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Uranium Price vs. Cumulative Use

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URANIUM PRICE VS. CUMULATIVE USE

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Abstract. Formulas are given for extrapolating uranium prices that could result from future trajectories for the cumulative use of native uranium. The logarithm of the extrapolated price is given by a monotonically increasing trend curve plus a sinusoidal oscillation calibrated to historical data. The trend curve as a function of cumulative extraction of native uranium accounts both for accessing lower ore grades and for the exploitation of more difficult to access richer ores as the more easily accessed richer ores are depleted. Accounting for both of these effects, the logarithm of the monotonic price trend is linear in the logarithm of cumulative extraction of native uranium, with least variance between observations and data a power law slope of $1/4.5$ up to the point where a limit on the accessibility of the remaining highest grade ores is reached. (However, a slope of $1/5.6$ gives an almost equally good fit.) As an example, a ratio 4 of maximum depth of other mines to maximum depth of current uranium mines is used as a measure of the accessibility limit. This limit is first reached when the background trend curve uranium price reaches \$143 per kg of elemental uranium in U.S. dollars inflation adjusted to year 2007 prices (\$US2007). Thereafter, the accessibility limit gradually reduces the cumulative amount of native uranium extracted at a given cost below that computed from the power law, multiplying it by a factor of 0.59 when the trend price reaches 300 \$US2007/kg. Increases of nuclear energy produced per kilogram of uranium mined with increasing uranium costs are also accounted for. A fraction of global nuclear energy users can develop a higher nuclear energy production rate per kilogram of mined uranium, e.g. by re-using the fissile material in spent fuel. Resulting cumulative cost changes as a function of cumulative nuclear energy use are presented in graphical and tabular form for a variety of input parameters.

1. Background

Most of the world's nuclear reactor fuel discharge awaits a decision on whether or not its remaining fissile content will be re-used in new reactor fuel. Only Finland and Sweden have committed to specific locations for permanently isolating spent fuel from the environment, and even in these cases the spent fuel could be recovered for re-use if it eventually became economically advantageous to do so. With opening the U.S. Yucca Mountain repository not being favored by the U.S. administration, spent nuclear fuel also has no well defined fate in the country with the world's largest stock thereof. A small fraction of reactor discharges have been fabricated into mixed uranium and plutonium oxides (MOX) and burned in water-cooled reactors, but the fate of reactor discharges from MOX fueled reactors also remains open. Only a minute fraction of reprocessed nuclear reactor discharges has been burned in reactors cooled by liquid sodium, mostly in the now shut down Phénix reactor.

Finland and Sweden have chosen to put spent nuclear fuel in copper containers in a non-oxidizing environment deep underground in crystalline rock (Finnish Energy Industries, 2009; Ström et al., 2008). The Finnish repository is designed to maintain integrity even if sealed only after about one quarter of the strontium-90 and cesium-137 in the latest emplaced fuel remains, and well before the passage of the c. 433 year half life of americium-241. Provided that suitable host rock is chosen, these countries' approach provides high confidence of long term isolation of radioactive materials. A similar solution is technically feasible in most other countries, including the United States.

However, the question remains of whether it is desirable to invest billions of dollars in facilities designed to isolate spent nuclear fuel for millennia if it may become economically advantageous sometime in the current or next few centuries to re-use the fissile material for energy production, as global sources of the most readily accessible high grade uranium ore are depleted. If fuel re-use may become economically advantageous, then the question arises of how long it should be planned to store spent reactor fuel in a retrievable mode before it can be expected to become a valuable resource. Given the conceptual importance of this question, a number of authors have made estimates of the economic impact of depletion of more readily extractable uranium sources. Conclusions vary widely, depending on the time horizon examined and the methodology used (IAEA, 2001; MacDonald, 2003; Dittmar 2009; and Schneider and Sailor, 2007, and references therein).

With the end of the cold war and the recent access of India to the global uranium market, the ready transportability of uranium concentrates and fuels suggests looking at uranium resources on a global basis. This simplifies the analysis. A summary of recent attempts at this (Schneider and Sailor, 2007) concentrates primarily on log-linear estimates of the form $\ln q = s_0 + s_1 \ln p$, where q is the cumulative historical global extraction of native uranium and p is inflation-adjusted production cost. Values of s_1 in the various approaches reviewed by Schneider and Sailor range from 0.5 to 3.5. The value 0.5 results from considering the estimates in a so-called Red Book (e.g. NEA, 2003) of

global resources extractable up to three different costs as sufficiently complete to support such an analysis. The value $s_1 = 3.5$ is the slope drawn by Schneider and Taylor of the high-ore-grade portion of a curve drawn by Defeyes and MacGregor (1980) on a log-log scale of the increments in earth crustal amounts of uranium between various uranium ore concentrations. As an estimate of the values of s_0 to accompany each value of s_1 , Schneider and Sailor quote a reference point (p_0, q_0) from the 2003 Red Book report of (\$40/kgU, 2.523 Mtonne) for cost per kilogram of elemental uranium and millions of metric tons of elemental uranium.

2. Quantity vs. Ore/Uranium Ratio

To re-examine the logic behind using various choices of the quantity-cost relations described by Schneider and Sailor, we ask three questions. First, what slope results from a systematic fit to the underlying estimates of amounts of various uranium ore grades given by Defeyes and MacGregor on a log-log scale?

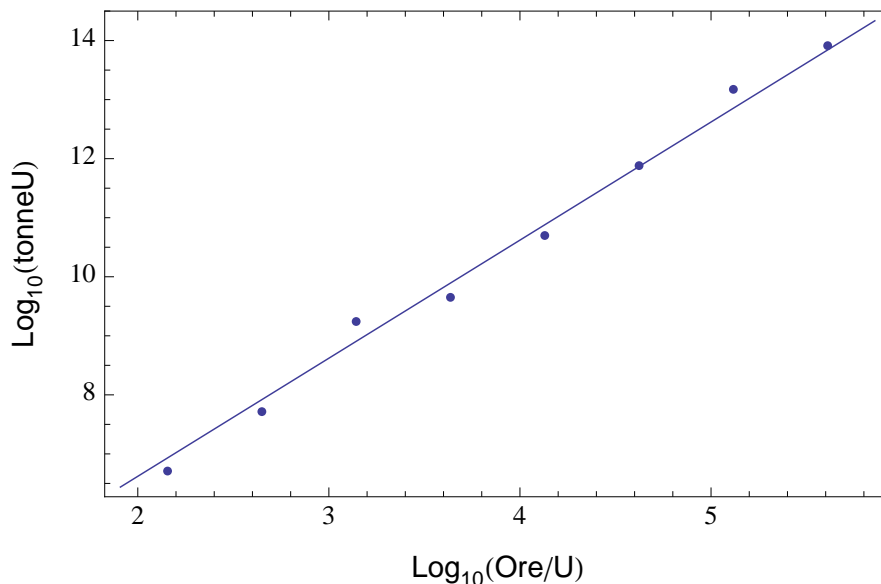


Fig. 1. Least squares “power law” fit to Defeyes and MacGregor’s estimates, here of cumulative crustal amount of uranium amounts in the top 1 km of the earth’s crust on the ordinate, up to the ore/uranium content ratios on the abscissa.

Second, how do the types of estimates given by Defeyes and MacGregor translate into mining practice? Third, to what extent are Red Book figures for amounts at a given cost useful for calibrating uranium price extrapolations? In the discussions denoted (1), (2), and (3) below, these questions are addressed in the context of extraction of uranium from the earth’s crust up to the point

where the alternative of extraction of uranium from seawater might become economically competitive, so only the rising portion of a fit to the Deffeyes and MacGregor estimates is relevant.

(1) For the ore concentrations of interest here, a fit to the rising portion of Deffeyes and MacGregor’s estimates is given in Fig. 1. The variable on the abscissa is the ore/uranium (ore/U) ratio r , which is the inverse of the ore grade. This choice of ore/U for the abscissa variable in Fig. 1 makes the slope $d \ln q / d \ln r$ positive. The least squares fit slope is $s_1 = 2.08$. For the rest of this paper, we round the value of s_1 from 2.08 to 2 for mathematical convenience, since greater precision is not justified by the approximate nature of the input information.

(2) If the cost of uranium were proportional to the ore/U ratio *and* miners responded to depletion of higher grade ores only by moving on to lower grade ores, then a cumulative mined quantity v vs. r relation of the form $\ln v = s_0 + s_1 \ln r$ would be appropriate. However, in fact the response appears to have been both to use lower grade ores and to go after higher grade ores that are less readily accessible.

