automate passenger prescreening. CAPPS was used to classify each passenger as either a *selectee* or a *nonselectee* based on passenger information and flight characteristics. CAPPS clears nonselectee passengers; selectee passengers are those who remain uncleared. In other words, CAPPS determines who is *not* a risk. The resulting passenger profiles were then used to screen a portion of the checked baggage by explosive detection systems (EDSs) before the baggage was loaded onto aircraft. At this time, only EDSs were used to screen checked baggage of selectee passengers, and the checked baggage of nonselectee passengers received no additional security attention. However, small airports generally did not have EDSs, so originating selectee bags were often unscreened. There were no further differences in screening procedures between selectee and nonselectee passengers. The overall selectee rate (proportion of passengers on a flight who were designated as selectees) was approximately 5-10 percent, with the selectee rate varying from zero on short commuter flights to 50 percent on certain international flights.

#### Aviation Security since September 11, 2001

The incidents of September 11, 2001 led to the passing of the Aviation Transportation and Security Act (ATSA) on November 19, 2001. This act established the majority of recent changes regarding the aviation security operations at the nation's airports, and, furthermore, established the Transportation Security Administration (TSA) as the government agency responsible for aviation security (this was formerly under the jurisdiction of the FAA). Eventually, the TSA will be absorbed by the Department of Homeland Security.

The ATSA established guidelines at the nation's airports for all areas of aviation security. Improved operations are defined in the areas of perimeter security, checked baggage screening, passenger prescreening, and cargo screening. However, the ATSA did not outline specific requirements or deadlines for passenger prescreening and perimeter security. It also set a requirement for the federalization of all screening personnel with a deadline of November 19, 2002 (this deadline was met). Finally, it established that all checked bags are to be screened for explosives by an EDS by December 31, 2002. This deadline was later extended until December 31, 2003. Prior to this deadline, all bags were to be screened either by EDSs, explosive trace detection (ETD) devices, or alternative technologies, including positive passenger baggage matching (PPBM) or hand search. Note that PPBM removes a checked bag from an aircraft if the passenger to whom the bag belongs has not boarded. This procedure was not used before September 11, 2001. It has been determined that the use of PPBM results in an average delay of approximately one minute per flight, much lower than expected.<sup>4</sup>

There have been several challenges in meeting the deadline for the 100 percent baggage screening mandate. The TSA has estimated that approximately 2,500 EDSs are required to be installed in the nation's airports for 100 percent baggage screening. At least 90 percent of the airports screened all checked bags with EDS or ETD devices by December 31, 2002, with 1,100 EDSs and 5,000 ETD devices in use.<sup>5</sup> The main challenge associated with the screening requirement is the installation of EDSs. These devices weigh approximately nine tons and are typically installed in airport lobbies. The infrastructure in many airports cannot support several heavy EDSs and the cost of installation is often higher than the \$1 million purchase price for EDS devices. Moreover, there is often not space to install EDSs where they can be integrated in the baggage handling system to most efficiently screen bags. As a result, they are installed wherever there is space, such as airport lobbies. Thus, more screeners are needed to operate the EDSs. Moreover, there has been a historically high turnover rate associated with security screeners, which presents challenges for training purposes.

There is a high cost associated with these aviation security measures. The ATSA required the federalization of 28,000 security screening personnel to operate the EDSs as well as other security devices. However, ETDs are mainly used to screen checked bags, and since ETDs are more labor intensive than EDSs, more screening personnel are required. As of February 5, 2003, the TSA employed 62,000 screeners. Furthermore, the TSA's budget in 2002 was approximately \$8.5 billion. Of these costs, approximately \$1.6 billion was used to screen checked bags and more than \$3 billion was used to install 1,100 EDSs. It has been projected that the cost of screening checked bags will be \$4.5 billion in 2004.<sup>6</sup> Additionally, the passenger security fee created by the TSA will only pay for a portion of these costs. This fee generated \$987 million in 2002, and is expected to generate \$1.66 billion in 2003 and \$1.74 billion in 2004.<sup>7</sup> In other words, approximately half of the TSA's entire budget, which is to be used to protect against all types of incidents in airports and on aircraft, is being used to protect against a single type of attack against the aircraft and the passenger fees are not sufficient to pay for the cost of aviation security.

Other changes in aviation security since September 11, 2001 include the standardization of how passengers are randomly selected for additional screening across the nation's airports. The TSA is developing CAPPS II, an enhanced system for automatically prescreening passengers. CAPPS II will partition passengers into three risk groups (as opposed to two in the previous system). Currently, CAPPS II is being tested by Delta airlines.

#### International Aviation Security

The aviation security operations at airports outside of the United States are often strikingly different from those in the United States in both attitude and procedure. Since information is extremely limited in this area, international security operations are presented for Israeli and European airports. However, a direct comparison between the air systems in Europe and Israel to the United States is difficult because the number of airports and passenger volume differ greatly. These countries typically have three to four major airports, as compared to four hundred in the United States. In 2002, 54 million airline passengers traveled in Europe as compared to 560 million airline passengers who traveled in the United States.<sup>8</sup> The European Civil Aviation Conference establishes the standards and recommended practices for aviation security on flights within and to Europe.

On July 22, 1968, aviation security became a priority for El Al airlines when three Palestinian terrorists hijacked an El Al aircraft en route from Rome to Tel Aviv. This hijacking differed from previous hijacking attempts in that a political statement was being made through the hijacking, and an airline associated with national identity was the target. Moreover, its purpose was in some sense to create a "traveling theater" of media coverage on the Israeli aircraft.<sup>9</sup> Because of the strong national affiliation of El Al airlines, Israel has had to develop reliable aviation security strategies before this was a priority for the United States and Europe. Dan Isaacharoff, the former head of security for El Al Airlines, indicates that the El Al approach to aviation security has been to focus on the *passengers* who may be a threat, rather than the *objects* that could be used to threaten or hijack an aircraft.<sup>10</sup> Additionally, the El Al aviation security strategy characterizes passengers who may be a threat in several categories (as opposed to the binary method used in the United States before September 11, 2001) ranging from naïve terrorists such as a passengers who could be unaware that they are carrying a bomb on an aircraft, to a terrorist such as a hijacker, or a suicide terrorist. In other words, passenger prescreening by the CAPPS-like system used in Israel has resulted in a secure and efficient aviation security strategy. El Al Airlines has not experienced a successful hijacking attempt since 1968.<sup>11</sup>

Aviation security in Europe changed significantly after the explosion on Pan Am Flight 103 in 1988, when a bomb in the checked bag of a passenger who was not on the aircraft resulted in the deaths of all 270 passengers. Positive passenger baggage matching (PPBM) was incorporated to the aviation security procedures for all flights in Europe. Furthermore, a multi-tiered system is used to identify high-risk passengers and screen them accordingly. In such a multi-tiered system, different procedures are used for passengers identified as low and high risk. Additionally, accurate and expensive devices are used to resolve alarms rather than to screen all passengers. X-ray devices are used to screen all checked baggage instead of EDSs. These x-ray devices have a screening capacity that is approximately an order of magnitude greater than that of EDSs, but similar x-ray devices for screening checked baggage have not been approved in the United States. EDS devices are used to resolve alarms triggered by x-ray devices and to screen high-risk passengers.

In both Europe and Israel, aviation security is mostly privatized, with approximately 85-90 percent of all security personnel in Europe being provided by private companies,<sup>15</sup> mainly because of the additional level of accountability and efficiency that private companies provide. In Israel, the government's role in aviation security is to set and enforce the standards through testing and quality control, rather than designing and implementing security procedures. This can be contrasted with the United States, whose new security procedures are implemented by security screening personnel who are now entirely composed of federal workers.

Both Israeli and European airports offer trusted traveler programs in order to screen registered low-risk passengers about whom sufficient information is known. In both cases, passengers must renew their trusted traveler status annually. The trusted traveler program used at Ben Gurion Airport in Tel Aviv boasts an average check-in time of fifteen minutes for trusted travelers compared to 120 minutes for other passengers. The International Air Transport Association is developing a trusted traveler program called S-Travel that uses a biometric smart card to verify passenger identities. This approach will also be used for employee access control. Another biometrically based trusted traveler program is being used on a trial basis in the Amsterdam Schiphol Airport. This program uses an iris scan instead of hand geometry measurements and is also being tested in Canadian airports, at Frankfurt, and at John F. Kennedy Airport in New York.<sup>16</sup>

#### PART THREE

#### Mathematical Issues in Aviation Security

In this section, basic terminology used in the problems in the following sections is defined, and the performance measures—possible objectives in aviation security problems—are described. Although the definitions and performance measures deal with quantitative measures, they are described in qualitative terms in this section without using mathematical notation.

The main goal of aviation security is to minimize the impact of willful human intent. For this reason, a passenger or his baggage represents a *threat* when the passenger is planning such an event of willful human intent. Similarly, a naïve terrorist who unknowingly has a bomb placed in his baggage, for example, can also be represented as a threat.

The following definitions are needed to describe the partial baggage screening problems. The TSA employs a *computer-aided passenger prescreening system* (CAPPS) to provide a binary risk assessment of all passengers. Passengers are termed *selectees* if CAPPS is unable to clear them from being a potential risk to the system. The *selectee rate* is the fraction of passengers (between zero and one) on a given flight who are selectees (i.e., number of selectees on a given flight divided by the total number of passengers on the flight). *Selectee baggage* (i.e., checked baggage belonging to selectees) is classified as either *screened* or *unscreened*, based on whether or not it has been processed through an airport baggage screening security device. Selectee baggage screened at the point at which it first enters the system (i.e., *origin*) is termed *originating* baggage. Any unscreened non-originating baggage at an airport en route to another airport is termed *baggage in transit*. A flight is said to be *uncovered* if one or more selectee bags on it have not been screened, while a flight is said to be *covered* if all selectee bags on the flight. **[This definition is unclear. Maybe it means, "Any number of uncovered passengers on an uncovered flight means the entire passenger load is considered as uncovered."?]** 

A baggage screening security device *deployment* for a set of airports is an allocation of baggage screening security devices to these airports and an assignment of selectee baggage that should be screened for the set of flight segments between these airports, where a *flight segment* (or *flight*) is a takeoff and landing of an aircraft from one airport to another. From this definition, the takeoff and landing of an aircraft from Airport A to Airport B with no intermediate stops counts as a single flight. A selectee is said to be on a *direct* route if his flight path is composed of one flight segment, or on a *connecting* route if his flight path is composed of two or more flight segments. Note that unless otherwise stated, all baggage screening occurs at origin (i.e., no baggage is screened in transit along a connecting route).

The following definitions are needed to describe the multilevel passenger prescreening problems. A *device* in this context is a process used to identify an attack. A device can consist of aviation security technology and/or airline personnel. Examples of devices include metal detectors, explosive detection systems, and hand searches by airport security officials. The device *capacity* is an integrated upper limit of how many passengers or bags can be processed by a device in a given period of time. A *risk group* is a pre-assigned subset of devices through which a passenger may be processed before boarding an aircraft. A computer-aided passenger prescreening system (CAPPS) assigns each passenger an *assessed threat value*, which quantifies the risk associated with the characteristics of the passenger. For the purpose of this paper, assessed to the budget only if there are passengers assigned to the risk group. The fixed cost may be used for purchase and overhead costs for the devices associated with that risk group. The *marginal cost* associated with a risk group is the direct cost to screen each passenger or bag assigned to the risk group.

There are several objectives that can be considered for each of the problems. The partial baggage screening problems address how to deploy a certain number of EDSs to screen some checked baggage in order to minimize the risk associated with the unscreened bags. Three security performance measures have been identified in conjunction with the FAA for quantifying risk and measuring baggage screening security device

utilization for a given set of flights carrying both selectee and nonselectee passengers: the uncovered flight segment (UFS), the uncovered passenger segment (UPS), and the uncovered baggage segment (UBS). The UFS considers the number of uncovered flights, the UPS considers the number of passengers on uncovered flights, and the UBS considers the number of unscreened selectee bags. The UFS measure captures the number of flights departing an airport station that may be subject to this particular type of unscreened checked baggage risk. The UPS measure captures the number of passengers on flights carrying one or more unscreened selectee bags, hence it quantifies the number of passengers departing an airport station that may also be subject to this type of risk. The UBS measure represents the number of unscreened selectee bags. The UFS and UPS measures provide two alternative ways to represent unscreened checked baggage risk on a set of flights, while the third measure focuses entirely on unscreened baggage in the system (hence it does not directly measure any type of risk on the flights). Note that there may be other types of risk on a flight that are not considered in this analysis. The following example illustrates the relationships between these performance measures.

**Example 1:** Suppose that for a given station, there are three departing flights, each to a distinct destination. Moreover, suppose that the number of passengers and selectee bags on these flights is:

Flight 1: 100 passengers, 5 selectee bags

Flight 2: 40 passengers, 2 selectee bags

Flight 3: 20 passengers, 1 selectee bag

Suppose that at most five selectee bags can be screened for these flights. Note that screening any five of the eight selectee bags results in a UBS of 3 (since uncovered bags are the number of bags that are left unscreened). Screening the selectee bags on flights 2 and 3 results in a UFS of 1 and a UPS of 100, minimizing the UFS measure. Screening the selectee bags on flight 1 results in a UFS of 2 and a UPS of 60, minimizing the UPS measure. Therefore, depending on which measure is chosen, the security of the system can be determined to be optimal in one of two distinct ways.

The multilevel screening problems capture the *false clear* rate associated with various strategies. A false clear represents an event where a passenger who represents a threat is not flagged by the aviation security procedures. The false clear rate is the proportion (in the long-run) of passengers who represent a threat that are cleared by aviation security procedures. Likewise, a *false alarm* is an event where a passenger who does not represent a threat is flagged by the security system. The false alarm rate is the proportion (in the long-run) of passengers who do not represent a threat that are not cleared by aviation security procedures. The following example demonstrates how device false clear rates can be used to determine the false clear rate of an entire security procedure.

**Example 2:** Suppose that the aviation security procedures use a single device in the following way. The device screens each passenger twice, and passengers are allowed to board only if they are cleared both times. If an alarm is sounded by the first pass through the device, the passenger is not screened a second time. The device has a false clear rate of 0.10 and a false alarm rate of 0.05. Assume that there is independence between each pass through the security device (i.e., whether an alarm sounds during the first pass through is not correlated with whether the alarm sounds on the second pass through).

First, the passengers who are a threat will be considered. The probability that they will pass the security procedures is determined by whether or not they are cleared by both passes through the security devices. This is equivalent to

= (Probability cleared on first pass) (Probability cleared on second pass)

$$= (0.10)(0.10) = 0.01$$

In other words, the procedure has an overall false clear rate of 1%, which means that 99% of passengers who are a threat will be flagged by the security procedures. This overall false clear rate is much lower than the false clear rate of the device by itself.

Now the passengers who are not a threat will be considered. They are flagged by security if they either are flagged on the first pass, in which case they will not be screened by the device a second time, or they are cleared on the first pass and flagged on the second pass. This is equivalent to

= (Probability flagged on first pass) + (Probability cleared on first pass)(Probability flagged on second pass

= 0.05 + (1-0.05)(0.05) = 0.0975

In other words, the overall false alarm rate is 9.75%, almost twice as large as the individual false alarm rate. In other words, 90.25% of passengers who are not a threat pass the security procedures.

#### PART FOUR

#### Partial Baggage Screening Problems

Baggage screening security devices and operations at airports throughout the United States provide an important defense against terrorist actions targeted at commercial aircraft. Determining how and where to assign and deploy such devices can be quite challenging. Moreover, even after such systems are in place, it can be very difficult to measure their effectiveness. This section formulates problems that model multiple sets of flights originating from multiple stations (e.g., airports, terminals), where the objective is to optimize a baggage screening performance measure subject to a finite amount of security resources. These measures include uncovered baggage segments (UBS), uncovered flight segments (UFS) and uncovered passenger segments (UPS). These problems are then formulated to identify optimal baggage screening security device deployments (i.e., determine the number and type of baggage screening security devices that should be placed at different airports, and determining which baggage should be screened with such devices). An example is provided to illustrate these results using data obtained from the Official Airline Guide (OAG). This section summarizes the work presented by Jacobson, Virta, McLay, and Kobza.<sup>17</sup>

Determining how to optimally deploy different types of baggage screening security devices throughout airports in the United States is of critical importance to national security. Limited available baggage screening security device capacities and time restrictions initially led to the implementation of procedures that require the screening of only a fraction of all passenger baggage. However, recent congressional mandates required that by the end of 2002, all checked bags were to be screened by federally approved baggage screening security devices. Until sufficient baggage screening capacity is deployed at airports throughout the nation, only a targeted fraction of all checked baggage can be screened. Moreover, as new, more effective (and potentially more expensive) baggage screening technologies are developed and reach the market, a natural hierarchy of baggage screening device will surface. For example, explosive detection systems (EDS) are the preferred method for checked baggage screening.<sup>18</sup> However, in airports with smaller volumes, the cost of such devices makes it difficult to justify their deployment. Therefore, less expensive explosive trace detection (ETD) devices are often the only devices that are used in such airports.

To describe the problems, several additional definitions and assumptions are needed. First, airport activity can vary on a daily basis, as well as within a given day. Therefore, the problems are based on activity during an airport's *peak period*, defined as the sixty-minute period during the day in which the largest number of originating passengers enters the airport. This assumption results in a worst-case scenario analysis, hence ensures that the number of baggage screening devices deployed is sufficient to optimally address baggage checked screening requirements during any period of the day. Assume that during an airport's peak period, there is at most one flight scheduled to each of the other airports in the system of airports under study. Also, assume that all peak periods coincide so that any selectee bags on a flight into a hub airport, regardless of arrival time, can transfer on any flight departing from the hub airport during the hub's peak period (i.e., once the peak hours are determined, the actual arrival and departure time of the flights are not considered). Though this assumption is unlikely to hold in practice, it once again allows for the system of airports to be studied under a worst-case scenario. Therefore, all averages discussed in this section correspond to averages during each airport's peak period.