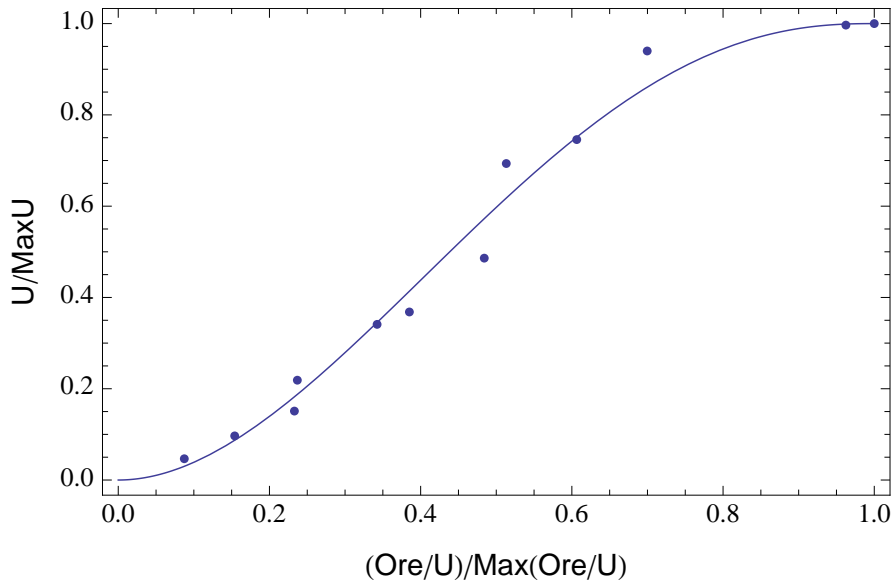


Fig. 2. Cumulative amounts of uranium estimated to be mined annually, from information on 12 mines with actual or planned operations from 2008–2012 for mines up to the ore/uranium ratios on the abscissa, with both the data on the ordinate and abscissa divided by that for the upper right-most point.

The dots in Fig. 2 show the fraction of uranium mined up to a given ore/U ratio as a function of the ore/U ratio, both normalized to the largest values thereof in the plotted data set (from WNA, 2009). If mining progressively moved to lower ore grades with higher ore/U ratios, the data shown in Fig. 2

should hug the zero line until the normalized ore/U ratio nears 1.0 and then abruptly rise. Evidently this is not the case.

The curve fit shown in Fig. 2 was made as follows. Assume that the lowest cost ores are successively extracted, where the cost is $c = ar + d^k$ as a function of the difficulty d of extracting a given amount of ore and of the ore/uranium ratio r . For example, if the cost of extraction a given grade of ore increased linearly with d , e.g. due to the need to dig ever deeper, then we would have the exponent $k = 1$. Let $v_r(c, r) = v_{\text{norm}} \int_{r_1}^{c/a} r(c - ar)^{m_r(s_1-1)} dr$ where solving $c = ar + d^k$ gives $(c - ar)^m = d$ if $m = 1/k$. Here v_{norm} is a scale factor that will be calibrated to historical experience, and r_1 is the lowest ore/U ratio available. The total uranium extracted historically up to the point where the extraction cost is c is $v(c) = v_r(c, c/a)$. The fit shown on Fig. 2 is for $(\partial v_r / \partial c) / (dv/dc)$, which is the incremental extraction up to ore grade r divided by the total incremental extraction dv as c increases by dc , in the limit of small dc . Working out the math gives the result $(\partial v_r / \partial c) / (dv/dc) = 1 - (1 + mx)(1 - x)^2$ with $x = r/r_{\text{max}}$ and r_{max} the maximum ore/uranium ratio for the mines in the data set. The least squares result for the adjustable fitting parameter is $m = 2.47$.

Mines where uranium is extracted with substantial amounts of other minerals can have much higher ore/U ratios because the economics are influenced by the co-product's value, and are thus excluded from Fig. 2. Also omitted from the data and fit shown in Fig. 2 are an equal number of mines with the largest and smallest ore/U ratios. The number of largest and lowest ore ratios so dropped was chosen to minimize the standard deviation at 0.052 of the residuals between the fit and the data shown in Fig. 2. If all of the data are included as in Fig. 3, the value of the fitting parameter is $m = 10.95$, but the standard deviation is 0.21. In the complete data set there are clear anomalies from a set of Canadian mines producing a small fraction of global output with very high ore/U ratios and thus not appropriate to include all as separate data points, as well as some unusually large ore/U ratios at the high ore/U end of the distribution.

Omitting four pairs of largest and smallest ore/U mines includes only the highest of the five high ore/U ratio Canadian mines and yields $m = 3.55$ and a standard deviation of 0.053, nearly the same standard deviation as for the fit in Fig. 2. This could be an appropriate approach if the high ore/U Canadian deposits are not considered anomalous altogether but merely over-weighting the data fit if all included individually. With only three pairs dropped, the fitting parameter is 4.82 and the standard deviation is 0.099. Omitting smaller numbers of high ore/U mines than the number of low ore/U ratio mines gives larger fitting parameters than omitting balanced pairs with the same number of high ore/U ratios omitted. Only if all but the central four ore/U ratios are dropped is the fitting parameter nearly 1 at 1.04, and then the standard deviation is 0.12. The examples shown below are for m being elements of the set $\{1, 5/2, 7/2, 5\}$. The central two values are chosen to illustrate the implications of the data fits with the lowest standard deviations. The extreme values correspond respectively to more theoretical cases with linear cost vs. difficulty of access and with cost depending very weakly on the cumulative amount mined. For values of $m > 5$,

the dependence on cost vs. quantity mined is so weak that there is little practical importance if even higher values are used.

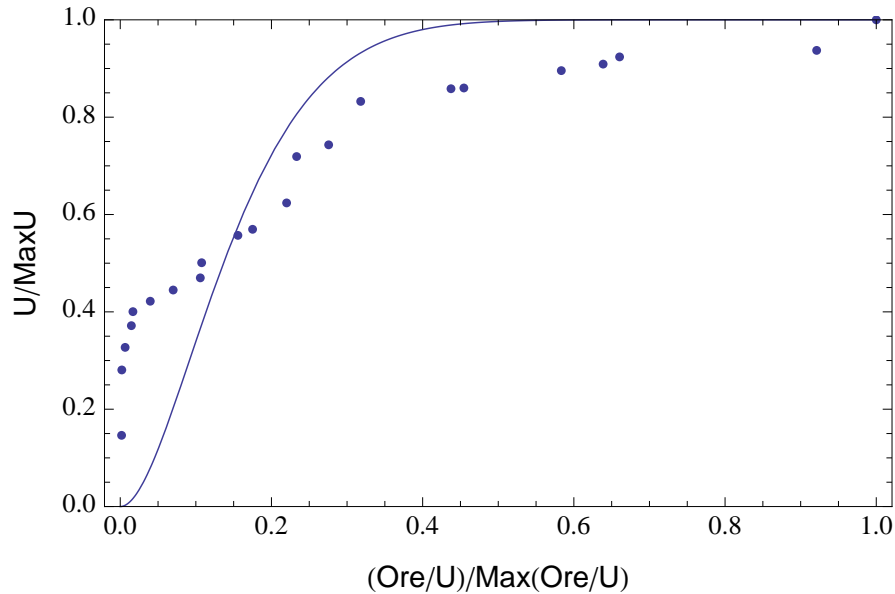


Fig. 3. Cumulative amounts of uranium estimated to be mined annually, from information on 22 mines with actual or planned operations from 2008–2012 for mines up to the ore/uranium ratios on the abscissa, with both the data on the ordinate and abscissa divided by that for the upper right-most point.

(3) There are several difficulties with using Red Book estimates of uranium resources to calibrate uranium price extrapolations, as pointed out in part by Schneider and Sailor. Dittmar (2009) elaborates on historical variations in successive Red Book estimates, albeit without providing an alternative quantitative analysis. One difficulty is apparent from an examination of the historical pattern of reports of the biennial Red Book estimates (Singer, 2001). While Red Book reporting categories have been held constant at \$40/kgU, \$80/kgU, and \$130/kgU, from 1977 to 2007, the U.S. consumer price index increased by a factor of 3.4 from July of 1977 to July of 2007. There has nevertheless been a tendency for the sum of the amounts in the various Red Book reporting categories to increase overall, despite the depletion of earlier resources by mining, not just shift from one cost category to another. A result of this tendency is shown in Fig. 4. That figure plots biennial estimates of resources up to 80 then-current U.S. dollars per kilogram divided by the cost per kilogram at \$80/kg adjusted with the U.S. consumer price index to year 2007 values. The input information was collected from individual Red Books through 2007 (e.g. NEA, 2008), though this is now more conveniently available through 2003 in a consolidated collection (NEA, 2006). The sums plotted in Fig. 4 include the Red Book reporting category “reasonably assured resources” as well as estimated “addi-

tional resources” (which is the sum of “EARI” and “EARI” from 1979 on). These sums also include cumulative production to date on the assumption that most production was likely to have come from the face value \$80/kg or lower category. (See comments below on the relationship between the \$/kg categories and market prices.)

If the Red Book estimation process provided useful insight into absolute amounts of the original geological endowment as a function of cost, then the points in Fig. 4 should show signs of asymptotically flattening out to a constant value as time passes and the knowledge base increases. However, extrapolation of the trend shown in Fig. 4 apparently would not produce any upper limit on original geologic endowment at a particular inflation-adjusted cost, as there is as yet no visible tendency in Fig. 4 that would support such a finding. The estimates given in the Red Books may be useful guidelines to the likely fractions of the uranium market production supplied by various countries over the next few decades (c.f. IAEA, 2001), but evidently not for longer term analysis of uranium price trends.