The following assumptions were made for this research. The average number of selectee bags on any flight is known. These values can be obtained using four parameters between any two airports: the average number of bags per passenger, the capacity of flights, the selectee rate, and the enplanement rate. Each passenger is assumed to have no more than two connecting flights. This assumption is reasonable, since the "hub and spoke" system used by most major commercial airlines in the United States facilitates such routing situations. Airline reservation systems have the capacity to collect the information needed to provide values for these parameters. However, given that schedules are constantly changing and being updated, this information will also change accordingly. Therefore, information needs to be regularly updated to provide the most current parameter values. Moreover, since peak period averages and information are being used, the baggage screening security device deployments that arise from a particular schedule should be reasonable for a variety of schedules, provided that the peak period flight schedules are similar.

The three problems that model the baggage screening security device deployment problem are defined using the UBS, UFS, and UPS performance measures described in Part Three. These three problems are described in more mathematical detail in the Appendix. For all three problems, the following information is known:

- a set of airports
- a set of individual baggage screening devices
- a cost for each of the baggage screening devices
- a baggage screening capacity for each of the baggage screening devices
- a set of flights departing from each airport
- the number of direct originating selectee bags on each flight
- the number of connecting originating selectee bags on each flight and the set of flights to which they transfer
- the total number of passengers on flight
- a baggage screening security device allocation budget

The problems are designed to optimally determine where to assign the different baggage screening security devices and which selectee baggage to screen, where optimality is based on minimizing either the number of uncovered baggage segments, the number of uncovered flight segments, or the number of uncovered passenger segments (hence resulting in three different problems). The following is a list of decision variables for these problems:

- the number of units of each baggage screening security device type deployed at each airport
- the number of uncovered originating direct selectee bags traveling between any two airports
- the number of uncovered originating connecting selectee bags originating and arriving at any two
  airports and connecting with a third airport
- the number of uncovered non-originating connecting selectee bags originating and arriving at any two airports and connecting with a third airport
- the set of flights that are uncovered based on the previous three variables

The objective functions for the three problems are as follows. The Multiple Airport Direct and Connecting Baggage Problem (MADCBP) finds the maximum number of baggage segments that can be screened and, hence, minimizes the UBS measure. The Multiple Airport Direct and Connecting Flight Problem (MADCFP) finds the maximum number of flight segments that can be screened. In other words, the maximum value of the number of flight segments that can be covered, which satisfies the conditions of the MADFP, will minimize the UFS. The Multiple Airport Direct and Connecting Passenger Problem (MADCPP) finds the maximum number of passenger segments can be covered, minimizing the UPS measure.

Each of these problems determines which (feasible) allocation of baggage screening security devices results in the minimum UBS, UFS, or UPS value. Furthermore, each of the problems determines which bags are screened given the allocation and whether the baggage screening security device deployment is within budget. Since the number of uncovered flight segments and the number of uncovered passenger segments are affected only by covered flights, then the formulations presented here do not consider partial screening of a flight. That is, if there is not enough capacity available to cover a flight, then a baggage screening security device is not used to screen additional bags. Therefore, it is unlikely that all the available baggage screening capacity will be used. All three problems have the same set of constraints. The constraints for minimizing the number of uncovered flight segments and uncovered passenger segments include:

- the purchase cost plus the first year annual operating and maintenance budget of the baggage screening devices must not exceed the given budget
- the baggage screening security device capacity plus the number of uncovered originating direct and connecting selectee bags leaving any airport (all unscreened non-originating connecting selectee bags leaving an airport remain unscreened)
- the number of uncovered originating selectee bags on a route cannot exceed the average number of originating selectee bags for that route
- a flight segment is uncovered if any unscreened baggage exists on that flight segment from either unscreened baggage in transit or unscreened originating baggage (this constraint is only necessary when minimizing the uncovered flight segment or uncovered passenger segment performance measures)

The following example illustrates the problems described earlier using data extracted from the Official Airline Guide (OAG). Optimal baggage screening deployments are obtained based on the UBS, UFS, and UPS security measures. The data extracted from the OAG is for a single airline carrier and the flights by that carrier between a set of ten airports in the United States. See Table 1 for a list of these airports. While the OAG provides some explicit data for the example (e.g., the number of peak bags per hour at an airport), other data must be created to support this example (e.g., the number of bags in transit).

Airport	Airport Code	Airport Location
A1	ATL	Atlanta, GA
A2	CLE	Cleveland, OH
A3	CLT	Charlotte, NC
A4	DTW	Detroit, MI
A5	ERI	Erie, PA
A6	FAY	Fayetteville, NC
A7	GSO	Greensboro, NC
A8	ITH	Ithaca, NY
A9	ORF	Norfolk, VA
A10	PIT	Pittsburgh, PA

Table 1: Airports Under Study

Table 2 lists the nineteen flights that exist between the ten airports under study, as well as the total number of available seats between the city pairs, the number of total (selectee and non-selectee) originating bags, and the selectee rate on each flight. Note that the true selectee rates (available from the TSA) are security sensitive information, hence they cannot be reported here. Therefore, a broad range of selectee rates are used so that the problems can be solved and the analysis procedure can be illustrated.

Let the peak hour at each airport be the hour of the day in which the largest number of bags is recorded in the OAG (e.g., 12:00 PM). To approximate the number of bags on a flight, let the bags at an airport during the peak hour be distributed proportionally among all flights departing the airport within one hour of the peak hour (e.g., 12:00 PM to 1:00 PM) based on the number of seats on each flight. The number of selectee bags on each flight is the number of bags on the flight multiplied by the selectee rate of that flight, and rounded to the nearest integer.

To determine the number of selectee bags in transit on a flight, first assume that all peak hours coincide so that any selectee bags on a flight into a hub airport can transfer on any flight departing from the hub airport (i.e., once the peak hours are determined, disregard the actual arrival and departure time of the flights). In practice, a transferring selectee bag at a hub airport can originate from any airport and transfer to any another airport. Specifically, a bag can transfer from an airport under study to an airport not under study, or from an airport not under study to an airport under study. For this example, assume that a selectee bag in transit must have originated from an airport under study and must transfer to an airport under study. This effectively assumes that there is a degree of interchangeability between various non-hub airports.

Assume there are four types of baggage screening security devices with associated capacity and total cost, as given in Table 3. Note that the actual rates and costs are sensitive data information, hence could not be used and reported here.

By inspection, the minimum budget required for 100 percent selectee coverage is \$7 million. That is, for each airport, deploy those devices with sufficient screening capacity to screen all originating selectee bags at the lowest cost. As the budget decreases from \$7 million to \$0, deployment choices must be made to minimize UBS, UFS, or UPS. These results are illustrated in Figures 2-4, and were found by solving the problems formulated using CPLEX, mathematical programming optimization software developed by ILOG, Inc. Figure 2 depicts the optimal UBS solution, the UBS value for the optimal UFS solution, and the UBS value for the optimal UPS solution, all as a function of budget. From this figure, the optimal UPS and the optimal UFS solution tends to

result in smaller UBS values than the optimal UFS solution. This suggests a stronger relationship between UPS and UBS than UFS and UBS.

<b>F1</b>	D (			
Flight	Route	Number of	Number of	Selectee Rate
Depart	Arrive	Seats	Originating Bags	
ATL	CLT	100	17	0.30
ATL	PIT	126	21	0.30
CLE	CLT	112	49	0.22
CLT	ATL	112	33	0.15
CLT	FAY	85	25	0.15
CLT	GSO	112	33	0.15
CLT	ORF	37	11	0.15
CLT	PIT	85	25	0.25
DTW	PIT	100	31	0.22
ERI	PIT	85	55	0.09
FAY	CLT	85	56	0.16
GSO	CLT	126	39	0.13
ITH	PIT	100	66	0.09
ORF	CLT	85	27	0.25
ORF	PIT	85	27	0.20
PIT	ATL	112	37	0.10
PIT	CLE	30	10	0.10
PIT	CLT	85	28	0.25
PIT	ERI	37	12	0.15

Table 2: Flight Information

Table 3: Baggage Screening Security Device Information

Device Type	Capacity Bags/hour	Total Cost
1	5	\$550,000
2	10	\$600,000
3	15	\$750,000
4	25	\$1,100,000

Figure 3 depicts the UFS value as a function of budget for the optimal UPS solution and the optimal UFS solution. For this example, the optimal UPS solution provides a good UFS value compared to the optimal UPS solution value. Figure 4 depicts the UPS value as a function of the budget for the optimal UFS solution and the optimal UPS solution. It is clear that the UFS and the UPS solutions are significantly different, since UFS favors the screening of smaller flights, resulting in fewer passengers covered, while UPS favors the screening of larger flights, resulting in a greater number of passengers covered.