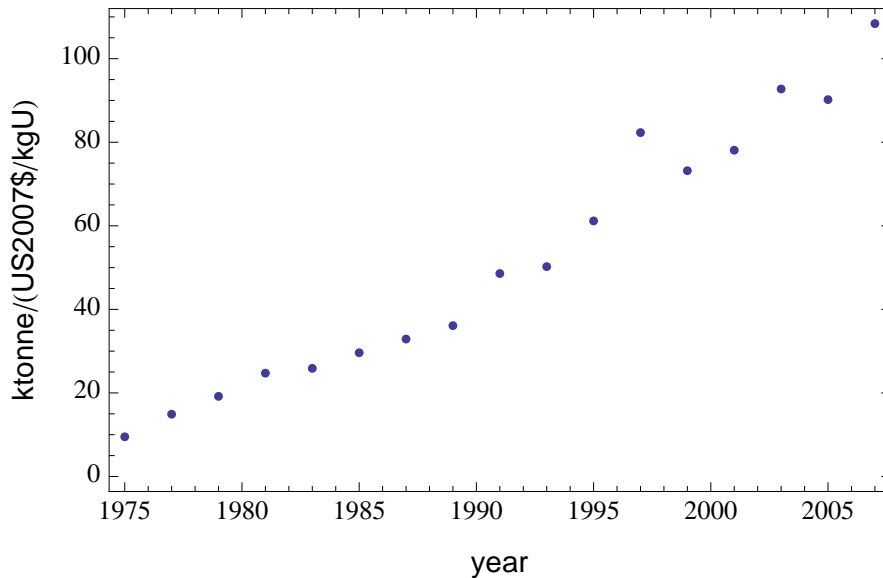


Fig. 4. Estimates of original geological endowment up to face value \$80/kg, divided by the inflation-adjusted value price per kg of \$80/kg. Points plotted at 1981, 1985, and 1987 are from the 1982, 1986, and 1988 Red Books, the only ones issued in even numbered years.

To address directly whether Red Book figures are likely to be underestimates of actual resources, Singer (2001) included the following question amongst those posed in a session scheduled for that purpose at one of the technical committee meetings on Uranium Resources, Production and Demand which included representatives from twenty-five countries, mostly uranium producing countries:

“Have ‘conservative’ biases towards underestimation which are referenced in some of the older literature been removed by recent changes in reporting procedures and coverage, or have they perhaps been overcompensated for.” (For a complete list of the questions posed, see Singer, 2001). With the possible exception of some central Asian countries interested at the time in increasing uranium production and exports, the general response from the assembled experts was that the estimates reported for compilation in the Red Books were conservative. A stated reason for this was quite simply that responsible parties reporting back on successive occasions to their countries would prefer to bring back “good news” that estimates had increased than “bad news” that previous estimates were too high.

Schneider and Sailor suggest that amounts reported in the higher cost categories in the 2003 Red Book are preferentially underestimated compared to fits based on Deffeyes and MacGregor estimates calibrated to the \$40/kgU category. For such a fit to Red Book reports gives a value s_1 of only 0.5 for the coefficient in a log-linear fit of the type described above.

Another difficulty with using Red Book estimates in connection with uranium price estimates is that uranium purchase costs have averaged well over \$40/kgU. This despite the fact that, for example, the 2.523 Mtonne figure for \$40/kgU inferred by Schneider and Taylor from the 2003 Red Book would suggest that more uranium remained available in that cost category than had ever been mined to that date. The methodology used to define the cost categories in the Red Books is not carefully described in each addition. The \$40/kgU category likely reflects only a portion of overall costs, such as ongoing operations costs, not fully accounting for the levelized cost of initial capitalization and de-commissioning, taxes, and profits, that are variously included in time-averaged contract market prices.

3. Uranium Price Oscillation

For all of the reasons just noted, we chose here to calibrate and extrapolate uranium prices based on actual historical prices and cumulative amounts mined, rather than on numbers in Red Book resource categories. As historical uranium prices have by no means increased monotonically with cumulative mining, it is essential to account for price variations around an underlying trend. The fit shown in Fig. 5 does this using the formula

$$\ln(p/p_0) = (\ln(v/v_0))/b + A \text{Sin}[2\pi(t - \tau_2)/\tau_1] \quad (1)$$

where $b = m + 2$.

The logic behind the term $(\ln(v/v_0))/b$ is described in the next section. The points on Fig. 5 represent delivered U.S. contract prices from von Hippel (2009) except for 2009 (WISE, 2009) and before 1973 (NEA, 2006), for which spot market prices were scaled to match the nearest available year’s contract price. All prices hereinafter are inflation adjusted to \$US2007 using July U.S. consumer price indices. The functional form of Eq. (1) corrects the underlying production

cost trend derived from the fits in Figs. 1 and 2. For the $m = 5/2$ case, the resulting damped sinusoidal correction has a phase shift $\tau_2 = \text{Julian year } 1971.4$, oscillation period $\tau_1 = 36.8$ years, and natural logarithm amplitude of 0.89. The following discussion concentrates on the underlying price trend without this oscillating correction, but it should be kept in mind that even if the oscillations are eventually damped they could be of substantial amplitude for many decades if the type of behavior illustrated in Fig. 5 continues well into the future.

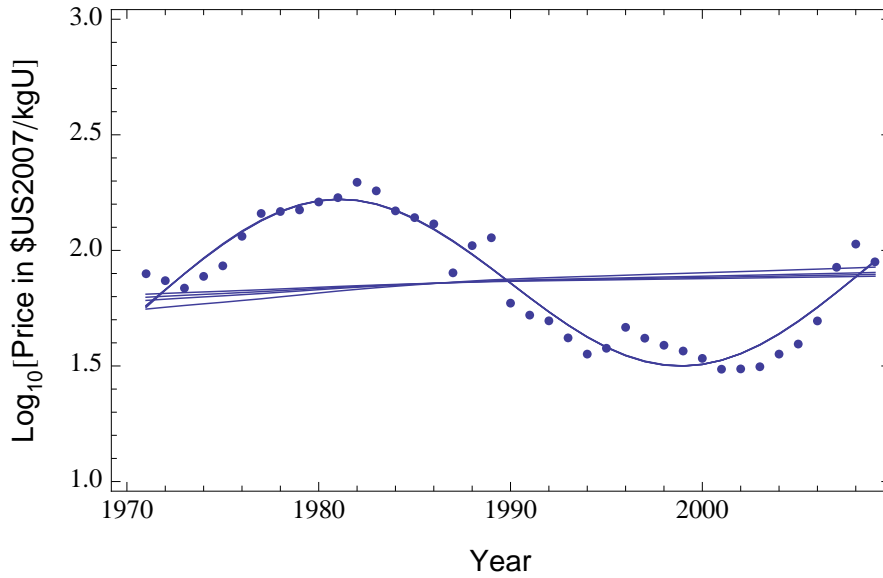


Fig. 5. Uranium prices background trend (monotonically increasing curves) and with damped sinusoidal oscillation of the log of the price (non-monotonic curves). The monotonically increasing curves with the larger slopes correspond to the lower values of m with m chosen from the set $\{1, 5/2, 7/2, 5\}$. The four data fits for these values of m for the non-monotonic curves overlap so closely that they are visually indistinguishable.

The price oscillation in Fig. 5 reflects periods of uranium overproduction compared to consumption rates before 1980 and underproduction during the 1990s. This production pattern reflected both the failure of nuclear energy use to grow as earlier anticipated during the 1980s and the interaction between civilian and military uses of nuclear energy during and for some time after the cold war. Both the technology of uranium mining and the stringency of occupational safety and environmental regulation of uranium mining have evolved considerably as well. There remains considerable uncertainty about how these various influences on the temporal variation of uranium prices will work out in the future. The point here is that inflation-adjusted uranium prices appear on the average to have been larger than $\$40/\text{kgU}$, suggesting a possible disconnect between the cost categories as used in the recent Red Books and the various

mechanisms that determine actual market prices. Another implication of the information shown in Fig. 5 is that it could well be several decades before the influence of uranium resource depletion can be clearly seen in the temporal history of uranium prices. Caution is thus in order concerning long-term capital investments in technologies aimed at reducing the amount of mined uranium used per unit of nuclear energy production on the assumption that very recent price data reflect a strong underlying cost escalation.