This example illustrates the effectiveness of and the relationship between each of the three performance measures. From Figures 2-4, the optimal UPS solution results in reasonable UFS values, while the UBS values are significantly lower than or similar to the UBS values for the optimal UFS solution. Also, at higher budgets, the optimal UFS solution did not yield acceptable UPS values, which were more than 1.4 times the value of UFS. While UBS ensures that the greatest number of selectee bags will be screened, it provides limited insight into the other performance measures (i.e., there is no guarantee that any flight segment or any passenger segment will be covered). Therefore, these results suggest that the UPS solution provides a reasonable baggage screening strategy for this airline carrier and these ten airports.

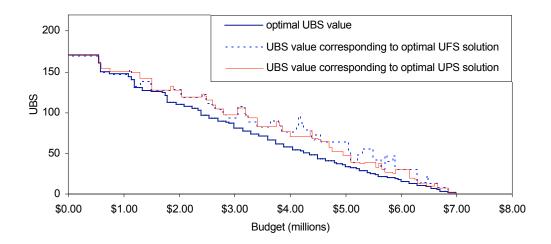


Figure 2: Uncovered Baggage Segments

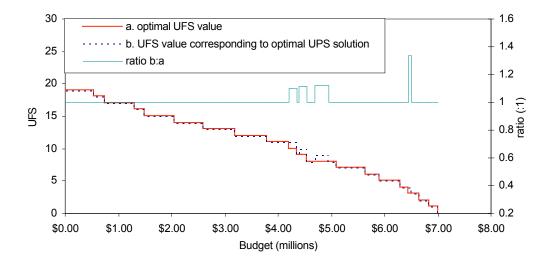


Figure 3: Uncovered Flight Segments

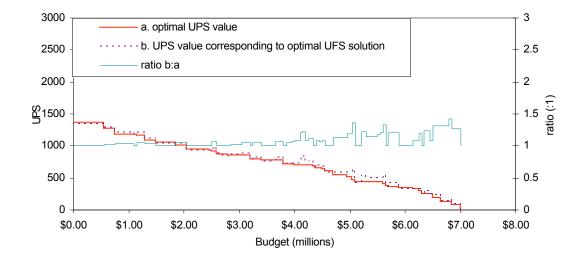


Figure 4: Uncovered Passenger Segments

#### PART FIVE

#### **Multilevel Prescreening Problems**

Since the September 11, 2001 terrorist attacks, mandates by Congress have largely influenced the direction of new aviation security operations. In particular, Congress allocated approximately \$40 billion annually for the Department of Homeland Security, with a large portion of this budget allocated to purchase and deploy explosion detection system (EDS) devices at the nation's airports. It has been suggested that a policy of 100 percent checked baggage screening is not cost effective and that enhancing the binary screening paradigm to a multilevel screening system would be a more effective approach to process airline passengers.<sup>19</sup> Therefore, multiple levels of security for processing passengers may be more effective than treating all passengers the same, from a security standpoint.

This section formulates two problems that model multilevel passenger prescreening. Multilevel screening considers no fewer than three levels of security procedures to screen passengers, as opposed to the binary system in place prior to September 11, 2001. This section summarizes a special case of the ongoing research efforts by McLay, Jacobson, and Kobza.

This section defines problems that model the multilevel prescreening problems using the false clear performance measure described in Part Three. In these problems, the overall security level is maximized, defined by the probability that a passenger who represents a threat is flagged by the security procedures. This is equivalent to minimizing the false clear rate. These two problems are described in more mathematical detail in the Appendix. For both problems, the following information is known:

- a set of passengers, each with an assessed threat valued assigned by CAPPS
- a set of security risk groups
- a set of security screening device types and a capacity associated with each device type
- the total number of each security device type
- a set of security devices and a procedure associated with each risk group
- the overall false clear rate associated with each risk group
- fixed and marginal costs associated with each risk group
- a budget

•

The first problem is the Multilevel Budget Allocation Problem (MBAP). The objective is to assign all of the passengers to the risk groups such that the overall security level is maximized (the overall false clear rate is minimized), subject to budget and assignment constraints. The decision variables needed for this problem include:

- the set of passengers that are assigned to a particular risk group
- the set of risk groups that have at least one passenger assigned to it (dependent on the previous variable)

The risk groups are defined in terms of the fixed and marginal costs, where these costs are determined by the set of devices that correspond to the risk groups. The particular devices associated with each risk group and their capacity are not used by this problem but may be indirectly used to calculate these costs. The following constraints are needed for the MBAP:

- every passenger must be assigned to exactly one risk group
- the sum of the products of the number of passengers assigned to each risk group and the marginal cost associated with that group plus the fixed costs of the risk groups that have at least one passenger assigned to them must not exceed the given budget

The Multilevel Passenger Assignment Problem (MPAP) is similar to the MBAP except that the device capacities are the limiting factors. The objective is to assign all of the passengers to risk groups such that the overall security level is maximized (the overall false clear rate is minimized), subject to device capacity and assignment constraints. The decision variables needed for this problem include:

the set of passengers that are assigned to a particular risk group

The risk groups are defined in terms of security screening devices and their capacities. The fixed and marginal costs associated with each group as well as the budget are not used by this problem. The following constraints are needed for the MPAP:

• every passenger must be assigned to exactly one risk group

• for any given device type, the sum of the passengers assigned to each risk group that uses this device type must not exceed the given capacity (based on the number of devices available)

The assessed threat values and the false clear rates can be estimated with data available from CAPPS and the TSA. The assessed threat values provide risk assessment measures for each passenger and are determined by the TSA. The security level of each risk group is based on security procedures of each device used to screen passengers in that group, and it is assumed that a passenger is cleared if all security screening devices used by the risk group to which the passenger is assigned clears the passenger in question.

The following example illustrates the problems described earlier using data extracted from the OAG for a single airline carrier at distinct stations in the United States. The data provided by the OAG includes the number of passengers during the peak operation period for originating passengers on domestic flights (the peak thirty minutes of the busiest six-hour period), and the number of seats on each flight departing during the peak hour of operation.

The following assumptions of the data were made. An 80 percent enplanement rate (i.e., the number of passengers divided by the number of available seats) was assumed on each flight. Additionally, it is assumed that all passengers have exactly one checked and one carry-on bag. Using this data, each problem was formulated and the resulting problems were solved and analyzed using CPLEX.

The data set was chosen based on finding the peak thirty-minute interval. This interval was chosen based on the expected number of arriving passengers when the passengers arrived according to a uniform distribution between thirty and ninety minutes before the departure time of their flights, the passenger arrival interval recommended by the airline of consideration for domestic flights. The Appendix contains a list of the flight data for this data set. Once each of the data sets was chosen, the exact arrival times were simulated for each passenger. There are 3,664 passengers over 71 flights for this example.

Three distributions for the assessed threat values of the passengers are considered. Initially, an identical passenger distribution was considered with all assessed values being equal to one. This models the case when no information is known about the passengers. The next passenger distribution considered is that with uniformly random assessed threat values, scaled between zero and one. The final distribution corresponds to the Severe threat levels determined by the Department of Homeland Security. For this distribution, it is assumed that 80.0% of passengers have an assessed threat value of 0.1, 19.5% have an assessed threat value of 0.3, and the remaining 0.5% have an assessed threat value of 0.9.

Table 4 contains device data used for the two example problems. The device values, including the cost information, false clear rate, and capacity, are estimated using information from Butler and Poole.<sup>20</sup> Table 4 is divided into three areas: checked baggage, personal, and carry-on baggage screening devices. The risk groups that use each device are based on the CAPPS II risk group descriptions by the TSA. The yearly costs are calculated based on the purchase costs, the expected lifetime of the device, and the yearly maintenance costs. The fixed costs per thirty minutes (FC') figure is based on the yearly fixed costs divided by the hours of operation per year (360 days a year, six peak hours per day). This value was then normalized by the capacity to find a fixed cost per passenger. The fixed cost values for the MBAP problems were multiplied each by half of the total number of passengers considered.

The values associated with each risk group are summarized in Table 5. The security level was computed from the false clear rates associated with the devices assuming that a passenger who represents a threat is detected if any single device detects that passenger. Additionally, it was assumed that it is equally likely for a threat to be in a checked bag, carry-on bag, or on a person. This is a reasonable assumption since no data exists that suggests a distribution among these means of attack. The marginal cost of a risk group is equivalent to the sum of all of the marginal costs of the devices associated with this group. The fixed costs were computed in a similar way. Since all risk groups use EDS, the metal detector, and the x-ray machine, these fixed costs were subtracted from the budget beforehand and only the additional fixed costs were considered. Since group 3 uses trace technology twice—Open Bag Trace (for checked baggage) as well as Open Bag Trace/Detailed Search (for carry-on baggage)—the fixed cost for this equipment was only assessed once.