4. Uranium Cost Trend vs. Cumulative Uranium Use

When extrapolating the above results to much larger quantities of cumulative uranium mined, eventually a limit will be reached on the lengths to which miners will go to reach the highest concentration ores. Currently the deepest shaft and open pit mines are $\ell = 4$ times deeper than the deepest uranium shaft and uranium open pit mines respectively. (While drilling techniques are used for jet washing of high grade ores and for in situ leaching, these are not the dominant methods of uranium recovery and are thus not considered here for this reference value of ℓ . However, it should be kept in mind that such methods provide another avenue for extraction of resources not economically accessible by shaft and open pit mining.) Allowing for such limitation on the lengths miners will go to recover the highest concentration ores, Appendix A shows that the relationship between the amount v of uranium that can be recovered up to a cost and that cost c is given, with $b = m + 2$, by

$$c/c_m = (v/v_m)^{1/b} \quad \text{for } v \leq v_m \quad (2)$$

$$c/c_m = \left(1 + \sqrt{1 + (2(v/v_m) - mn)n/(bm^2)}\right) m/n \quad \text{for } v \geq v_m \quad (3)$$

Here $n = m + 1$, $c_m = c_0 \ell^{1/m}$, and $v_m = v_0 \ell^{b/m}$.

The difficulty of extraction of uranium is not, of course, just a function of mine depth. The type of materials in the overburden and the ore also impact costs. While Dittmar (2009) points out that the size of deposits also affects costs, if lower concentration ores tend to occur in larger deposits then the effective value of ℓ may be larger than the nominal value of $\ell = 4$ used here. However, initially only the small fraction of the inaccessible ores that have the highest uranium concentrations are excluded when the accessibility limit is encountered, so the results are not very sensitive to the value chosen for this parameter until after much larger amounts of cumulative uranium mining.

A plot of the cost trend c as a function of cumulative uranium use v is given in Fig. 6. This uses year 2007 reference values of $v_0=2.37$ Mtonne and $c_0 = 80\$/\text{kgU}$ as the underlying trend cost for 2007 from the data fits for $m = 5/2$ fit shown in Fig. 5. The solid curve in Fig. 6 shows the result from Eqs. (2) and (3). The dashed curve in Fig. 6 shows the continuation of the formula $c = c_m(v/v_m)^{1/b}$, i.e. without the limitation on accessing the highest concentration ores imposed for $v > v_m$.

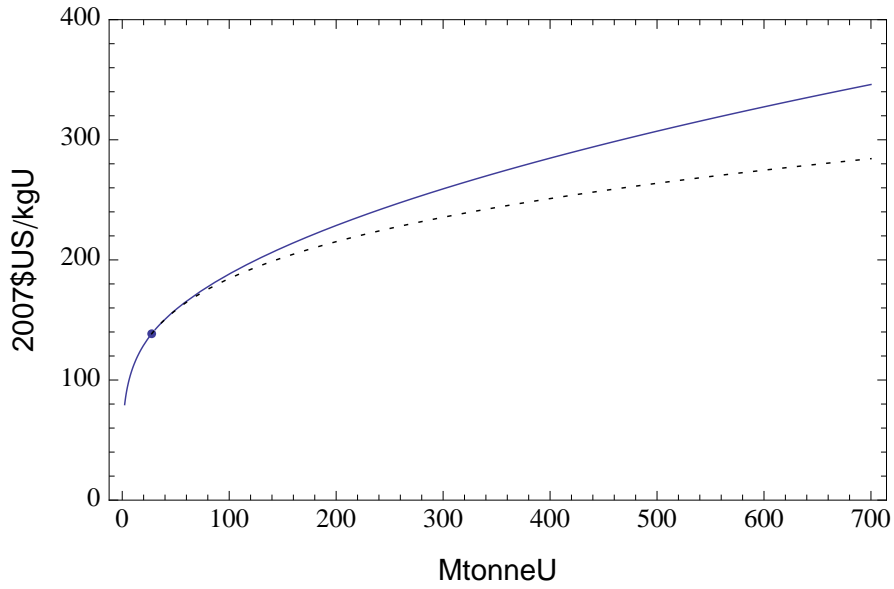


Fig. 6. Uranium cost trend vs. cumulative uranium use with (solid curve) and without (dashed curve) accessibility limit correction included.

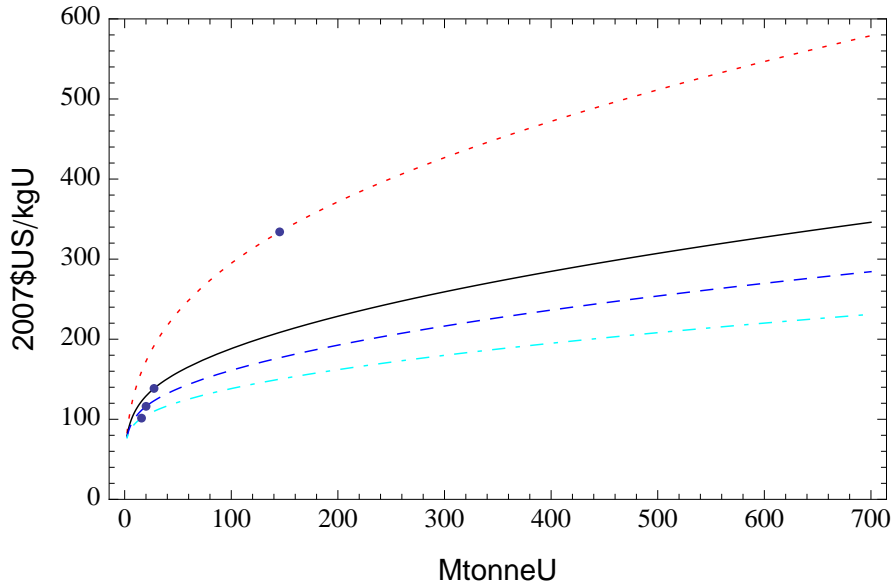


Fig. 7. Uranium cost trend vs. cumulative uranium use for $m = 5$ (dot-dashed), $m = 7/2$ (dashed), $m = 5/2$ (solid), and $m = 1$ (dotted).

Plots of $c(v)$ for values of m from the set $\{1, 5/2, 7/2, 5\}$ are shown in Fig. 7, along with the points $\{v_m, c_m\}$. The larger values of $\{v_m, c_m\}$ for the uppermost $m = 1$ case reflect a preference for more readily accessible lower concentration

ore with the stiffer cost penalty for mining less accessible high concentration ores. Thus the amount of uranium mined before the accessibility limit is first encountered is largest with $m = 1$.

5. Uranium Use Intensity of Nuclear Energy Production

The rate of use of “virgin” mined uranium dv/dt divided by the rate of production of nuclear power du/dt may well change as the price of mined uranium evolves over the long term. One option available for decreasing dv/du is to change the fraction of U-235 in the ore that is loaded into reactors after enrichment in that isotope. Another is to adjust the operation of a reactor so that more of the energy produced comes from plutonium-239 and other isotopes derived from fertile actinides. Spent fuel can also be recycled back into reactors after one or another type of treatment. We do not go into the details of this here, but rather simply examine the consequences of a linear relationship of the form $dv/du = (1 - c/c_b)/(1 - c_0/c_b)$, with c_0 and c_b being constants.

In the equation $dv/du = (1 - c/c_b)/(1 - c_0/c_b)$, the constant c_b is set to $c_b = c_0 + \Delta_c$, for different values of the parameter Δ_c . Here du/dt is the rate of nuclear power use at time t_0 when $c = c_0$, multiplied by the ratio of the uranium use rate to the rate of nuclear energy production at that time. Thus, when $c = c_0$ we have $dv/du = 1$. When $c = c_0 + F_C\Delta_c$, then $dv/du = 1 - F_C$. If uranium trend prices ever reach $c_0 + 0.9\Delta_c$, liquid sodium cooled reactors might be used to bring dv/du down to 0.1. If c rose to $c_0 + 2\Delta_c/3$, then another type of advanced converter reactor might be used to achieve $dv/du = 1/3$. For $c = c_0 + \Delta_c/3$, MOX fuel in water cooled reactors could be used to achieve $dv/du = 2/3$. Additionally or alternatively, devolatilizing spent fuel and mixing the resulting powder with virgin uranium for use in water-cooled reactors and variations in enrichment tails and fuel burn-up in water-cooled reactors might be used to help for $2/3; dv/du < 1$.

The method used here could be adapted to other relationships between dv/du and c based on a more detailed examination of various technological possibilities. However, the simple approach adopted here allows more generically for taking into account that ratios of uranium consumption rates to nuclear energy production rates may vary with uranium costs.

As is currently the case, it may be in the future that some countries pursue policies that lead to lower ratios of the rate of use of mined uranium to the rate of nuclear energy production than other countries do. To account for this, we set

$$dv/du = [F_N(1 - c_N/c_b) + F_R(1 - c_R/c_b)]/(1 - c_0/c_b) \quad (4)$$

Here $c_N(v)$ is given by Eqs. (2) and (3). Taken as a whole, countries following policies that lead to lower uranium to nuclear power use rate ratios are assumed to ramp up the use of the required technology exponentially as a function of v after cumulative global mined uranium exceeds a value v_1 , until it reaches a value v_2 . After that their ratio of mined uranium to nuclear energy use rates stays at constant value $(1 - c_3/c_b)/(1 - c_0/c_b)$ until cumulative use of global

mined uranium reaches the value v_3 at which $c_3 = c_0 + F_C \Delta_c = c(v_3)$. For $v > v_3$, the mined uranium intensity of nuclear energy for the entire world evolves as $dv/du = (1 - c_N/c_b)/(1 - c_0/c_b)$, with the increase in dv/cu driven by rising uranium costs rather than by whatever policy imperatives drove the lower mined uranium intensity in part of the world during the period when $v_1 < v < v_3$. Under these assumptions, the equation for $c_R(v)$ for is

$$c_R = \text{Max}[\text{Min}[c_1(c_3/c_1)^{(v-v_1)/(v_2-v_1)}, c_3], c_N] \quad (5)$$

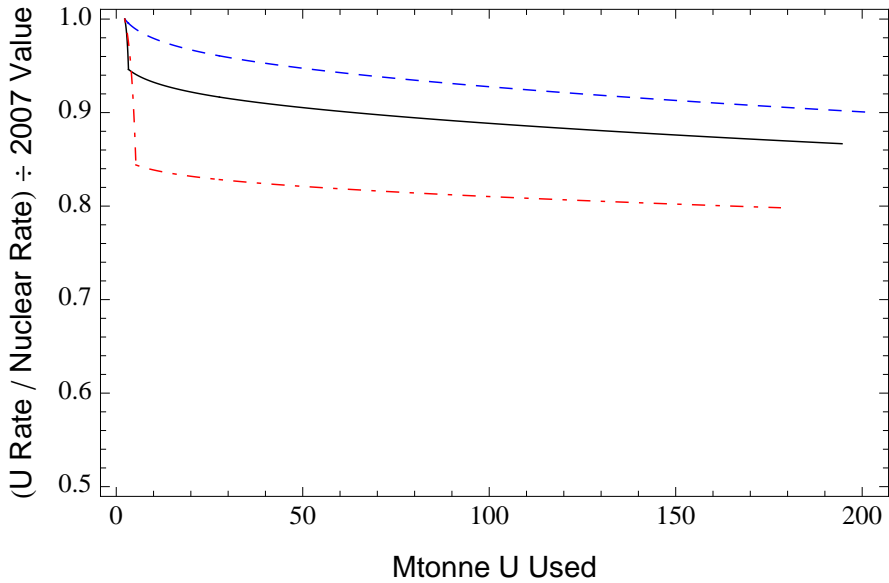


Fig. 8. dv/du curves from Eq. (4) for $F_R = 0$ (dashed), $F_R = 0.15$ (solid), and $F_R = 0.45$ (dotted). Fixed parameters for this figure are $m = 5/2$ and $\Delta_c = \$1500/\text{kgU}$, and $F_C = 1/3$.

Fig. 8 normalizes the amount of mined uranium used per unit of nuclear energy production by the global average for 2007, assuming that average for 2007 is the same as that computed from the 2007 Red Book for 2006. The abscissa in Fig. 8 is cumulative use of mined uranium. Here $v_1 = v_0$, the cumulative mined uranium for the reference year 2007, and $v_2 - v_1$ is the amount of uranium that would be used in $300 F_C F_R$ years at the rate of global use of mined uranium in 2007. For example, for the $F_C = 1/3$ and $F_R = 0.15$ solid curve case in Fig. 8, we have $300 F_C F_R = 15$. Nearly complete global market penetration of technologies with higher F_C values would be expected to take much longer, the time depending to some extent on how fast depletion of natural uranium resources affects uranium prices, as in the approach used here.

Analytic expressions for $u(v)$ that are quite accurate and for most parameter ranges of interest and convenient to implement using a variety of computational

platforms are given in Appendix B. However, in order to have negligible numerical imprecision over a wide parameter range, the equation for dv/du was integrated numerically for the results presented here.

Fig. 9 shows global fueling costs in billions of \$US2007 per terawatt year of nuclear-generated electricity (TWyre) from the above equations, for the same parameters used for Fig. 8. The conversion from uranium use rates in the reference year of TWyre of 4.59 MtonneU/TWyre is multiplied by the values of dv/du in Fig. 8 to convert cumulative uranium use to cumulative nuclear electric energy production when producing Fig. 9.

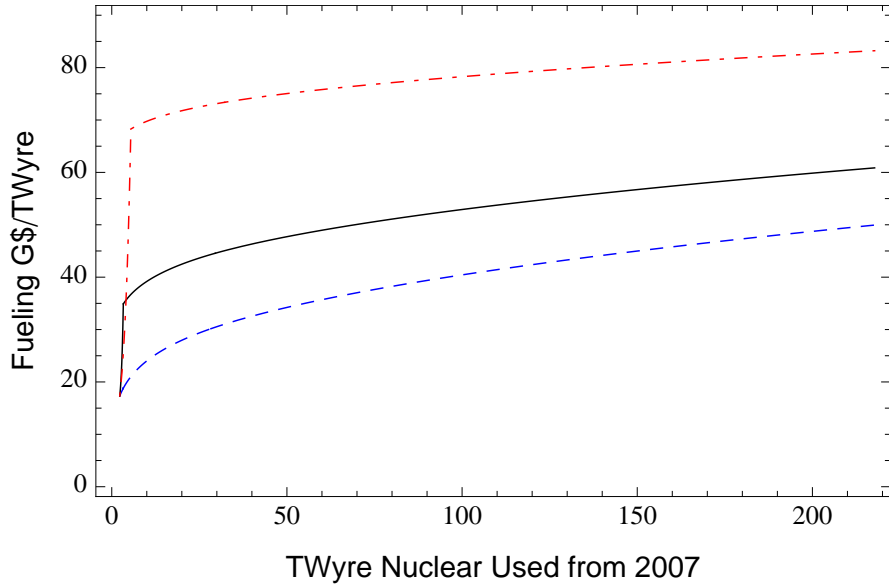


Fig. 9. Fueling cost rate curves for $F_R = 0$ (dashed), $F_R = 0.15$ (solid), and $F_R = 0.45$ (dotted).

The cumulative increments in fueling costs for the $F_R = 0.15$ and $F_R = 0.45$ cases over the $F_R = 0$ case from Fig. 9 are plotted vs. cumulative global nuclear electricity production in Fig. 10. If the higher ratios of nuclear electricity energy production to mined uranium use rates for the $F_R = 0.15$ and $F_R = 0.45$ cases are concentrated in particular groups of countries, then they make only a very modest contribution to fueling cost savings for the rest of the world's nuclear energy use over the cumulative global nuclear electric energy production range covered in Figs. 8 and 9. The primary effect is that, for whatever reason they choose to do so, the countries that contribute to setting higher global average values for F_R simply bear the cumulative extra fueling costs depicted in Fig. 10. If carried on indefinitely, this would eventually result in lowered uranium mining costs. However, with the parameters used for Figs. 7–9, lower annual fueling costs do not begin to pay back the accumulated extra costs for the $F_R = 0.45$ case until 2862 Mtonne of mined uranium has been used, i.e. only in an extremely

distant and highly hypothetical future. Moreover, if the cost of obtaining uranium from seawater is less than \$580/kgU, then the extra up front fueling costs would not start to be recovered until after the seawater source is exhausted, again hypothetically in an even more extremely distant future.

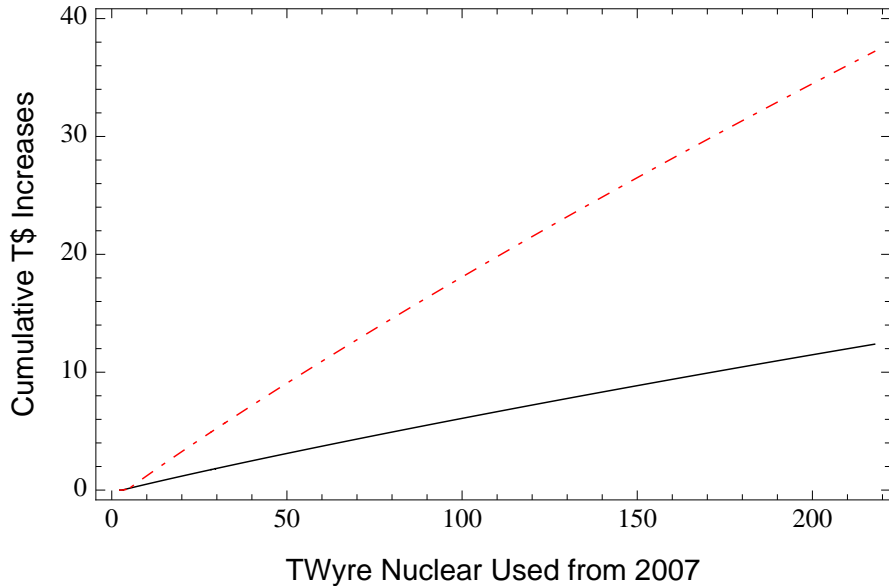


Fig. 10. Cumulative fueling cost increments over the $F_R = 0$ case for $F_R = 0.15$ (solid curve), and $F_R = 0.45$ (dot-dashed curve).