Device Type	False Clear	Yearly	FC'	MC	Units/hour	Risk Groups
	Rate	Costs				
EDS	0.12	125,000	0.4167	1.00	125	1, 2, 3
Open Bag Trace	0.15	10,000	0.1199	0.83	28	3
Metal Detector	0.30	2,000	0.0051	0.28	90	1, 2, 3
Hand Wand Inspection	0.20	80	0.0009	1.25	20	2, 3
X-Ray	0.20	28,000	0.0720	0.28	90	1, 2, 3
Detailed Hand Search	0.20	0	0	1.25	20	2
Open Bag Trace/	0.15	14,500	0.1199	1.29	18	3
Detailed Hand Search						

Table 4 <sup>.</sup>	Security	Screening	Device Data
	Decurry	Screening	Device Data

Table 5: Risk Group Data

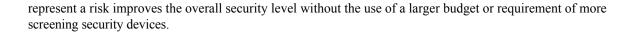
Group	FC (\$)	MC (\$)	L
1	0	1.56	0.793
2	164.88	2.81	0.927
3	22,130.56	4.93	0.964

The results for the MBAP example are presented in Figure 5 as a function of the total budget. It can be seen that the overall security level increases along with the budget for all three assessed threat distributions. For any given budget, the identical distribution never provided a better solution than either of the other two distributions. This indicates that a variation in assessed threat values—additional information that is able to distinguish passengers from one another—improves the overall security level. In other words, treating all passengers the same from a security standpoint and screening passengers with greater scrutiny purely randomly results in worse security than is otherwise possible.

The example for MPAP considers the device capacities. The number of devices available for this example was enough to screen all passengers unless otherwise noted. Initially, only risk groups 1 and 2 were considered by having no units of the devices solely used by risk group 3 available. The number of hand wands was then varied to see how the passengers were assigned with respect to this security device. Then only risk groups 1 and 3 were considered by having no detailed hand search available. The number of detailed hand search with openbag trace was then varied. Figure 6 depicts the overall security level for these two cases. Figure 7 depicts the overall security level when all three risk groups are used. For every unit of detailed hand search and detailed hand search with open bag trace available, two hand wand devices were available. All three cases considered for MPAP show that overall security level is lowest when the assessed threat values are identical. As for MBAP, treating all passengers the same from a security standpoint is not as effective as identifying passengers that may be high-risk.

All of these cases indicate that when the device availability is low, the severe assessed threat values provide the greatest level of security. The kink in the curves represents shifting from the medium and high risk passengers (assessed threat values of 0.9 and 0.3) to low risk passengers. For higher level of device availability, the random assessed threat values provide the greatest level of security. This can be explained by considering that when resources are scarce, being able to identify the few passengers of the highest risk provides the greatest level of security. When the assessed threat values are distributed more evenly, such as in the uniform case, more passengers are identified as being of high risk, and so more passengers must be screened with greater scrutiny to provide a greater change in the level of security.

In both the MBAP and MPAP example, it is seen that using CAPPS to identify high-risk passengers and screening these passengers with more effective, and possibly more expensive, security devices improves the overall security level. This can be compared to the case when all passengers are treated as indistinguishable and passengers are screened with greater scrutiny only at random. In other words, targeting passengers who may



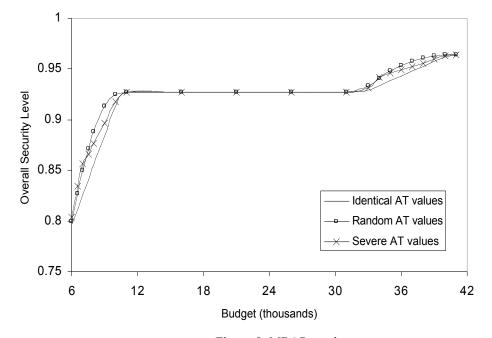


Figure 5: MBAP results

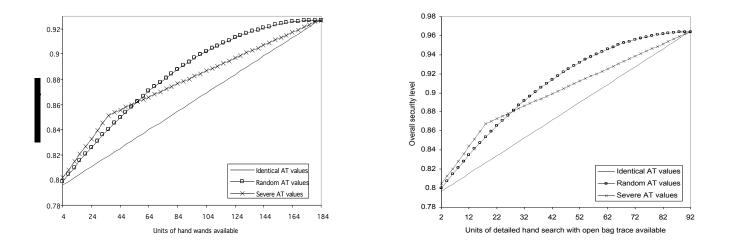


Figure 6: MPAP results when (a) risk groups 1 and 2 are used and (b) risk groups 1 and 3 are used

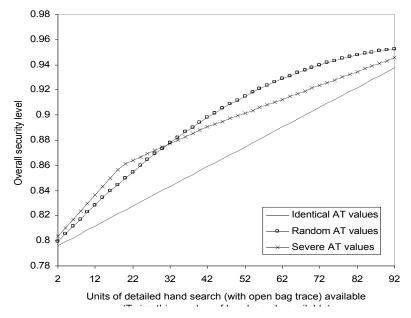


Figure 7: MPAP results when all risk groups are used

#### PART SIX

#### **Implications and Conclusions**

The critical need for improved aviation security strategies for passenger prescreening has driven the development of new and improved screening security devices. The optimal deployment of these devices is critical to take full advantage of each device's capabilities. The two approaches in this paper focus on partial baggage screening and multilevel screening strategies. The first approach targets *objects* that may be of risk and the latter approach targets *people* that may be of risk. The partial baggage screening problems introduced can be used to determine the optimal deployment of baggage screening security devices across a set of airports, based on three different security performance measures. The relationship between these three performance measures is an area of current research, along with the possibility of simultaneously optimizing all three of these measures, using multicriteria optimization tools. The multilevel screening problems introduced can be used to determine the optimal assignment of passengers to predefined risk groups in order to reduce the probability of a successful attack given that a passenger is a threat. The results of the multilevel screening examples indicate that using CAPPS to identify high-risk passengers can improve security procedures, even when additional devices or screening personnel are available.

To effectively use and implement the proposed problems, a large amount of data may be needed. Since some of this data, such as the flight routings of passengers, may be unavailable or difficult to obtain from the airlines, work is in progress to modify the problems to include only readily available or easily estimated data. Moreover, while the problems can provide useful deployment strategy information, the data that supports it must be estimated and is constantly changing. This may require deployment decisions to be constantly updated, based on such new information.

The examples provide an initial level insight into the optimal deployment of baggage screening security devices across a set of flights between a set of airports and into the optimal passenger assignment to risk groups. Work is in progress to extend these problems to cover a broader scope of baggage screening deployment issues, as well as to study such problems using multi-criteria tools, hence to better understand how to design more effective aviation security systems. Furthermore, an analysis of these problems under dynamic conditions may shed some light on how effective decisions can be made in real-time as passengers arrive for check-in. The analysis presented here provides a first step towards achieving such objectives.

#### APPENDIX

#### Mathematical Formulations for Partial Baggage Screening Problems

This section contains the decision problems and the corresponding integer programming models for the baggage screening problems using the three security performance measures described in Part Three. The difficulties associated with finding an optimal baggage screening security device deployment that satisfies physical and operational constraints (such as not exceeding budget levels) are also discussed. The decision problems formulated are all NP-complete.<sup>21</sup>

The following assumptions were made for this research. The flow of selectee baggage from airport *i* to airport *j* is denoted by  $f_{i,j}$  and can be further described to capture connections. This information is needed to account for selectee baggage in transit. For a given set of flights out of Airport A<sub>i</sub>, let  $g_{i,j}$  denote the average number of selectee bags on direct flights from Airport A<sub>i</sub> to Airport A<sub>j</sub>, and let  $m_{i,j,k}$  denote the average number of selectee bags originating at Airport A<sub>i</sub> with final destination Airport A<sub>k</sub>, connecting through Airport A<sub>j</sub>. Therefore,

 $f_{i,j} = g_{i,j} + m_{i,j,k} + m_{h,i,j}$   $i, j = 1, 2, ..., N, i \neq j.$ 

Note that the values for  $g_{i,j}$  and  $m_{i,j,k}$  can also be computed using information extracted from an airlines reservation system. However, the data collection requirement for such detailed, passenger-by-passenger information may be prohibitively expensive to secure, given the volume of traffic that must be accounted for.

Let R denote the number of baggage screening security device types available, each with a capacity ( $\lambda_r$ ) per sixty minute period, a purchase cost ( $p_r$ ), an average annual maintenance and operation cost (including airport rental space fees for the device) ( $o_r$ ), and a number of available devices of type r ( $s_r$ ), r = 1, 2, ..., R. Lastly, let B denote the budget available to cover the purchase of the baggage screening security devices and the first year of operating cost for a given set of airports.

The Multiple Airport Direct and Connecting Baggage Problem (MADCBP) is a decision problem for determining whether a feasible allocation of baggage screening security devices exists such that a minimum number of baggage segments are screened.