There are various ways to reduce by $F_C = 1/3$ the amount of uranium used per unit of nuclear energy produced. One of these is one round of aqueous/organic extraction of plutonium from nuclear reactor discharges to make MOX fuel. Other possible approaches include using a fleet composed mostly of water-cooled reactors fueled with native uranium, complemented by a fleet of reactors cooled with liquid sodium. The approach used here does not distinguish between various possibilities, but simply uses the above equations to parameterize the cost of reducing the amount of mined uranium per unit of nuclear energy produced. For $F_C = 1/3$, Table I lists the cumulative cost increase over the $F_R = 0$ case after cumulative production of 1000 TWyre beyond reference year 2007, for various values of m , Δ_c , and F_R . As in Figs. 7–9, this is with $v_1 = v_0$ and with the difference $v_2 - v_1$ scaled from the case shown in Fig. 8 in proportion to the product $F_C F_R$.

A large incremental cumulative nuclear energy production of 1000 TWyre was chosen as the point for listing the results in Table I. This is because it is only after a very substantial cumulative nuclear energy production that any of the parameter sets in that table lead to uranium trend price increases comparable to the nuclear energy production cost increment associated with $F_C = 1/3$. Hence

the sizeable numbers in Table I, noting that the units for entries in the last six columns are T\$US2007.

TABLE I
\$Trillion Cost Increment after 1000 TWyre for $F_C = 1/3$

F_R =Fraction of World Adopting $F_C = 1/3$ Early

b = $m + 2$	Δ_c \$/kgU	F_R :	0.15	0.30	0.45	0.60	0.75	0.90
7	2100	20	41	61	81	101	121	
7	1800	17	34	51	68	85	102	
7	1500	14	28	42	56	70	83	
7	1200	11	21	32	43	54	64	
7	900	8	15	23	30	38	45	
11/2	2100	20	39	59	78	98	117	
11/2	1800	16	33	49	66	82	98	
11/2	1500	13	26	40	53	66	79	
11/2	1200	10	20	30	40	50	60	
11/2	900	7	14	21	28	35	42	
9/2	2100	19	38	56	75	94	112	
9/2	1800	16	31	47	62	78	94	
9/2	1500	12	25	37	50	62	75	
9/2	1200	9	18	28	37	46	56	
9/2	900	6	12	18	24	31	37	
3	2100	16	31	47	62	78	94	
3	1800	13	25	37	50	63	75	
3	1500	9	18	28	37	47	57	
3	1200	6	12	19	25	32	38	
3	900	3	6	10	13	16	20	

To put the figure of 1000 TWyre in perspective, for Intergovernmental Panel on Climate Change energy futures scenarios A1G using the MESSAGE software the nuclear electric energy production rate for the year 2100 is 4.3 TWe (IPCC-WG3). The A1G scenario corresponds to a rapid draw down in coal use and concomitant growth in nuclear energy use. With a least squares logistic fit to the temporal evolution of the 1990–2100 nuclear energy use rates in this scenario, and using a thermal to electricity conversion ratio of 0.38, a total of 1000 TWyre is produced globally between 2007 and 2265. The year 2265 is before the original design date for possible cessation of convective cooling in the Yucca Mountain repository. So with such a design the repository would not yet be sealed off from convective cooling. Compared to the A1, A2, B1, and B2 scenarios the A1G scenario has the largest long term nuclear energy use and thus might seem the most favorable for adoption of technologies to reduce the rate of use of

mined uranium per unit of nuclear energy production. Nevertheless, even with the nuclear energy use rate in the extrapolated A1G scenario, for none of the parameter sets in Table I have uranium trend prices increased enough to make achieving $F_C = 1/3$ economically favorable on a fueling cost basis.

Table II shows results for the same cases as in Table I, but after cumulative nuclear electric energy production after 2007 of 200 TWyre. Estimated as described above, this production increment would be reached in 156 years, 154, and 106 years after 2007 in the A2, B2, and A1G scenarios, respectively. (The MESSAGE software outputs for the A1 and B1 scenarios have declining nuclear energy use in 2100 and thus suggest comparatively little long term depletion of global uranium resources and are not of particular interest here.)

The entries in bold type in Tables I and II are results for a reference case with $m = 5/2$, $c_0 + F_C \Delta_c = \$580/\text{kgU}$, and $F_R = 0.15$. The reference parameter value $m = 5/2$ is close to the value of 2.47 from the standard deviation minimization procedure described above. The total cost differential for France for full use of MOX vs. no reprocessing has been estimated as over 600 \$US2006/kgU (von Hippel, 2009). A year 2008 Euratom Supply Agency report estimated that by 2010 MOX fuel would be used in 15% of the world's reactors (EURATOM, 2009).

Is the figure in bold in Table II a best estimate of an actual outcome? On the one hand, it should be noted that a value of $m = 3.55$ resulted from use of additional mining data as described above in Section 3 and gave nearly the same standard deviation. Moreover, Schneider and Sailor a survey of price vs. cumulative amount mined for a large number of other minerals consistent with $b \geq 2.54$, the lowest value 2.54 being for cobalt. That survey included a number of extensively mined minerals, with scarcity variously larger and smaller than uranium, and including elements with a wide range of atomic weights. MacDonald (2003) made the same point about other minerals from a more qualitative perspective. Also, the estimated net cost differential for the more recently implemented MOX fuel use in Japan is over twice that for France (von Hippel 2009), resulting in part from different operations costs and in part from different accounting for the cost of decommissioning and of capital for construction. On the other hand, many of the reactors using MOX currently cannot take a full MOX load and would need to be converted, replaced, or augmented with other reactors in order to reach $F_R = 0.15$ globally.

Thus, unless the cost differential for MOX use globally is preferentially reduced substantially compared to other nuclear system costs, the entries in bold may represent a lower bound on the cumulative cost differentials for $F_R = 0.15$, but a value as large as F_R may not be durably achieved. For the other reference case parameters and smaller values of F_R , long-term cumulative costs scale roughly in proportion to F_R and so may be smaller if this level is not in fact reached globally after the type of phase-in schedule used for the examples here. Time will tell how closely the results with the parameter set $\{m, \Delta_c, F_R\} = \{5/2, \$1500/\text{kgU}, 0.15\}$ approximates an actual future outcome. The figure in bold type in Table II is not meant to predict an actual outcome, but rather to put in current context the overall scale of the numbers listed in that table.

TABLE II

\\$Trillion Cost Increment after 200 TWyre for $F_C = 1/3$

F_R =Fraction of World Adopting $F_C = 1/3$ Early								
b	Δ_c	$F_R :$						
$= m + 2$	\$/kgU	0.15	0.30	0.45	0.60	0.75	0.90	
7	2100	4.1	8.0	11.8	15.4	18.9	22.2	
7	1800	3.5	6.8	10.0	13.1	16.1	18.9	
7	1500	2.9	5.6	8.3	10.8	13.3	15.6	
7	1200	2.2	4.4	6.5	8.5	10.5	12.3	
7	900	1.6	3.2	4.8	6.2	7.7	9.0	
11/2	2100	4.0	7.9	11.6	15.2	18.6	21.9	
11/2	1800	3.4	6.7	9.8	12.9	15.8	18.6	
11/2	1500	2.8	5.5	8.1	10.6	13.0	15.3	
11/2	1200	2.2	4.3	6.3	8.3	10.2	12.0	
11/2	900	1.6	3.1	4.6	6.0	7.3	8.6	
9/2	2100	3.9	7.7	11.4	14.9	18.2	21.4	
9/2	1800	3.3	6.5	9.6	12.6	15.4	18.1	
9/2	1500	2.7	5.3	7.9	10.3	12.6	14.8	
9/2	1200	2.1	4.1	6.1	8.0	9.8	11.5	
9/2	900	1.5	2.9	4.3	5.7	7.0	8.2	
3	2100	3.6	7.2	10.6	13.8	16.9	19.8	
3	1800	3.0	6.0	8.8	11.5	14.1	16.5	
3	1500	2.5	4.8	7.1	9.2	11.3	13.2	
3	1200	1.8	3.6	5.3	6.9	8.5	9.9	
3	900	1.2	2.4	3.5	4.6	5.6	6.6	

Table III gives results as in Table I except for values of $F_C = 2/3$. Table IV gives the same information for $F_C = 9/10$, except that the $m = 5$ case adds little that is useful and is omitted. These values of F_C could result from use of advanced converter reactors or some mix of these with deep actinide burners and water-cooled reactors. Large scale use of such technologies has been proposed in part for adapting to uranium resource depletion in case of rapid expansion of nuclear energy use, e.g. as in the strongest nuclear energy growth scenario described above.

TABLE III

\\$Trillion Cost Increment after 1000 TWyre for $F_C = 2/3$

F_R =Fraction of World Adopting $F_C = 2/3$ Early

b	Δ_c	$F_R :$					
$= m + 2$	\$/kgU	0.15	0.30	0.45	0.60	0.75	0.90
7	2100	43	85	126	167	207	247
7	1800	36	72	107	142	176	210
7	1500	30	59	88	117	145	173
7	1200	23	46	69	92	114	136
7	900	17	34	50	67	83	99
11/2	2100	42	83	124	164	204	243
11/2	1800	35	70	105	139	173	206
11/2	1500	29	58	86	114	142	169
11/2	1200	22	45	67	89	111	132
11/2	900	16	32	48	64	80	96
9/2	2100	41	81	121	161	200	238
9/2	1800	34	68	102	136	169	201
9/2	1500	28	56	83	111	138	164
9/2	1200	22	43	64	86	107	128
9/2	900	15	30	46	61	76	91
3	2100	37	74	111	147	183	219
3	1800	31	61	92	122	153	183
3	1500	24	49	73	97	122	146
3	1200	18	36	54	73	91	110
3	900	12	24	36	48	61	73

The underlying trend cost at c. \$80/kgU of native uranium supply was only about 3.4% of the median year 2004 busbar cost (Kooimey and Hultman, 2007) of nuclear energy in the United States. By way of comparison, the capital cost of the BN 800 liquid sodium reactor under construction in Russia is expected to be about forty percent higher per unit of electric power capacity than that of a comparable capacity water-cooled reactor (Cochran et al., 2010). This is the only liquid sodium reactor nearing completion based on experience with a comparable size and type of such reactor in the same country. Russia has sufficient weapons grade plutonium to fuel its new BN 800 for many years and thus does not need to pay the additional spent fuel reprocessing costs that would also be required for large scale deployment of such reactors.

While actinide burning in liquid sodium reactors could allow a higher packing density in a deep underground nuclear waste depository, it is not clear that there is a net fuel cost savings from this approach after accounting for the capital, operating, decommissioning, low level radioactive waste management, and

fuel fabrication costs associated with nuclear fuel re-use. The expected capital cost of the new BN-800 plus the costs of reprocessing and fuel fabrication for such reactors does not directly translate into an estimate of how much uranium prices would have to increase to make such an approach economically competitive from a fuel use standpoint. However, these costs may give some indication of where in Tables III and IV the parameter Δ_c might lie if liquid sodium reactors were to play a major role in reducing the rate of use of mined uranium in countries that use them.

TABLE IV

\$Trillion Cost Increment after 1000 TWyre for $F_C = 9/10$

F_R =Fraction of World Adopting $F_C = 9/10$ Early

s	Δ_c	$F_R :$					
$= m + 2$	\$/kgU	0.15	0.30	0.45	0.60	0.75	0.90
11/2	2100	57	113	168	222	275	326
11/2	1800	48	96	143	189	234	277
11/2	1500	40	79	118	155	192	228
11/2	1200	31	62	92	122	151	180
11/2	900	23	45	67	89	110	131
9/2	2100	56	111	165	219	270	321
9/2	1800	47	94	140	185	229	272
9/2	1500	39	77	115	152	188	223
9/2	1200	30	60	89	118	147	175
9/2	900	22	43	64	85	106	126
3	2100	52	103	154	204	254	302
3	1800	43	86	129	171	213	253
3	1500	35	69	104	138	172	205
3	1200	26	53	79	105	131	156
3	900	18	36	54	72	90	108

6. Implications

It was once widely thought that increases in uranium costs with the cumulative mining of as little as about 20 Mtonne or less of elemental uranium would force a substantial move in the direction of lowering intensities dv/du of mined uranium use for nuclear energy use. It is now increasingly widely understood that such a level of cumulative uranium use is unlikely to be the primary driving force behind deployment of technologies that appreciably reduce dv/du . The results in this paper are consistent with the latter conclusion.

It may nevertheless be the case that some countries adopt or continue using such technologies for other reasons. Such reasons can include hopes for reducing the difficulty of long-term spent nuclear fuel management, concerns about the

reliability of access to the global uranium market, maintaining a recessed rather than overt capability for more rapidly manufacturing nuclear explosives, or simply the institutional inertia of pre-existing programs. With respect to global uranium prices, lower uranium intensities of nuclear energy in some countries have the effect of slowing the rise of uranium trend costs for other countries. The results in the previous section give such effects on the global uranium market being quite modest until very large amounts of uranium have been consumed globally. The basic reason for this conclusion derives from the central observation made in this paper. That is that uranium mining appears to respond to depletion of higher concentration uranium ores not only by moving on to lower concentration ores, but also by using previously inconveniently accessible higher concentration ores. As a result, there is probably an extended range of values of cumulative uranium use v over which the trend in inflation-adjusted costs of mining elemental uranium increases as $c \propto 1/b$ with $b > 3$.

A weak dependence of uranium costs on cumulative use parameterized by $b > 3$ suggests that reducing the rate of increase of global uranium use through technologies like aqueous/organic extraction of nuclear reactor discharge to produce MOX fuel in some countries is not likely to have much effect on the cost of electricity in other countries. This conclusion holds unless such practices are fairly widely practiced, and then not until well over 20 Mtonne of native elemental uranium has been consumed. The principle effect on the overall cost of nuclear fuel loaded into reactors would instead be to increase those costs in countries using less native uranium per unit of nuclear energy produced, as long as these costs remain higher than just using native uranium.

The comments made here apply only to direct implications of uranium price increases. As noted above, there are other economic, technological, safety, environmental, security, and public policy reasons besides uranium prices that influence approaches taken to nuclear fuel management. Much more detailed work on the interplay between these factors is needed to make full use of the insights developed here.

Appendix A. Uranium Price vs. Cumulative Use

Denote cumulative uranium extracted up to a cost c by v where, to a good approximation with $s_1 = 2$, we have the derivative of crustal quantity with respect to ore/U ratio $dq/dr \propto r$. Then

$$v/v_{\text{norm}} = \int_0^{c/a} (c - ar)^m, \ell c_0^m] r dr \quad (6)$$

The lower limit of the integral is set to 0 for mathematical convenience, since for the parameters of interest here this makes negligible difference from using the lowest naturally occurring values of $r_1 \sim 5$. Let $y = ar/c$. For $c \leq c_m$ with $c_m = c_0 \ell^{1/m}$, the result is

$$v/v_{\text{norm}} = c^m \int_0^1 \text{Min}[(1 - y)^m (c/a)^2 y dy = c^{m+2} / (a^2 (m + 1)(m + 2)) \quad (7)$$

i.e. $v \propto c^{m+2}$ For $c \geq c_m$, the result is

$$v/v_{\text{norm}} = \int_0^{r_m} d_m r dr + \int_{r_m}^1 (c - ar)^m r dr \quad (8)$$

where $r_m = (c - c_m)/a$. Letting $z = 1 - ra/c$ gives

$$v/v_{\text{norm}} = d_m^2 r_m^2 / 2 + c^m (c/a)^2 \int_0^{c_m/c} z^m (1 - z) dz \quad (9)$$

Defining v_{norm} such that $v_m/v_{\text{norm}} = c_m^m (c_m/b)^2 / ((m+1)(m+2))$, substituting in $d_m = c_m^m$ and $r_m = (c - c_m)/a$, and dividing by $(m+1)(m+2)(v_m/v_1)/2$ gives

$$2(v/v_m)/((m+1)(m+2)) = ((c/c_m) - 1)^2 + 2(c/c_m)/(m+1) - 2/(m+2) \quad (10)$$

Solving this quadratic equation for c/c_m gives Eq. (3) in Section 3 above.

Appendix B. Equations for Uranium Intensity

This appendix gives an analytic approximation to the solution $v(u)$ of Eq. (4) with the definitions in Eqs. (2), (3), and (5). This is for $v_0 = v_1 \leq v \leq v_3$ and $v_2 < v_m < v_3$. This is the case of greatest interest given that observational data is available up to the start of Japanese use of MOX fuel shortly after a substantial expansion of French use of MOX fuel, allowing us to set the reference cumulative uranium use v_0 equal to the cumulative uranium use v_1 at which the substantial re-use of the fissile content of spent nuclear fuel starts. The extension of the method given here to cases with $v_0 < v_1$ is straightforward to derive if needed. It is also straightforward to extend the method given here to cases with such an extended phase-in of increasing re-use of spent fuel that cumulative use v_2 at the end of that phase-in is greater than the cumulative uranium use v_m at which uranium mining first meets the ‘‘depth limit’’ described above. However, this case is not of much practical interest and so the relevant results are not given here.

The results given here for $v \geq v_m$ are exact, given the value u_m of cumulative nuclear energy use when $v = v_m$. For the reference case with results indicated in bold type in Tables I and II, the results for $v < v_m$, and hence the inferred value of u_m , are accurate to 0.6%. The solution is obtained as follows.