#### The Multiple Airport Direct and Connecting Baggage Problem (MADCBP)

- A set of N airports  $A = \{A_1, A_2, \dots, A_N\}$
- a set of M individual baggage screening devices,  $D = \{d_1, d_2, \dots, d_M\}$
- a cost for each element of D,  $C(d) \in Z^+$ ,  $d \in D$
- a baggage screening capacity for each element of D,  $\lambda(d) \in Z^{\dagger}$ ,  $d \in D$
- a baggage screening security device allocation budget  $B \in Z^+$
- the number of originating selectee bags on direct flights from each Airport A<sub>i</sub> available to be screened, q<sup>1</sup><sub>i</sub>∈ Z<sup>+</sup>, i = 1, 2,..., N
- the number of originating selectee bags on connecting flights from each Airport A<sub>i</sub> a available to be screened,  $q_i^2 \in Z^+$ , i = 1, 2,..., N
- the minimum number of selectee bags to be screened,  $\alpha \in Z^+$

*Question:* Is there a set of N subsets of baggage screening security devices  $D_i \subseteq D$ , i = 1, 2, ..., N, where  $D_i \cap D_j = \emptyset$ , for all i, j = 1, 2, ..., N,  $i \neq j$ , and a number of bags to screen  $\hat{q}_i^1 \leq q_i^1$  and  $\hat{q}_i^2 \leq q_i^2$  such that

$$C(d) \le B, \ \hat{q}_i^1 + \hat{q}_i^2 \le \lambda(d), \ i = 1, 2, \dots, N, \text{ and } (\hat{q}_i^1 + 2\hat{q}_i^2) \ge \alpha?$$

Theorem 1: The MADCBP is NP-complete.<sup>22</sup>

Instance:

The term  $\hat{q}_i^1 + \hat{q}_i^2 \leq \lambda(d)$  captures whether the capacity of the baggage screening security devices

deployed exceeds the number of selectee bag segments screened, while the term  $(\hat{q}_i^1 + 2\hat{q}_i^2) \ge \alpha$  captures

the minimum number of selectee bags that can be screened. Note that each selectee bag on a connecting flight, if screened at origination, will be covered for two flight segments, thus impacting the number of covered baggage segments twice (hence the coefficient two in this expression).

The Multiple Airport Direct and Connecting Flight Problem (MADCFP) is a decision problem for determining whether a feasible allocation of baggage screening security devices exists such that a minimum number of flight segments,  $\chi$ , can be screened. Therefore, the maximum value of  $\chi$  that satisfies the conditions of the MADCFP will minimize UFS.

#### The Multiple Airport Direct and Connecting Flight Problem (MADCFP)

- N, A, D, C(d) for  $d \in D$ ,  $\lambda(d)$  for  $d \in D$ , and B from the instance of MADCBP
  - a set of flights departing Airport  $A_i$ ,  $F_i$ , i = 1, 2, ..., N
  - the number of direct originating selectee bags on flight  $f \in F_i$ ,  $i = 1, 2, ..., N, Q^d(f)$
  - the number of originating connecting selectee bags on flight  $f \in F_i$ ,  $i = 1, 2, ..., N, Q^{\circ}(f)$
  - the minimum number of flight segments to be screened,  $\chi \in Z^+$

*Question:* Is there a set of N subsets of baggage screening security devices  $D_i' \subseteq D$ , i = 1, 2, ..., N, where  $D_i' \cap D_j' = \emptyset$ , for all i, j = 1, 2, ..., N,  $i \neq j$ , and a subset of flights that are covered  $F_i' \subseteq F_i$  at each Airport A<sub>i</sub>, i

=1, 2,..., N (i.e., all selectee baggage on each  $f \in F_i'$ , i = 1, 2, ..., N, is screened) such that  $C(d) \leq C(d) \leq C(d) \leq C(d)$ 

B, 
$$Q^{d}(f) + Q^{o}(f) \leq \lambda(d)$$
, for each  $i = 1, 2, ..., N$ , and  $|F_{i}'| \geq \chi$ ?

Theorem 2: The MADCFP is NP-complete.

The Multiple Airport Direct and Connecting Passenger Problem (MADCPP) is a decision problem for determining whether a feasible allocation of baggage screening security devices exists such that a minimum number of passenger segments can be covered. Therefore, the maximum value of  $\delta$  which satisfies the conditions of the MADCPP will minimize UPS.

#### The Multiple Airport Direct and Connecting Passenger Problem (MADCPP)

- N, A, D, C(d) for d∈ D, λ(d) for d∈ D, and B from the instance of MADCBP,
  F<sub>i</sub>, i = 1, 2,..., N; Q<sup>d</sup>(f), f∈F<sub>i</sub>, i =1, 2,..., N; Q<sup>o</sup>(f), f∈F<sub>i</sub>, i =1, 2,..., N; from the instance of MADCFP
- the total number of passengers on flight  $f \in F_i$ , i = 1, 2, ..., N,  $P(f) \in Z^+$ ,
- the minimum number of passenger segments to be screened,  $\delta \in \mathbb{Z}^+$ .

*Question:* Is there a set of N subsets of baggage screening security devices  $D_i \subseteq D$ , i = 1, 2, ..., N, where  $D_i \cap D_j = \emptyset$ , for all i, j = 1, 2, ..., N,  $i \neq j$ , and a subset of flights that are covered  $F_i \subseteq F_i$  at each Airport A<sub>i</sub>, i

= 1, 2,..., N, such that  $C(d) \le B$ ,  $Q^{d}(f) + Q^{o}(f) \le \lambda(d)$ , for each i = 1, 2,..., N, and

 $P(f) \ge \delta$ ?

Theorem 3: The MADCPP is NP-complete.

Note that MADCBP, MADCFP, and MADCPP all remain NP-complete even if the flights are restricted to only direct flights.

The integer programming models for obtaining optimal baggage screening security device deployments are described. The resulting models are designed to optimally determine where to assign the different baggage screening security devices and which selectee baggage to screen, where optimality is based on minimizing either

Instance:

Instance:

the number of uncovered baggage segments, the number of uncovered flight segments, or the number of uncovered passenger segments (hence resulting in three different integer programming models).

In these three models, all decision variables are either non-negative integers or binary. The following is a list of decision variables for these models.

- X<sub>i,r</sub> = the number of units of baggage screening security device type r = 1, 2,..., R, deployed at Airport A<sub>i</sub>, i = 1, 2,..., N, where R denotes the number of different types of baggage screening security devices
- $U_{i,j}$  = the number of uncovered originating direct selectee bags leaving Airport A<sub>i</sub> for Airport A<sub>j</sub>, i,j = 1, 2,..., N,  $i \neq j$
- $T_{i,j,k}$  = the number of uncovered originating connecting selectee bags leaving Airport A<sub>i</sub> enroute to connecting Airport A<sub>j</sub> with final destination Airport A<sub>k</sub>, i,j,k = 1, 2,..., N, i \neq j \neq k
- $V_{i,j,k}$  = the number of uncovered non-originating connecting selectee bags originating at Airport A<sub>i</sub>, leaving Airport A<sub>j</sub> with final destination Airport A<sub>k</sub>, i,j,k = 1, 2,..., N, i \neq j \neq k
- $F_{i,j} = 1(0)$  if the flight from Airport A<sub>i</sub> to Airport A<sub>j</sub> is (un)covered, i, j = 1, 2, ..., N,  $i \neq j$

The objective functions, corresponding to the three security performance measures, are defined. The objective function for minimizing the number of uncovered bag segments is

The objective function for minimizing the number of uncovered flight segments is

ستع Minimize (1-F<sub>i,i</sub>)

The objective function for minimizing the number of uncovered passenger segments is

Minimize  $(\mathbf{c}_{i,j} \bullet \sigma_{i,j}) (1-\mathbf{F}_{i,j})$ 

Since the number of uncovered flight segments and the number of uncovered passenger segments are affected only by covered flights, then the formulations presented here do not consider partial screening. That is, if there is not enough capacity available to cover a flight, then a baggage screening security device is not used to screen additional bags. Therefore, it is unlikely that all the available baggage screening capacity will be used. The constraints for minimizing the number of uncovered flight segments and uncovered passenger segments include (note that the terms in parenthesis represent a relaxation to allow screening in transit, if permitted):

i. The purchase cost plus the first year annual operating and maintenance budget of the baggage screening devices must not exceed the given budget

 $[p_r + o_r] X_{i,r} \leq B$ 

ii. The baggage screening security device capacity plus the number of uncovered originating direct and connecting selectee bags leaving Airport A<sub>i</sub> (plus the number of uncovered non-originating selectee bags leaving Airport A<sub>i</sub>) must equal or exceed the average number of originating selectee bags leaving Airport A<sub>i</sub> (plus any uncovered connecting selectee bags)

$$\lambda_r X_{i,r} + U_{i,j} + T_{i,j,k} + V_{h,i,j}$$

 $g_{i,j} + m_{i,j,k} + T_{h,i,j}, \quad i = 1, 2, ..., N$ 

- iii. The number of uncovered originating selectee bags on a route cannot exceed the average number of originating selectee bags for that route.
  - (a)  $U_{i,j} \le g_{i,j}$   $i, j = 1, 2, ..., N, i \ne j$

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- (b)  $T_{i,j,k} \le m_{i,j,k}$  i, j, k = 1, 2,..., N, i  $\ne j \ne k$
- iv. When baggage screening in transit is not permitted (as in most cases), then all unscreened non-originating connecting selectee bags leaving Airport A<sub>j</sub> remain unscreened, and V<sub>i,j,k</sub> is equal to T<sub>i,j,k</sub>, hence (a) is used. If baggage screening in transit is permitted, then V<sub>i,j,k</sub> is not fixed, but rather bounded by the number of uncovered connecting selectee bags on the route, hence (b) is used.