Let $V = (v - v_0)/v_m$ and $U = (u - u_0)/v_m$ where u_0 is the value of u when $v = v_0 = v_1$. Then $dv/du = dV/dU$, and the starting condition for integration of the equation for dV/dU is $V = 0$ when $U = 0$. The integration of dV/dU is divided into three regions with upper boundaries at (A) $V_2 = (v_2 - v_0)/v_m$, (B) $V_m = 1 - V_0$ where $V_0 = v_0/v_m$, and (C) $V_3 = (v_3 - v_0)/v_m$.

(A) $0 \leq V \leq V_2$: The equation for dV/dU can be written as

$$dV/dU = (F_N(1 - \epsilon(V_0 + V)^\sigma) + F_R(1 - \epsilon e^{\zeta V}))/ (1 - \epsilon\rho) \quad (11)$$

Here $\sigma = 1/s$, $\epsilon = c_m/c_b$, $\rho = c_0/c_m$, and $\zeta = (v_m/(v_2 - v_1))\text{Ln}(c_3/c_1)$. The value of ϵ for the above-mentioned reference case is 0.088, so it is not surprising that the absolute value of the fractional difference between the solution expanded through first order in ϵ with $V = U + \epsilon G_1$ and the exact solution is less than $\epsilon^2 = 0.008$, as noted above. The result through first order in ϵ is

$$G_1 = \rho U - F_N((V_0 + U)^{\sigma+1} - V_0^{\sigma+1})/(\sigma + 1) + (F_R \rho / \zeta)(e^{\zeta U} - 1) \quad (12)$$

The value of $U_2 = U(V_2)$ solved through first order in ϵ is

$$U_2 = V_2(1 - \epsilon\rho) + \epsilon F_N((V_0 + V_2)^{\sigma+1} - V_0^{\sigma+1})/(\sigma + 1) + \epsilon F_R(\rho/\zeta)F_R(e^{\zeta U} - 1) \quad (13)$$

(B) $V_2 \leq V \leq V_m$: For this case $dV/dU = F_N(1 - \epsilon(V_0 + V)^\sigma) + \epsilon F_2$ where $F_2 = F_R c_3/c_m$. Setting $V = U + \epsilon G_2$ gives, to first order in ϵ ,

$$G_2 = G_{12} + (\rho - F_2)(U - U_2) - F_N((U_2 + U)^{\sigma+1} - U_2^{\sigma+1})/(\sigma + 1) \quad (14)$$

where $G_{12} = G_1|_{U=U_2}$. The value $U_m = U(V_m)$ is $U_m = 1 - V_0 - \epsilon G_2|_{U=V_m}$

(C) $V_m \leq V \leq V_3$: For this case

$$dV/dU = (1 - \epsilon F_N(m/n)(1 + \sqrt{1 + (2V_0 + 2V - mn)n/(bm^2)}) - \epsilon F_2)/(1 - \epsilon\rho) \quad (15)$$

This can be written as $dV/(F_3 - F_D\sqrt{\mu V + S}) = dU/(1 - \epsilon\rho)$ where $F_D = \epsilon F_N m/n$, $F_3 = 1 - F_D - \epsilon F_2$, $\mu = 2n/(bm^2)$, and $S = 1 + (2V_0 - mn)n/(bm^2)$. Integrating both sides and multiplying by $\phi/2$ where $\phi = F_D/F_3$ gives

$$(U - U_m)\phi F_D \mu / 2 = (1 - \phi R) - \text{Ln}(1 - \phi R) - (1 - \phi R_m) + \text{Ln}(1 - \phi R_m) \quad (16)$$

where $R = \sqrt{\mu V + S}$ and $R_m = \sqrt{\mu V_m + S}$. Taking the negative of the inverse of the exponential of both sides of this result gives $W e^W = Q$ where $W = \phi R - 1$ and

$$Q = (\phi R_m - 1)e^{-((U - U_m)F_D \mu \phi / 2 + 1 - \phi R_m)} \quad (17)$$

The solution W of the equation $W e^W = Q$, the Lambert W (i.e. ProductLog) function, has the series expansion $W = \sum_j (Q)^j (-j)^{j-1} / j!$. The solution for $V(U)$ is $V = ((1 + W)^2 / \phi^2 - S) / \mu$. This solution is valid up to a value of U of $U_3 = U(V_3)$ where

$$U_3 = U_m + 2(\phi(R_m - R_3) + \text{Ln}[(1 - \phi R_m)/(1 - \phi R_3)])(1 - \epsilon\rho) / (\mu \phi^2 F_3) \quad (18)$$

with $R_3 = \sqrt{\mu V_3 + S}$.

The solution for $U > U_3$ can be found by the same method but with $F_N = 1$ and $F_R = 0$, since for such large values of U the increasing price of uranium alone drives the decrease in the amount of uranium used per unit of nuclear power production. This continues until extraction of uranium from seawater becomes more economical, after which the amount of uranium used per unit of nuclear power is constant as long as the price of uranium from seawater is constant. If it is expected that seawater uranium will more economical than mined

uranium before cumulative use v_3 and it then becomes clear that reprocessing is uneconomic and is abandoned, then the condition of constant dV/dU can be imposed when cumulative uranium use becomes large enough that extraction of uranium from seawater becomes economically competitive. After such a point the formulas given here are no longer needed.

References

- BLS , 2010. (U.S. Department of Labor Bureau of Labor Statistics), “Consumer Price Index,” <ftp://ftp.bls.gov/pub/special.requests/cpi/cpi.txt> (February), accessed June 7, 2010.
- COCHRAN, T. P., H. R. FEIVESON, W. PATTERSON, G. PSHOKIM, M. W. RAMANA, M. SCHNEIDER, T. SUZUKI, F. VON HIPPLE, 2010. “Fast Breeder Reactor Programs: History and Status,” International Program on Fissile Materials (February).
- DEFFEYES, K. S. and I. D. MacGREGOR, 1980. “World Uranium Resources,” *Sci. Am.* **242**, 1, 66.
- DITTMAR, M., “How (un)reliable are the Red Book Uranium Resource Data,” in *The Future of Nuclear Energy: Facts and Fiction*, arXiv.1421v1 [physics.soc-ph] 8 Sept.
- EURATOM SUPPLY AGENCY, 2009., “Annual Report 2008,” ISSN 0257-9138, Office of the European Union.
- FINNISH NUCLEAR INDUSTRIES, [2006]. “Nuclear Waste Management in Finland,” http://www.energia.fi/en/publications/nuclear_waste.pdf, (2007), accessed June 7, 2010.
- IAEA 2001 (International Atomic Energy Agency) “Analysis of Uranium Supply to 2050,” STI/PUB/1104, Vienna.
- IPCC-WG3 Technical Support Unit [2010]. “Appendix SRES Version 1.1 Note,” http://sres.ciesin.columbia.edu/htmls/version1_1_notes.html and <http://www.grida.no> (no publication date listed; accessed March 25, 2010).
- KOOMEY, J. and N. E. HULTMAN, 2007. “A Reactor-level Analysis of Busbar Costs for US Nuclear Plants, 1970–2005,” *Energy Policy* **35**, 5360.
- MACDONALD, C., 2003. “Uranium: Sustainable Resource or Limit to Growth?” World Nuclear Association Annual Symposium, 3–5 September, 2003.
- NEA, 2004. (Nuclear Energy Agency), “Uranium 2003: Resources, Production, and Demand,” NEA-05921, Organization for Economic Cooperation and Development/ Nuclear Energy Agency.
- NEA, 2006. (Nuclear Energy Agency), “Forty Years of Uranium Resources, Production, and Demand,” NEA-06096, Organization for Economic Cooperation and Development/ Nuclear Energy Agency.
- NEA, 2008. (Nuclear Energy Agency), “Uranium 2007: Resources, Production, and Demand,” NEA-6345, Organization for Economic Cooperation and Development/ Nuclear Energy Agency.
- SCHNEIDER, E. A. and W. C. SAILOR. 2008. “Long-term Uranium Supply Estimates,” *Nuclear Technology*, **162**, 379.

- SINGER, C. E., 2001. "An Analytic Uranium Sources Model," in IAEA-TECDOC-1258, *Assessment of Uranium Deposit Types and Resources—A Worldwide Perspective: Proceedings of a Technical Committee Meeting organized by the International Atomic Energy Agency and the OECD Nuclear Energy Agency and held in Vienna, 10–13 June, 1997*, p. 27 (December).
- STRÖM, A., J. ANDERSSON, K. SKAGIUS, and A. WINGBERT, 2008. "Site Descriptive Modelling During Characterization for a Geological Repository for Nuclear Waste in Sweden," *Applied Geochemistry*, **23**, 1747.
- VON HIPPEL, F., 2009. "Why Reprocessing Persists in Some Countries and not in Others: The Costs and Benefits of Reprocessing," Non-proliferation Education Center (April), <http://www.npec-web.org>, accessed March 25, 2010.
- WISE, 2010. (World Information Service on Energy) "Uranium Mining Issues: 2009 Review," (January), accessed March 15, 2010.
- WNA, 2009. (World Nuclear Association), "World Uranium Mining," <http://www.world-nuclear.org/info/inf23.html> (November), accessed March 1, 2010.