(a) 
$$V_{i,j,k} - T_{i,j,k} = 0$$
 i, j, k = 1, 2,..., N, i  $\neq j \neq k$ ,

(b) 
$$V_{i,j,k} - T_{i,j,k} \le 0$$
 i, j,  $k = 1, 2, ..., N$ ,  $i \ne j \ne k$ .

*\_\_* 

v. A flight segment is uncovered if any unscreened baggage exists on that flight segment from either unscreened baggage in transit or unscreened originating baggage, hence, F<sub>i,j</sub> is forced to zero by the minimizing function if any selectee bags on a flight from Airport A<sub>i</sub> to Airport A<sub>j</sub> are left uncovered, and forced to one otherwise. Note this constraint is only necessary when minimizing the uncovered flight segment or uncovered passenger segment performance measures.

$$f_{i,j}(1\text{-}F_{i,j})\text{-}U_{i,j}\text{-} \qquad T_{i,j,k}\text{-} \qquad V_{h,i,j} \geq 0 \quad i,\,j=1,\,2,\ldots,\,N, \qquad i\neq j$$

The size of the resulting integer programming models is polynomial in the number of airports (N) and the number of baggage screening security device types (R). In the worst case, with flights between each airport and baggage in transit at each airport, there are  $N(2N^2 - 5N + R + 3)$  integer variables and N(N-1) binary variables. In addition, there are  $N(N-1)^2$  simple bound constraints (i.e., variables constrained by an integer value). Lastly, the number of constraints (excluding simple bounds) is  $N^3 - 3N^2 + 3N + 1$  when minimizing the uncovered baggage segment performance measure and  $N^3 - 2N^2 + 2N + 1$  when minimizing the uncovered flight segment or uncovered passenger segment performance measures.

To approximate the number of transferring selectee bags on a flight, for each flight into the hub airport, distribute the selectee bags on the incoming flight among the flights departing the hub airport based on the number of seats on each departing flight, and round to the nearest integer. If, due to rounding, the sum of the transferring selectee bags is larger (smaller) than the total number of selectee bags from the originating airport to the hub airport, then round down (up) the value(s) previously rounded up (down) with a fractional component nearest 0.5, as needed. Also, any value of transferring selectee bags greater than one should be rounded down before any value less than one is rounded down. If the origin of the flight into the hub airport is the same as the destination of the departing flight out of the hub airport, then that portion of selectee bags are direct route bags.

These results were found by solving the integer programming models formulations using CPLEX 7.0. The approximate run-times for minimizing UBS, UFS, and UPS were 1800, 700, and 800 CPU seconds, respectively, using an Intel® Pentium® III Xeon™ processor (approximately 550MHz).

#### **Mathematical Formulation for Multilevel Screening Problems**

This section introduces a general framework for multilevel security screening optimization problems, as well as two optimization models that incorporate multilevel security screening strategies. For each of these problems, the discrete optimization model and associated integer programming model are described.

The Multilevel Budget Allocation Problem (MBAP) is first stated as an optimization problem and then formulated as an integer programming model. The MBAP is formally stated.

The Multilevel Budget Allocation Problem (MBAP)

Instance:	-	A set of N passengers, each characterized by an assessed threat value AT <sub>1</sub> , AT <sub>2</sub> ,, AT <sub>N</sub>
		with $0 < AT_i \le 1, i = 1, 2,, N$

- a set of M risk groups
- a fixed cost associated with each risk group FC<sub>1</sub>, FC<sub>2</sub>, ..., FC<sub>M</sub>
- a marginal cost associated with each risk group MC<sub>1</sub>, MC<sub>2</sub>, ..., MC<sub>M</sub>
- the total budget B
- the security level of each risk group,  $L_i$  where  $0 \le L_i \le 1$ , i = 1, 2, ..., M, where  $L_i = 1 FCR_i$ , where FCR<sub>i</sub> is the false clear rate associated with risk group *i*
- the *risk level* of each risk group,  $R_i$ , *i* =1, 2,...,M, a function equivalent to the proportion of the assessed threat values of the passengers assigned to risk group *i*

*Question:* How can the N passengers be assigned to the M risk groups, denoted by  $A_1, A_2, ..., A_M$ , where  $A_i$  represents the passengers who are assigned to risk group *i* and each passenger is assigned to exactly one risk

group, such that the budget constraint is satisfied (i.e.,  $\sum_{i=1}^{M} MC_i |A_i| + \sum_{i \in A_i|>0} FC_i \le B$ ) and the overall

security level (i.e., 
$$\sum_{i=1}^{M} L_i R_i$$
) is maximized?

Theorem 4: The MBAP is NP-complete.<sup>23</sup>

The assessed threat values and the false clear rates can be estimated with data available from CAPPS and the TSA. The assessed threat values provide risk assessment measures for each passenger (scaled between zero and one) and are determined by the TSA. The risk level is an increasing function of the partition of passengers assigned to risk groups, where  $R_i=0(1)$  if no (all) passengers are assigned to risk group *i* and can alternatively be written as

$$R_i = \frac{\sum_{j \in \mathcal{A}_i} AT_j}{\sum_{j=1}^N AT_j} \tag{1}$$

The security level of each risk group (scaled between zero and one) is based on security procedures of each device used to screen passengers in that risk group. The risk level for risk group *i* may be defined as the probability that risk group *i* contains a passenger who is a threat given that the passenger population contains a passenger who is a threat. The overall security level may be defined as the probability a threat is detected given the passenger population contains a passenger who is a threat. Since each of the passengers in risk group *i* are screened individually, the probability that a risk group *i* passenger who is a threat is identified is  $L_i$ . Define the following events:

- D = a threat is detected in the passenger population
- T = the passenger population contains a threat
- $C_i$  = risk group *i* contains a passenger who is a threat, *i* = 1, 2,..., M

By conditioning on which risk group contains the threat, the overall security level can be expressed as

$$P(D | T) = \sum_{i=1}^{M} P(D | C_i, T) P(C_i | T)$$
  
=  $\sum_{i=1}^{M} L_i R_i$ 

The MBAP can also be formulated as an integer program (2) with binary decision variables  $y_{ij}=1$  (0) if passenger *j* is (not) assigned to risk group *i*, *i* = 1, 2,..., M, *j* = 1, 2,..., N, and  $x_i = 1$  (0) if there is (not) at least one passenger assigned to risk group *i* = 1, 2,..., M.

$$\max \sum_{i=1}^{M} L_{i}R_{i} = \frac{1}{\sum_{j=1}^{N} AT_{j}} \sum_{i=1}^{M} \sum_{j=1}^{N} L_{i}AT_{j}y_{ij}$$
  
subject to 
$$\sum_{i=1}^{M} \sum_{j=1}^{N} MC_{i}y_{ij} + \sum_{i=1}^{M} FC_{i}x_{i} \le B$$
(2)

$$\sum_{i=1}^{M} y_{ij} = 1, j = 1, 2, ..., N$$
  

$$x_i - \frac{1}{N} \sum_{j=1}^{N} y_{ij} \ge 0, i = 1, 2, ..., M$$
  

$$y_{ij} \in \{0,1\}, i = 1, 2, ..., M, j = 1, 2, ..., N$$
  

$$x_i \in \{0,1\}, i = 1, 2, ..., M$$

In (2), the objective is to maximize the overall security level, which is captured by the sum of the products of the security and risk levels using the formulation for  $R_i$  given in (1). The first constraint is the budget feasibility constraint. The second set of constraints ensures that each passenger is assigned to exactly one risk group. The third set of constraints ensures the fixed costs are included for all nonempty risk groups. The last two sets of constraints indicate that  $x_i$  and  $y_{ij}$  are 0-1 binary variables.

Note that for the special case of all the risk groups being used, the budget is reduced by the sum of the fixed costs, which simplifies the first constraint to

$$\sum_{i=1}^{M} \sum_{j=1}^{N} MC_{i} y_{ij} \le B - \sum_{i=1}^{M} FC_{i}$$

For this case, the third and fourth set of constraints and the variables  $x_1, x_2, ..., x_M$  can be deleted from (1). However, this special case remains NP-hard to solve.

The Multilevel Passenger Assignment Problem (MPAP) is similar to the MBAP except that the device capacities are the limiting factors. The MPAP is first formally stated as a discrete optimization problem and then formulated as an integer programming model. The objective is to maximize the probability that a threat is detected, given a threat exists for a set of devices used by each risk group and a finite capacity associated with each device type.

The Multilevel Passenger Assignment Problem (MPAP)

Instance:

- A set of N passengers, each characterized by an assessed threat value  $AT_1, AT_2, ..., AT_N$ with  $0 < AT_i \le 1, i = 1, 2, ..., N$
- a set of M risk groups
- a set of V devices types, where device k has capacity  $c_k$ , k = 1, 2, ..., V
- a subset of risk groups associated with each device type,  $D_1, D_2, ..., D_V$ , and, equivalently, a subset of device types that are associated with each risk groups,  $D'_1$ ,  $D'_2$ ,..., $D'_M$
- the security level of each risk group,  $L_i$  where  $0 \le L_i \le 1$ , i = 1, 2, ..., M, where  $L_i = 1 FCR_i$ , where FCR<sub>i</sub> is the false clear rate associated with risk group *i*
- the *risk level* of each risk group,  $R_i$ , i = 1, 2, ..., M, a function equivalent to the proportion of the assessed threat values of the passengers assigned to risk group *i*

*Question:* How can the N passengers be assigned to the M risk groups, denoted by A<sub>1</sub>, A<sub>2</sub>,..., A<sub>M</sub>, where A<sub>i</sub> represents the passengers who are assigned to risk group *i*, such that each passenger is assigned to exactly one risk group, each device is within its capacity (i.e.,  $\sum_{i \in D_k} |A_i| \le c_k$ , k = 1, 2, ..., V), and the overall security

level (i.e.,  $\sum_{i=1}^{M} L_i R_i$ ) is maximized?

Theorem 5: The MPAP is NP-complete.

As with the MBAP, the assessed threat values, the security levels of the devices and risk groups, and the risk levels can be estimated with data available from CAPPS and the TSA. The security level of a risk group may be defined as the conditional probability that an arbitrary passenger in the group who poses a threat is detected. The security level of each risk group could also be defined directly from the devices associated with each group, and there are several ways to determine the security level of each group. For example, if the device decisions are independent, the security level of risk group i,  $L_i$ , could be expressed as the probability of a threat being detected by any device:

 $L_i = 1 - \prod_{k \in D'_i} (1 - P\{\text{threat detected by device type } k\})$ 

Another example is to model the security level of each risk group as a "k-out-of-n" system, where a threat must be detected by k or more devices in order to be detected within a risk group.

The MPAP can also be formulated as an integer programming model (3) with binary decision variables  $x_{ij} = 1$  (0) if passenger *j* is (not) assigned to risk group *i*, *i* = 1, 2,..., M, *j* = 1, 2,..., N

$$\max \sum_{i=1}^{M} L_{i}R_{i} = \frac{1}{\sum_{j=1}^{N} AT_{j}} \sum_{i=1}^{M} \sum_{j=1}^{N} L_{i}AT_{j}y_{ij}$$
  
subject to  $\sum_{i \in D_{k}} \sum_{j=1}^{N} x_{ij} \le c_{k}$ ,  $k = 1, 2, ..., V$   
 $\sum_{i=1}^{M} x_{ij} = 1, j = 1, 2, ..., N$   
 $x_{ij} \in \{0,1\}, i = 1, 2, ..., M, j = 1, 2, ..., N$  (3)

In (3), the objective is to maximize the overall security level (i.e., minimize the overall false clear rate). The first set of constraints captures the device capacity constraints (i.e., a passenger can be assigned to a risk group only if none of the capacity constraints of the devices associated with the risk group are exceeded). The second set of constraints ensures that each passenger is assigned to exactly one risk group. The final set of constraints indicates that  $x_{ij}$  are 0-1 binary variables.

The flight data for these problems are found in Table 6. The thirty-minute arrival window of consideration was 8:50 - 10:19 AM. This is based on an enplanement rate of 80 percent, but the arrival window for some of the flights did not entirely coincide with this window, so the enplanement rate for individual flights may have been less than 80 percent.

#### Flight Data for Multilevel Screening Problems

Flight	Departure Time	Seats	Passengers	Flight	Departure Time	Seats	Passengers
1	900	48	6	37	930	158	63
2	900 900	40 112	0 15	38	930 930	194	78
3	900 900	112	15 17	38 39	930 930	194 282	113
4	900 900	120	24	39 40	930 930	202 400	160
4 5	900 905	400	24 80	40 41	930 930	400 400	160
6	905 909	400 19	5	41	930 931	400 194	78
7	909	30	8	43	931	208	83
8	914	19	6	44	935	70	28
9	914	30	10	45	935	128	51
10	914	70	22	46	940	70	28
11	914	128	41	47	944	70	28
12	914	158	51	48	944	158	63
13	914	158	51	49	945	128	51
14	914	158	51	50	945	128	51
15	914	194	62	51	945	128	51
16	915	100	33	52	945	194	78
17	915	128	43	53	945	208	83
18	915	132	44	54	945	303	121
19	915	158	53	55	946	112	45
20	916	112	39	56	946	132	53
21	920	48	19	57	946	158	63
22	920	100	40	58	947	112	45
23	920	112	45	59	947	128	51
24	920	128	51	60	947	128	51
25	920	128	51	61	947	179	72
26	920	132	53	62	948	70	28
27	920	158	63	63	1000	128	34
28	920	208	83	64	1000	158	42
29	920	282	113	65	1000	158	42
30	921	179	72	66	1000	158	42
31	921	179	72	67	1000	179	48
32	925	194	78	68	1000	282	75
33	930	30	12	69	1005	19	4
34	930	112	45	70	1005	19	4
35	930	158	63	71	1010	100	13
36	930	158	63				

Table 6: Flight Data for Multilevel Prescreening Example

#### Notes

<sup>1</sup> Federal Aviation Administration, "FAA Forecast Continued Drop in Air Traffic This Year, Strong Recovery in 2003," Technical Report, Washington, DC, 2002.

<sup>2</sup> These facts as well as the data in Figure 1 are from National Research Council, *Airline Passenger Security* Screening: New Technologies and Implementation Issues (Washington, DC: National Academy Press, 1996).

<sup>3</sup> L.O. Malotky, "Introduction to the Sixth International Civil Aviation Security Conference," *Journal of Testing* and Evaluation 22 (1994): 235-237.

<sup>4</sup> A. Barnett, R. Shumsky, M. Hansen, A. Odoni, and G. Gosling, "Safe at Home? An Experiment in Domestic Airline Security," *Operations Research* 49 (2001): 181-195.

<sup>5</sup> K.M. Mead, "Key Challenges Facing the Transportation Security Administration," Report Number CC-2002-180 (Washington, DC: Office of Inspector General, Department of Transportation, 2002). See also K.M. Mead, "Aviation Security Costs, Transportation Security Administration," Report Number CC-2003-066 (Washington, DC: Office of Inspector General, Department of Transportation, 2003).

<sup>6</sup> Mead, "Aviation Security Costs."

<sup>7</sup> J.M. Loy, Testimony before the Subcommittee on Senate Commerce, Science, and Transportation, Washington, DC, February 5, 2003.

<sup>8</sup> European Regions Airline Association, *Performance: January–December 2002*, April 2003,

http://www.eraa.org/performance\_jandec02.pdf (accessed May 2, 2003).

<sup>9</sup> B. Hoffman, "Aviation Security and Terrorism: An Analysis of the Political Threat to Air Cargo Integrators," *Terrorism and Political Violence* 10 (1998): 54-69.

<sup>10</sup> National Research Council, Airline Passenger Security Screening.

<sup>11</sup> Associated Press, "Hijacking Attempt on El Al Flight Foiled," November 17, 2001.

<sup>12</sup> Hoffman, "Aviation Security and Terrorism."

<sup>13</sup> National Research Council, Airline Passenger Security Screening.

<sup>14</sup> Associated Press, "Hijacking Attempt on El Al Flight Foiled."

<sup>15</sup> M. Robinson, "Make Airline Security as Efficient as Post Office?" Human Events Online, 2001,

http://www.humaneventsonline.com (accessed March 24, 2003).

<sup>16</sup> A summary of international trusted traveler programs can be found in R.W. Poole, Jr., and G. Passantino, "A Risk-Based Airport Security Policy," *Reason Public Policy Institute, Policy Study No. 308*, 2003.

<sup>17</sup> S.H. Jacobson, J.L. Virta, L.A. McLay, and J.E. Kobza, "Integer Programming Models for Deployment of Airport Baggage Screening Security Devices," Technical Report, University of Illinois at Urbana-Champaign, 2003.

<sup>18</sup> Mead, "Aviation Security Costs."

<sup>19</sup> V. Butler and R.W. Poole, Jr., "Rethinking Checked-Baggage Screening," *Reason Public Policy Institute, Policy Study No. 297*, 2002.

<sup>20</sup> Ibid.

<sup>21</sup> See Michael R. Garey and David S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness* (San Francisco: W.H. Freeman and Company, 1979).

<sup>22</sup> For proofs of Theorems 1-3, see Jacobson, Virta, McLay, and Kobza, "Integer Programming Models."

<sup>23</sup> For proofs of Theorems 4 and 5, see McLay, Jacobson, and Kobza (2003).